F-Sombor and Modified F-Sombor Indices of Certain Nanotubes

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Abstract. In this study, we introduce the F-Sombor index, modified F-Sombor index and their exponentials of a graph. Furthermore, we present exact expressions for these F-Sombor indices and their exponentials of certain nanotubes.

Keywords: F-Sombor index, modified F-Sombor index, nanotube

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1. Introduction

Let G be a finite, simple, connected graph with vertex set V(G) and edge set E(G). The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. For definitions and notations, we refer to the book [1].

In Chemistry, topological indices have been found to be useful in discrimination, chemical documentation, structure-property relationships, structure-activity relationships and pharmaceutical drug design. There has been considerable interest in the general problem of determining topological indices, see [2].

The first F-index [3] and second F-index [4] of a graph $G$ are defined as

$$F_1(G) = \sum_{u \in V(G)} \left[ d_G(u)^2 + d_G(v)^2 \right], \quad F_2(G) = \sum_{u \in V(G)} d_G(u)^2 \cdot d_G(v)^2.$$ 

Recently some F-indices were studied in [5, 6, 7]. We put forward the F-Sombor index of the graph $G$ and defined it as

$$FSO(G) = \sum_{u \in V(G)} \sqrt{\left(d_G(u)^2 + d_G(v)^2\right)^2}.$$ 

We propose the modified F-Sombor index of the graph $G$, defined as

$$= FSO(G) = \sum_{u \in V(G)} \sqrt{\left(d_G(u)^2 + d_G(v)^2\right)^2}.$$ 

Recently, some Sombor indices were studied in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

We introduce the F-Sombor exponential of the graph $G$ and defined it as
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\[ FSO(G, x) = \sum_{u \in V(G)} x^{\sqrt{d_G(u)^2 + d_G(v)^2}}. \]

We define the modified F-Sombor exponential of a graph \( G \) as

\[ mFSO(G) = \sum_{u \in V(G)} \frac{1}{\sqrt{(d_G(u))^2 + (d_G(v))^2}}. \]

In this paper, the F-Sombor and modified F-Sombor indices and their corresponding exponential versions of certain nanotubes are determined.

2. \( HC_5C_7 \) Nanotubes

The chemical graphs \( G \) of nanotube \( HC_5C_7 \) structure have \( 4pq \) vertices and \( 6pq - p \) edges are shown in Figure 1.

![Figure 1: 2-D lattice of nanotube \( HC_5C_7 \) [p, q]](image)

In the above structure, we obtain that \{\( d_G(u), d_G(v) \): \( uv \in E(G) \} \) has two edge set partitions.

\[ E_1 = \{uv \in E(G) | d_G(u)=2, d_G(v)=3\}, \quad |E_1|=4p, \]
\[ E_2 = \{uv \in E(G) | d_G(u)=3, d_G(v)=3\}, \quad |E_2|=6pq - 5p. \]

**Theorem 1.** The F-Sombor index of a nanotube \( HC_5C_7 \) is

\[ FSO(G) = 54\sqrt{2}pq + (4\sqrt{97} - 45\sqrt{2})p. \]

**Proof:** Applying definition and edge set partition of \( G \), we conclude

\[ FSO(G) = \sum_{uv \in E(G)} \sqrt{(d_G(u))^2 + (d_G(v))^2} \]
\[ = (2^2 + 3^2)4p + (3^2 + 3^2)(6pq - 5p) \]
\[ \text{gives the desired result after simplification.} \]

**Theorem 2.** The modified F-sombor index of a nanotube \( HC_5C_7 \) is given by

\[ mFSO(G) = \frac{2}{3\sqrt{2}} pq + \left(\frac{4}{\sqrt{97}} - \frac{5}{9\sqrt{2}}\right)p. \]

**Proof:** Applying definition and edge set partition of \( G_1 \), we conclude

\[ mFSO(G) = \sum_{u \in E(G)} \frac{1}{\sqrt{(d_G(u))^2 + (d_G(v))^2}} \]

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\[
F(S) = \left(\frac{1}{\sqrt{2^4+3^4}}\right)4p + \left(\frac{1}{\sqrt{3^4+3^4}}\right)(6pq - 5p)
\]

After simplification, we get the desired result.

