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Decomposition of Complete Tripartite Graphs into Cycles and Paths of Length *k*

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Abstract. Let C_k and P_k respectively denote a cycle and a path on k vertices. In this paper, we give necessary and sufficient conditions for the existence of $(k; p, q)$-decomposition of $K_{r,s,t}$ when k is prime.

Keywords: Cycle, Path, Complete Tripartite Graph, Decomposition of Graphs.

AMS Mathematics Subject Classification (2010): 05C38, 05C51

1. Introduction

Here, we consider finite undirected simple graphs. Let $K_{n_1,n_2,\ldots,n_r}$ denote a complete r-partite graph with part sizes $n_1 n_2, \ldots, n_r$ where each $n_i > 0$ is an integer. A partition of a graph G into edge-disjoint subgraphs $G_1, G_2, G_3, \ldots, G_n$ such that their union gives G is called a *decomposition* of G. Let C_k and P_k respectively denote a cycle and a path on k vertices. They are also called k-cycle and k-path, respectively. We say that the complete tripartite graph $K_{r,s,t}$ has a $\{C_k, P_k\}_{p,q}$- decomposition of $K_{r,s,t}$ if $K_{r,s,t}$ can be decomposed into p copies of C_k and q copies of P_k for all possible values of p and q. Throughout this paper, the partite sets of the complete tripartite graph $K_{r,s,t}$ with $1 \leq r \leq s \leq t$, are assumed to be $\{a_1, a_2, \ldots, a_r\}$, $\{b_1, b_2, \ldots, b_s\}$ and $\{c_1, c_2, \ldots, c_t\}$. The problem of finding necessary and sufficient conditions to decompose complete bipartite graphs into k-cycles was solved by Sotteau[14]. Cavenagh[5] has shown that $K_{m,m,m}$ can be decomposed into k-cycles if and only if $k \leq 3m$ and k divides $3m^2$. Billington [2] gave necessary and sufficient conditions for the existence of a decomposition of any complete tripartite graph into 3-cycles and 4-cycles. Mahmoodian and Mirzakhani [11] proved the existence of a C_5-decomposition of $K_{r,s,t}$ whenever the necessary conditions are satisfied and two of the partite sets have equal size, except when $r = s \equiv 0 (mod\ 5)$ and $t \equiv 0 (mod\ 5)$. The authors of [1,3,6,7] also studied this problem. Billington, et al. [4] gave necessary and sufficient conditions for the existence of path and cycle decomposition of complete equipartite graphs with 3 and 5 parts. Jeevadoss and Muthusamy [9] gave necessary and sufficient conditions for the existence of $\{P_{k+1}, C_k\}_{p,q}$- decomposition of $K_{m,n}$ and K_n, when $m \geq \frac{k}{2}$, $n \geq \left\lceil \frac{k+1}{2} \right\rceil$ for $k \equiv 0 (mod 4)$ and when $m, n \geq 2k$ for $k \equiv 2 (mod 4)$.

Ganeshmurthy and Paulraja [8] gave necessary and sufficient conditions for the existence of a complete $\{C_3, C_6\}$-decomposition of $K_{a,b,c}$, $a \le b \le c$. Priyadarsini and Muthusamy [13] proved the existence of decomposition of $K_{r,s,t}$ into paths and cycles of length 3. Manikandan and Paulraja [12] gave necessary and sufficient conditions for the existence of C_p-decomposition of some regular graphs.

In this paper, we study the existence of $\{C_k, P_k\}$-decomposition of $K_{r,s,t}$, so we abbreviate the notation as $(k; p, q)$-decomposition. The obvious necessary condition for such existence is $k(p+q) = |E(G)|$. We only consider cases where the vertices r, s and t are of even degree in which the case $q \ne 1$ is also obvious, since the endpoints of a single path in the decomposition would have to have odd degree. Hence, all the vertex degrees are even and $q \ne 1$. We prove that $K_{r,s,t}$ has a $(k; p, q)$- decomposition where at least any two of the partite sets are divisible by k when k≥ 5 is prime. Let $K_{r,s,t}$ be the complete tripartite graph with parts r, s and t, where $r = \{a_1, a_2, a_3, \dots, a_r\}$, $s = \{b_1, b_2, b_3, \dots, b_s\}$ and $t = \{c_1, c_2, c_3, \dots, c_t\}$.

