*Annals of Pure and Applied Mathematics*
*Vol. 27, No. 2, 2023, 73-78*
*ISSN: 2279-087X (P), 2279-0888(online)*
*Published on 29 June 2023*
www.researchmathsci.org
*DOI: http://dx.doi.org/10.22457/apam.v27n2a05908*

Annals of
Pure and Applied
Mathematics

# Domination Product Connectivity Indices of Graphs

***V.R.Kulli***

Department of Mathematics
Gulbarga University, Gulbarga 585 106, India
E-mail:vrkulli@gmail.com

*Received 20 May 2023; accepted 28 June 2023*

***Abstract.*** In this study, we introduce the domination product connectivity index and the reciprocal domination product connectivity index of a graph. Furthermore, we compute these indices for some standard, French windmill, friendship graphs.

***AMS Mathematics Subject Classification (2010):*** 05C10, 05C69

***Keywords:*** domination product connectivity index, reciprocal domination product connectivity index, graph.

## 1. Introduction

In this paper, $G$ denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. we refer [1, 2], for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [3].

The domination degree $d_d(u)$ [4] of a vertex $u$ in a graph $G$ is defined as the number of minimal dominating sets of $G$ which contains $u$.

Recently, some domination indices were studied, for example, in [5, 6, 7, 8].

We introduce the domination product connectivity index of a graph $G$, defined as

$$DP(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_d(u) d_d(v)}}.$$

Recently, some connectivity indices were studied, for example, in [9, 10, 11, 12].

Considering the domination product connectivity index, we introduce the domination product connectivity polynomial of a graph $G$ and defined it as

$$DP(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_d(u) d_d(v)}}}.$$

We also introduce the reciprocal domination product connectivity index of a graph $G$, defined as

$$RDP(G) = \sum_{uv \in E(G)} \sqrt{d_d(u) d_d(v)}.$$

We define the reciprocal domination product connectivity polynomial of a graph $G$ as

$$RDP(G,x)=\sum_{uv\in E(G)} x^{\sqrt{d_d(u)d_d(v)}}.$$

In this paper, we determine the domination product connectivity index, reciprocal domination product connectivity index and their corresponding polynomials of some standard graphs, French windmill graphs and friendship graphs.

## 1. Domination product connectivity index

### 1.1. Results for some standard graphs

**Proposition 1.** If $K_n$ is a complete graph with *n* vertices, then

$$DP(K_n)=\frac{n(n-2)}{2}.$$

**Proof:** If $K_n$ is a complete graph, then $d_d(u)$ =1. From definition, we have

$$DP(K_n)=\sum_{uv\in E(K_n)}\frac{1}{\sqrt{d_d(u)d_d(v)}}=\frac{n(n-1)}{2}\frac{1}{\sqrt{1\times 1}}=\frac{n(n-1)}{2}.$$

**Proposition 2.** If $S_{n+1}$ is a star graph with $d_d(u)$ =1, then $DP(S_{n+1})=n.$

**Proposition 3.** If $S_{p+1,q+1}$ is a double star graph with $d_d(u)$ =2, then

$$DP(S_{p+1,q+1})=\frac{p+q+1}{2}.$$

**Proposition 4.** Let $K_{m,n}$ be a complete bipartite graph with $2\leq m\leq n$. Then

$$DP(K_{m,n})=\frac{mn}{\sqrt{mn+m+n+1}}.$$

**Proof:** Let $G=K_{m,n}$, $m, n\geq 2$ with $d_d(u)=m+1$

$= n+1$, for all $u\in V(G)$.

From definition, we have

$$DP(K_{m,n})=\frac{1}{\sqrt{d_d(u)d_d(v)}}=\frac{mn}{\sqrt{(m+1)(n+1)}}=\frac{mn}{\sqrt{mn+m+n+1}}.$$

In the following proposition, by using definition, we obtain the domination product connectivity exponential of $K_n$, $S_{n+1}$, $S_{p+1,q+1}$ and $K_{m,n}$.