By using definitions and cardinalities of the edge partitions of \(G\), we obtain the F-Sombor and modified F-Sombor exponentials of \(G\) as follows:

**Theorem 3.** The F-Sombor exponential of \(G_1\) is

\[
FSO(G_1, x) = 4px^{\sqrt{7}} + (6pq - 5p)x^{\sqrt{7}}.
\]

**Theorem 4.** The modified F-Sombor exponential of \(G_1\) is

\[
mFSO(G_1, x) = \frac{1}{2}(4px^{\sqrt{7}} + (6pq - 5p)x^{\sqrt{7}}).
\]

**3. SC\(_5C_7[p, q]\) nanotubes**

The chemical graphs \(H\) of nanotube SC\(_5C_7[p, q]\) structure have \(4pq\) vertices and \(6pq - p\) edges are shown in Figure 2.

![Figure 2: 2-D lattice of nanotube SC\(_5C_7[p, q]\)](image)

In the above structure, we obtain that \(\{d_H(u), d_H(v); uv \in E(H)\}\) has three edge set partitions.

- \(E_1 = \{uv \in E(H) \mid d_H(u)=2, d_H(v)=2\}\), \(|E_1|=q\).
- \(E_2 = \{uv \in E(H) \mid d_H(u)=2, d_H(v)=3\}\), \(|E_2|=6q\).
- \(E_3 = \{uv \in E(H) \mid d_H(u)=3, d_H(v)=3\}\), \(|E_3|=6pq - p - 7q\).

**Theorem 5.** The F-Sombor index of \(H\) is

\[
FSO(H) = 54\sqrt{2}pq - 9\sqrt{2}p + (6\sqrt{97} - 50\sqrt{2})q.
\]

**Proof:** Applying definition and edge set partition of \(H\), we conclude

\[
FSO(H) = \sum_{uv \in E(H)} \sqrt{(d_H(u))^2 + (d_H(v))^2}
\]

\[
= \left(\sqrt{2^4 + 2^4}\right)q + \left(\sqrt{2^4 + 3^4}\right)6q + \left(\sqrt{3^4 + 3^4}\right)(6pq - p - 7q)
\]

gives the desired result after simplification.
Theorem 6. The modified $F$-sombor index of a nanotube $SC_5C_7[p, q]$ is given by

$$mFSO(H) = \frac{2}{3\sqrt{2}}pq - \frac{1}{9\sqrt{2}}p + \left(\frac{1}{4\sqrt{2}} + \frac{6}{\sqrt{9}q} - \frac{7}{9\sqrt{2}}\right)q$$

Proof: Applying definition and edge set partition of $H$, we conclude

$$mFSO(H) = \sum_{u \in E(G)} \frac{1}{\sqrt{(d_G(u))^2 + (d_G(v))^2}}$$

$$= \left(\frac{1}{\sqrt{2^2+2^2}}\right)q + \left(\frac{1}{\sqrt{2^2+3^2}}\right)6q + \left(\frac{1}{\sqrt{3^2+3^2}}\right)(6pq - p - 7q).$$

After simplification, we obtain the desired result.

By using definitions and cardinalities of the edge partitions of $H$, we obtain the $F$-Sombor and modified $F$-Sombor exponentials of $G_1$ as follows:

Theorem 7. The $F$-Sombor exponential of $H$ is

$$FSO(H,x) = qx^{\sqrt{2}} + 6qx^{\sqrt{3}} + (6pq - p - 7q)x^{\sqrt{2}}.$$ 

Theorem 8. The modified $F$-Sombor exponential of $H$ is

$$mFSO(H,x) = qx^{\sqrt{2}} + 6qx^{\sqrt{3}} + (6pq - p - 7q)x^{\sqrt{3}}.$$ 

4. Conclusion

In this study, we have computed the $F$-Sombor and modified $F$-Sombor indices and their corresponding exponentials of certain nanotubes.

REFERENCES

F-Sombor and Modified F-Sombor Indices of Certain Nanotubes