To prove our results, we state the following:

Theorem 1. ([12]). For any prime $p \ge 11$, $m \ge 3$, $C_p | K_m * \overline{K_n}$ if and only if $n(m-1)$ is even and $p | m(m-1)n^2$.

2. $(k; p, q)$-decomposition of $K_{r,s,t}$ when $k \ge 5$ is prime

In this section we investigate the existence of $(k; p, q)$ −decomposition of complete tripartite graph when $k \ge 5$ is prime.

Let $(a_1 b_1 c_1 c_{(k-1)/2} a_1)$ and $P(b_1 c_1 b_2 c_2 c_{(k+1)/2})$ respectively denote the cycle C_k and path P_k of length k.

Construction 1. Let C_α and C_β be two cycles of length k, where

$$C_\alpha = \left(a_1 b_{i+(k-l)} a_2 c_{j+l} a_3 b_{i+(k-(l-1))} \dots a_r b_i c_j \dots c_{j+k-\left(\frac{k-(2r+1)}{2}\right)} a_1\right)$$

and

$$C_\beta = \left(a_1 b_{(i+1)+(k-l)} a_2 c_{(j+1)+l} a_3 b_{(i+1)+(k-(l-1))} \dots a_r b_{i+1} c_{j+1} \dots c_{(j+1)+k-\left(\frac{k-(2r+1)}{2}\right)} a_1\right).$$

$C_\alpha \cup C_\beta = P_\alpha \cup P_\beta$

where

$$P_\alpha = P\left(b_{i+(k-l)} a_1 b_{(i+1)+(k-l)} a_2 c_{j+l} a_3 b_{i+(k-(l-1))} \dots a_r b_i c_j \dots c_{j+k-\left(\frac{k-(2r+1)}{2}\right)}\right)$$

and

$$P_\beta = P\left(b_{i+(k-l)} a_2 c_{(j+1)+l} a_3 b_{(i+l)+(k-(l-1))} \dots a_r b_{i+1} c_{j+1} \dots c_{(j+1)+k-\left(\frac{k-(2r+1)}{2}\right)} a_1 c_{j+k-\left(\frac{k-(2r+1)}{2}\right)}\right)$$

Construction 2. The complete bipartite graph $K_{k,k}$ can be decomposed into k copies of P_k when $k \ge 5$ is prime as follows:

$P\left(b_i c_j b_{i+1} c_{j+(k-1)} b_{i+2} c_{j+(k-2)} b_{i+3} \dots b_{i+\left(\frac{k-1}{2}\right)} c_{j+\left(\frac{k-1}{2}\right)}\right)$ for $i = j = \{1,2,\dots,k\}$, where addition with the subscripts are taken modulo k.

Construction 3. The complete bipartite graph $K_{s,t}$ can be decomposed into P_k when at least one of s, t is even and divisible by k. If s is even and t is divisible by k, then we have

$$P\left(b_i c_j b_{i+1} c_{j+1} b_{i+2} c_{j+2} \dots c_{j+\left(\frac{k-3}{2}\right)} b_{i+\left(\frac{k-1}{2}\right)} c_{j+(k-1)}\right)$$

for i is odd, $j = 1, k+1, 2k+1, 3k+1, \dots$ and i is even, $j = \frac{k+1}{2}, \frac{3k+1}{2}, \frac{5k+1}{2}, \frac{7k+1}{2}, \dots$, where $1 \le i \le s$, $1 \le j \le t$ and addition with the subscripts i, j are taken modulo s, t respectively. Hence we get $s\left(\frac{t}{k}\right)$ copies of P_k. Similarly, if t is even, then we have $P\left(c_j b_i c_{j+1} b_{i+1} c_{j+2} b_{i+2} \dots b_{i+\left(\frac{k-3}{2}\right)} c_{j+\left(\frac{k-1}{2}\right)} b_{i+(k-1)}\right)$ for j is odd, $i = 1,\ k+1,\ 2k+1, 3k+1, \dots$ and j is even, $i = \frac{k+1}{2}, \frac{3k+1}{2}, \frac{5k+1}{2}, \frac{7k+1}{2}, \dots$, where $1 \le i \le s$, $1 \le j \le t$ and addition with the subscripts i, j are taken modulo s, t respectively.

Lemma 1. If k, r, s, t be positive integers such that $r, s, t \equiv 0 (mod\ k)$, and $k \ge 5$ is prime, then the complete tripartite graph $K_{r,s,t}$ with $r \le s \le t$, has $(k;\ p, q)$- decomposition.