**Proposition 5.** The domination product connectivity exponential of $K_n$, $S_{n+1}$, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $DP(K_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_d(u)d_d(v)}} = \frac{n(n-1)}{2} x^{\sqrt{1\times 1}} = \frac{n(n-1)}{2} x^1.$

(ii) $DP(S_{n+1}, x) = nx^1.$

(iii) $DP(S_{p+1,q+1}, x) = (p+q+1)x^2.$

(iv) $DP(K_{m,n}, x) = mnx^{\sqrt{mn+m+n+1}}.$

## 1.2. Results for French Windmill graphs

The French windmill graph $F_n^m$ is the graph obtained by taking $m \geq 3$ copies of $K_n$, $n \geq 3$ with a vertex in common. The graph $F_n^m$ is presented in Figure 1. The French windmill graph $F_3^m$ is called a friendship graph.

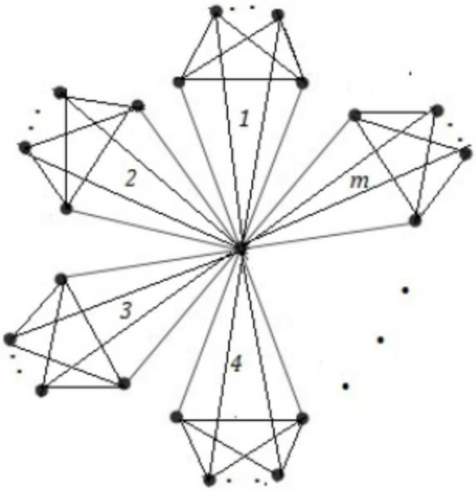


**Figure 1:** French windmill graph $F_n^m$

Let $F$ be a French windmill graph $F_n^m$. Then

$$d_d(u) = \begin{cases} 1, & \text{if } u \text{ is the center vertex} \\ (n-1)^{n-1}, & \text{otherwise} \end{cases}$$

**Theorem 1.** Let $F$ be a French windmill graph $F_n^m$. Then

$$DP(F) = \frac{m(n-1)}{\sqrt{(n-1)^{(m-1)}}} + \frac{[(mn(n-1)/2) - m(n-1)]}{(n-1)^{(m-1)}}.$$

**Proof:** In $F$, there are two sets of edges. Let $E_1$ be the set of all edges which are incident with the center vertex and $E_2$ be the set of all edges of the complete graph. Then

$$DP(F)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)d_d(v)}}$$

$$=\sum_{uv\in E_1(G)}\frac{1}{\sqrt{d_d(u)d_d(v)}}+\sum_{uv\in E_2(G)}\frac{1}{\sqrt{d_d(u)d_d(v)}}$$

$$=\frac{m(n-1)}{\sqrt{1\times(n-1)^{(m-1)}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{(n-1)^{(m-1)}\times(n-1)^{(m-1)}}}$$

$$=\frac{m(n-1)}{\sqrt{(n-1)^{(m-1)}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{(n-1)^{(m-1)}}.$$

**Corollary 1.1.** Let $F_3^m$ be a friendship graph. Then $DP(F_3^m)=\dfrac{2m}{\sqrt{2^{(m-1)}}}+\dfrac{m}{2^{m-1}}$.

## 2. Reciprocal domination product connectivity index

### 2.1 Results for some standard graphs

**Proposition 6.** If $K_n$ is a complete graph with $n$ vertices, then

$$RDP(K_n)=\frac{n(n-1)}{2}.$$

**Proof:** If $K_n$ is a complete graph, then $d_d(u)$ =1. From definition, we have

$$RDP(K_n)=\sum_{uv\in E(K_n)}\sqrt{d_d(u)d_d(v)}=\frac{n(n-1)}{2}\sqrt{1\times1}=\frac{n(n-1)}{2}.$$

**Proposition 7.** If $S_{n+1}$ is a star graph with $d_d(u)$ =1, then $RDP(S_{n+1})=n.$

**Proposition 8.** If $S_{p+1,q+1}$ is a double star graph with $d_d(u)$ =2, then

$$RDP(S_{p+1,q+1})=2(p+q+1).$$

**Proposition 9.** Let $K_{m,n}$ be a complete bipartite graph with $2\leq m\leq n$. Then

$$RDP(K_{m,n})=mn\sqrt{mn+m+n+1}.$$

**Proof:** Let $G=K_{m,n}$, $m, n\geq2$ with

$$d_d(u)=\begin{cases} m+1 \\ n+1, \text{for all } u\in V(G)\end{cases}$$

From definition, we have

$$RDP(K_{m,n})=\sum_{uv\in E(K_{m,n})}\sqrt{d_d(u)d_d(v)}$$

$$=mn\sqrt{(m+1)(n+1)}=mn\sqrt{mn+m+n+1}.$$

In the following proposition, by using definition, we obtain the domination product connectivity exponential of $K_n$, $S_{n+1}$, $S_{p+1,q+1}$ and $K_{m,n}$.