Proof: Let $r = s = t \equiv 0 (mod\ k)$, then the graph $K_{r,r,r}$ can be decomposed into $\left(\frac{r}{k}\right)^2 K_{k,k,k}$. The graph $K_{k,k,k}$ can be decomposed into C_k by Theorem 1. By Construction 1, we get 3k copies of P_k. Let $s = t$, then the graph $K_{r,s,s}$ can be decomposed into

$\left(\frac{r}{k}\right)\left(\frac{s}{k}\right) K_{k,k,k} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k} - \frac{r}{k}\right) K_{k,k}$.

Let $s < t$, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s,s} \cup K_{r,t-s} \cup K_{s,t-s}$

where $K_{r,t-s} = \left(\frac{r}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$ and $K_{s,t-s} = \left(\frac{s}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$. The graph $K_{k,k}$ can be decomposed into P_k by Construction 2. Hence the graph $K_{r,s,t}$ has the desired decomposition.

Lemma 2. If k, r, s, t and r' be positive integers such that $r \equiv r' (mod\ k)$, $s, t \equiv 0 (mod\ k)$ and $k \ge 5$ is prime, then the complete tripartite graph $K_{r,s,t}$ with $r \le s \le t$, has $(k;\ p, q)$- decomposition.

Proof. Case (i): r' is odd.

Subcase (a): $r' \le \frac{k+1}{2}$. Let r, s, t are all odd and $r = r'$, then the graph $K_{r',k,k}$ can be decomposed into k copies of C_k as follows:

$(a_1 b_{i+(k-l)} a_2 c_{j+l} a_3 b_{i+(k-(l-1))} a_4 c_{j+(l-1)} a_5 b_{i+(k-(l-2))} a_6 c_{j+(l-2)} \dots b_{i+(k-1)} a_{r'-1} c_{j+1}$
$a_{r'} b_i c_j b_{i+1} c_{j+(k-1)} b_{i+2} c_{j+(k-2)} \dots b_{i+\left(\frac{k-(2r+1)}{2}\right)} c_{j+\left(k-\left(\frac{k-(2r+1)}{2}\right)\right)} a_1)$

for $i = j = \{1,2,3,\dots,k\}$ and $l = \frac{r-1}{2}$. The remaining edges give $2r'$ copies of P_k and we have the following construction:

$P\left(c_j b_1 c_{j+1} b_2 c_{j+2} b_3 \ldots c_{j+\left(\frac{k-1}{2}\right)} b_{\frac{k+1}{2}}\right)$ and $P\left(b_{\frac{k+1}{2}} c_{j+\left(\frac{k+1}{2}\right)} b_{\frac{k+3}{2}} c_{j+\left(\frac{k+3}{2}\right)} \ldots c_{j-1} b_k c_j\right)$ for $j \equiv 0(mod\ 2) < k$, where addition with the subscripts are taken modulo k. Let $s = t \equiv 0(mod\ k)$, then the graph $K_{r',s,s}$ can be decomposed into
$\left(\frac{s}{k}\right) K_{r',k,k} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k} - 1\right) K_{k,k}$.

For $s < t$, the graph $K_{r',s,t}$ can be decomposed into $K_{r',s,s} \cup K_{r'+s,t-s}$,
Where $K_{r'+s,t-s}$ can be decomposed into $\left(\frac{t-s}{2k}\right) K_{r'+k,2k} \cup \left(\frac{s-k}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$.
The graph $K_{r'+k,2k}$ can be decomposed into $2(k + r')$ copies of P_k by Construction 3.
Let $r \equiv r'\ (mod\ k)$ and $s = t$, then the graph $K_{r,s,s}$ can be decomposed into

$$K_{r',s,s} \cup 2\left(\frac{s}{k}\right)\left(\frac{r-r'}{k}\right) K_{k,k}.$$

For $s < t$, the graph $K_{r,s,t}$ can be decomposed into
$K_{r,s,s} \cup \left(\frac{t-s}{2k}\right) K_{k+r',2k} \cup \left(\frac{t-s}{k}\right)\left(\frac{(r+s)-(k+r')}{k}\right) K_{k,k}$.
If r, s and t are even, then the graph $K_{r,s,s}$ can be decomposed into
$\left(\frac{s}{k}\right) K_{r',k,k} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k} - 1\right) K_{k,k} \cup 2\left(\frac{s}{k}\right)\left(\frac{r-r'}{k}\right) K_{k,k}$.
For $s < t$, $\quad K_{r,s,t} = K_{r,s,s} \cup \left(\frac{s}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k} \cup K_{r,t-s}$,
Where $K_{r,t-s}$ can be decomposed into $r\left(\frac{t-s}{k}\right)$ copies of P_k by Construction 3.