**Proposition 10.** The domination product connectivity exponential of $K_n$, $S_{n+1}$, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $RDP(K_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_d(u)d_d(v)}} = \frac{n(n-1)}{2} x^{\sqrt{1\times 1}} = \frac{n(n-1)}{2} x^1.$

(ii) $RDP(S_{n+1}, x) = nx^1.$

(iii) $RDP(S_{p+1,q+1}, x) = (p+q+1)x^2.$

(iv) $RDP(K_{m,n}, x) = mnx^{\sqrt{mn+m+n+1}}.$

## 2.2. Results for French Windmill graphs

**Theorem 2.** Let $F$ be a French windmill graph $F_n^m$. Then

$$RDP(F) = m(n-1)\sqrt{(n-1)^{(m-1)}} + [(mn(n-1)/2) - m(n-1)](n-1)^{(m-1)}.$$

**Proof:** In $F$, there are two sets of edges. Let $E_1$ be the set of all edges which are incident with the center vertex and $E_2$ be the set of all edges of the complete graph. Then

$$RDP(F) = \sum_{uv \in E(G)} \sqrt{d_d(u)d_d(v)} = \sum_{uv \in E_1(G)} \sqrt{d_d(u)d_d(v)} + \sum_{uv \in E_2(G)} \sqrt{d_d(u)d_d(v)}$$

$$= m(n-1)\sqrt{1\times(n-1)^{(m-1)}} + [(mn(n-1)/2) - m(n-1)]\sqrt{(n-1)^{(m-1)} \times (n-1)^{(m-1)}}$$

$$= m(n-1)\sqrt{(n-1)^{(m-1)}} + [(mn(n-1)/2) - m(n-1)](n-1)^{(m-1)}.$$

**Corollary 2.1.** Let $F_3^m$ be a friendship graph. Then

$$RDP(F_3^m) = 2m\sqrt{2^{(m-1)}} + m2^{m-1}.$$

## 3. Conclusion

In this paper, the domination product connectivity index, reciprocal domination product connectivity index and their corresponding polynomials of certain graphs are computed.

***Acknowledgements.*** The author is thankful to the reviewers for their suggestions for updating the paper.

**Conflict of interest.** As a sole author, there is no question of conflict of interest.

**Authors' Contributions.** The author did all the work for this paper.

## REFERENCES


1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, *Theory of Domination in Graphs*, Vishwa International Publications, Gulbarga, India (2010).
3. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2019) 66-91.
4. A.M.H.Ahmed, A.Alwardi and M.Ruby Salestina, On domination topological indices of graphs, *International Journal of Analysis and Applications*, 19(1) (2021) 47-64.
5. V.R.Kulli, Domination Nirmala indices of graphs, submitted.
6. V.R.Kulli, Domination Dharwad indices of graphs, submitted.
7. S.Raju, Puttuswamy and S.R.Nayaka, On the second domination hyper index of graph and some graph operations, *Advances and Applications in Discrete Mathematics*, 39(1) (2023) 125-143.
8. A.A.Shashidhar, H.Ahmed, N.D.Soner and M.Cancan, Domination version: Sombor index of graphs and its significance in predicting physicochemical properties of butane derivatives, *Eurasian Chemical Communications*, 5 (2023) 91-102.
9. V.R.Kulli, Connectivity neighborhood Dakshayani indices of POPAM dendrimers, *Annals of Pure and Applied Mathematics*, 21(1) (2019) 49-54.
10. V.R.Kulli, New connectivity topological indices, *Annals of Pure and Applied Mathematics*, 20(1) (2019) 1-8.
11. V.R.Kulli, Product connectivity E-Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 27(1) (2023) 7-12.
12. V.R.Kulli, Atom bond connectivity E-Banhatti indices, *International Journal of Mathematics and Computer Research*, 11(1) (2023) 3201-3208.