Subcase (b): $r' > \frac{k+1}{2}$. Let r, s, t are all odd. If $(k-1) \equiv 2(mod\ 4)$, then the graph $K_{r',k,k}$ can be decomposed into
$K_{r'-\left(\frac{k+1}{2}\right),k,k} \cup 2K_{\frac{k+1}{2},k}$.
The graph $K_{r',s,s}$ can be decomposed into $K_{r'-\left(\frac{k+1}{2}\right),s,s} \cup 2\left(\frac{s}{k}\right) K_{\frac{k+1}{2},k}$.

The graph $K_{r,s,s}$ can be decomposed into $K_{r',s,s} \cup 2\left(\frac{r-r'}{k}\right)\left(\frac{s}{k}\right) K_{k,k}$.

If r, s and t are all even, then the graph $K_{r,s,s}$ can be decomposed into
$\left(\frac{s}{k}\right) K_{r'-\left(\frac{k+1}{2}\right)} \cup 2K_{\frac{k+1}{2},s} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k} - 1\right) K_{k,k} \cup 2\left(\frac{r-r'}{k}\right)\left(\frac{s}{k}\right) K_{k,k}$.
For $s < t$, the graph $K_{r,s,t}$ can be decomposed into
$K_{r,s,s} \cup \left(\frac{t-s}{2k}\right) K_{r',2k} \cup \left(\frac{r-r'}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k} \cup \left(\frac{s}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$.
Hence the desired decomposition is obtained.
If $(k-1) \equiv 0(mod\ 4)$, then the graph $K_{r',k,k}$ can be decomposed into
$K_{r'-\left(\frac{k+3}{2}\right),k,k} \cup 2K_{\frac{k+3}{2},k}$.
The graph $K_{r',s,s}$ can be decomposed into $K_{r'-\left(\frac{k+3}{2}\right),s,s} \cup 2\left(\frac{s}{k}\right) K_{\frac{k+3}{2},k}$. The graph $K_{r,s,s}$ can be decomposed into $K_{r',s,s} \cup 2\left(\frac{r-r'}{k}\right)\left(\frac{s}{k}\right) K_{k,k}$.
If r, s and t are all even, then the graph $K_{r,s,s}$ can be decomposed into

$\left(\frac{s}{k}\right) K_{r'-\left(\frac{k+3}{2}\right)} \cup 2K_{\frac{k+3}{2},s} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k}-1\right) K_{k,k} \cup 2\left(\frac{r-r'}{k}\right)\left(\frac{s}{k}\right) K_{k,k}$.

For $s < t$, the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s,s} \cup \left(\frac{t-s}{2k}\right) K_{r',2k} \cup \left(\frac{r-r'}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k} \cup \left(\frac{s}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$.

Hence the required decomposition is obtained.

Case (ii): r' is even.

Subcase (c): $r' \leq \frac{k+1}{2}$. Either r, s, t are all odd or even and $(k-1) \equiv 0 (mod\ 4)$, then the graph $K_{r,s,s}$ can be decomposed into

$\left(\frac{s}{k}\right) K_{\frac{k+1}{2},k,k} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k}-1\right) K_{k,k} \cup 2\left(\frac{s}{k}\right) K_{k-1,k} \cup \left(\frac{2r-(3k-1)}{k}\right)\left(\frac{s}{k}\right) K_{k,k}$.

If (k-1)≡ $2(mod 4)$, *then the graph* $k_{r,s,s}$ can be decomposed into

$\left(\frac{s}{k}\right) K_{\frac{k+3}{2},k,k} \cup \left(\frac{s}{k}\right)\left(\frac{s}{k}-1\right) K_{k,k} \cup\ 2\left(\frac{s}{k}\right) K_{k-1,k} \cup \left(\frac{2r-(3k+1)}{k}\right)\left(\frac{s}{k}\right) K_{k,k}$.

For $s < t$, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s,s} \cup\ K_{k+r',t-s} \cup K_{(r+s)-(k+r'),t-s}$,

where $K_{k+r',t-s} = \left(\frac{t-s}{2k}\right) K_{k+r',2k}$ and $K_{(r+s)-(k+r'),t-s} = \left(\frac{(r+s)-(k+r')}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$.

Subcase (d): Let $r' > \frac{k+1}{2}$. Either all of r, s, t are odd or even, then the graph

$K_{r,s,t} = K_{r-r',s,s} \cup \left(\frac{t+s}{k}\right) K_{r',k} \cup \left(\frac{(r+s)-r'}{k}\right)\left(\frac{t-s}{k}\right) K_{k,k}$.

Hence obtained the required decomposition.

Lemma 3. If r, s, t and s' be positive integers such that $r, t \equiv 0(mod\ k), s \equiv s'(mod\ k)$ and $k \geq 5$ is prime, then the complete tripartite graph $K_{r,s,t}$ with $r \leq s \leq t$, has $(k;\ p, q) - decomposition$.

Proof. Case (i): r, s, t are all odd. Let s' is odd and $s \neq r + k$, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s-(k+s'),s-(k+s')} \cup \left(\frac{t-(s-(k+r'))}{k}\right)\left(\frac{(r+s)-(k+s')}{k}\right) K_{k,k} \cup \left(\frac{r}{k}\right) K_{k,k+s'} \cup \left(\frac{t-3k}{2k}\right) K_{k+s',2k} \cup K_{k+s',3k}$.

If s' is even, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s-(2k+s'),s-(2k+s')} \cup \left(\frac{r}{k}\right) K_{k,2k+s'} \cup K_{r,t-(s-(2k+s'))} \cup K_{s-(2k+s'),t-(s-(2k+s'))} \cup K_{2k+s',t}$.

Case (ii): r, s, t are all even. Let s' is odd and $s \neq r + k$, then $K_{r,s,t}$ can be decomposed into

$K_{r,s-(k+s'),s-(k+s')} \cup \left(\frac{t-(s-(k+s'))}{k}\right)\left(\frac{(r+s)-(k+s')}{k}\right) K_{k,k} \cup \left(\frac{r}{k}\right) K_{k,k+s'} \cup \left(\frac{t}{2k}\right) K_{k+s',2k}$.

If s' is even, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s-(2k+s'),s-(2k+s')} \cup \left(\frac{r}{k}\right) K_{k,2k+s'} \cup K_{r,t-(s-(2k+s'))} \cup \left(\frac{s-(2k+s')}{k}\right)\left(\frac{t-(s-(2k+s'))}{k}\right) K_{k,k} \cup K_{2k+s',t}$.

By using Construction 1, 2 and 3, the graph $K_{r,s,t}$ *has the* required decomposition.

Lemma 4. If k, r, s, t and t' be positive integers such that $r, s \equiv 0(mod\ k), t \equiv t'(mod\ k)$ and $k \geq 5$, is prime, then the complete tripartite graph $K_{r,s,t}$ with $r \leq s \leq t$, has $(k; p, q)$ – decomposition.

Proof: Either all of r, s and t are odd or even and $t \neq s + k$. If t' is odd, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s,s} \cup \left(\frac{r+s}{k}\right) K_{k,k+t'} \cup \left(\frac{r+s}{k}\right)\left(\frac{(t-s)-(k+t')}{k}\right) K_{k,k}.$

If t' is even, then the graph $K_{r,s,t}$ can be decomposed into

$K_{r,s,s} \cup \left(\frac{r+s}{k}\right) K_{k,k+t'-1} \cup \left(\frac{r+s}{k}\right) K_{k,k+1} \cup \left(\frac{r+s}{k}\right)\left(\frac{(t-s)-(2k+t')}{k}\right) K_{k,k}.$

By using Construction 1, 2 and 3, the graph $K_{r,s,t}$ *has the* required decomposition.

Main Theorem. Let p and q be non negative integers and let r, s, t be positive integers. The complete tripartite graph $K_{r,s,t}$ with $r \leq s \leq t$, has $(k; p, q)$- decomposition when $k \geq 5$ is prime if and if $k(p + q) = rs + st + tr, q \neq 1$. These necessary conditions are sufficient when at least any two of the parts are divisible by k.

Proof: This follows from the Lemmas 1, 2, 3 and 4.

3. Conclusion

The objective of this manuscript is to find the decomposition of complete tripartite graphs into cycles and paths for all possible values of p and q when k≥ 5 *is* prime. I find the necessary conditions for such existence and also proved these necessary conditions are sufficient when at least any two of the parts are divisible by k.

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Conflicts of Interest. This is a single authored paper. There is no conflict of Interest.

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