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Properties of Partial Complement Mycielskian Graph $\mu^{|}(G)$

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Abstract. For a graph G with vertex set $V(G) = \{v_1, v_2, v_3, \dots \dots v_n\}$ the partial complement Mycielskian of graph G is the graph $\mu^{|}(G)$ with vertex set $\mu^{|}(V(G)) = V \cup U \cup \{x\}$ corresponding to each vertex v_i in $V(G)$. Introduce a new vertex u_i and let U = $\{u_i{:}1 \leq i \leq n, v_i \in V$ and $i = 1,2,3, \dots \dots n\}$ is a set disjoint from V, take another vertex x and add edges from x to all vertices in U. $E(\mu^{|}(G)) = E(G) \cup \{v_i v_j \,/ v_i v_j \notin E(G)\} \cup \{xu/u \in U\}$. In this article, we explore the basic properties of the complement of the Mycielskian graph.

Keywords: Mycielskian Graph, Partial complement Mycielskian, connected graphs, isomorphic graphs.

AMS Mathematics Subject Classification (2010): 05C07, 05C75

1. Introduction

All graphs considered in this paper are finite and connected without loops or multiple edges. Let $G = (V, E)$ be a graph with order n and size m. If $u, v \in V(G)$ are said to be adjacent if $e = uv \in E(G)$. The degree of vertex $v \in V(G)$ is the number of edges incident to v in G denoted by $deg_G(v)$. A graph G is said to be connected if there exists a path between any two vertices. Let $\bar{G}$ denote the complement of graph G, where the order and size of $\bar{G}$ are n and $\binom{n}{2} - m$, respectively. For $u, v \in V(G)$, the distance $d(u, v)$ is the length of the shortest path connecting the vertices u, v. Vertex independent number denoted by $\alpha(G)$ is a maximum number of vertices, no two of which are adjacent. The vertex covering number denoted by $\beta(G)$ is a minimum number of vertices that cover all edges of G. Edge independent number denoted by $\alpha'(G)$ is a maximum number of edges, no two of which are adjacent. Edge covering number denoted by $\beta'(G)$ is a minimum number of edges that cover all edges of G [1,2,3,4,6,7,9]. A circuit C in a graph G is said to be an Eulerian circuit if C contains every edge of G. A connected graph that contains an Eulerian circuit is called an Eulerian graph. A cycle in a graph G that contains every vertex of G is called the Hamiltonian cycle of G. A connected graph that contains a Hamiltonian cycle is called a Hamiltonian graph. A graph G is said to be an eulerian if and only if every vertex is of even degree [8].

Motivated by the research on Mycielskian graph and Partial Complement Mycielskian graph, we found some results on the properties of Eulerian, Vertex Independent and Vertex Covering Number, Edge Independent and Edge Covering Number, Maximum degree, Minimum degree, isomorphic of partial complement Mycielskian graph.

2. Mycielskian graph

In order to get a triangle-free graph with a small clique number and high chromatic number, a graph was introduced by Mycielski called the Mycielskian graph denoted by $\mu(G)$ and defined as: For a given graph G with vertex set $V(G) = \{v_1, v_2, v_3, \dots \dots v_n\}$ is the graph obtained by applying the following steps:

1. Corresponding to each vertex v_i in $V(G)$, introduce a new vertex u_i and let $U = \{u_i: 1 \leq i \leq n\}$.
   Add edges from each vertex u_i of U to the vertex v_j if $v_i v_j \in E(G)$.
2. Take another vertex x and add edges from x to all vertices in U.

The new graph thus obtained is called the Mycielski graph of G and is denoted by $\mu(G)$ [5].

3. Partial complement Mycielskian graph

For a graph G with vertex set $V(G) = \{v_1, v_2, v_3, \dots \dots v_n\}$ the partially complement Mycielskian of graph G is the graph $\mu^|(G)$ with vertex set $\mu^|(V(G)) = V \cup U \cup \{x\}$ corresponding to each vertex v_i in $V(G)$. Introduce a new vertex u_i and let U $= \{u_i{:}1 \leq i \leq n, v_i \in V \text{ and } i = 1,2,3, \dots \dots n\}$ is a set disjoint from V, take another vertex x and add edges from x to all vertices in U. $E(\mu^|(G)) = E(G) \cup \{v_i v_j / v_i v_j \notin E(G)\} \cup \{xu/u \in U\}$.

4. Main results

Theorem 1. No of vertices of partial complement Mycielskian graph $\mu^|(G)$ is $2n + 1$.

Proof: Let G be a graph with n vertices and m edges, then the resulting partial complement Mycelskain graph has $2n + 1$ vertices, i.e. The graph has n original vertices of G and n duplicate vertices with one more vertex x adjacent to n duplicate vertices, hence has $n + n + 1 = 2n + 1$ number of vertices.

Theorem 2. Degree of vertices of partial complement Mycielskian graph $\mu^|(G)$

i. $deg_{\mu^|(G)}(v_i) = n - 1$

ii. $deg_{\mu^|(G)}\left(v_i^|\right) = n - \deg(v_i)$

iii. $deg_{\mu^|(G)}(x) = n$.

Proof: i) Suppose the degree of $v_i \in V(G)$ is k then by definition of partial complement Mycelskain graph, the duplicate of non-adjacent vertices of v_i contributes $n-k-1$ degree in $v_i \in \mu^{|}(V(G))$ implies degree of v_i in $\mu^{|}(G) = n-k-1+k = n-1$.

ii) Suppose if the degree of $v_i \in V(G)$ is k, then the duplicate vertices of v_i which is $v_i^{|}$ will be adjacent to non-adjacent vertices of v_i, since $v_i^{|}$ are non-adjacent in partial complement Mycielskian graph and x is adjacent to $v_i^{|}$ implies $deg_{\mu^{|}(G)}\left(v_i^{|}\right) = n-k-1+1 = n-\deg(v_i)$

iii) Since all the duplicate vertices $v_i^{|} \in V(G)$ are adjacent to x, the degree of x will be n.

Theorem 3. The Number of edges of partial complement Mycielskian graph $\mu^{|}(G)$ is $n^2 - m$.

Proof: Let G be a graph with n vertices and m edges, also, the partial complement Mycielskian graph $\mu^{|}(G)$ has $2n+1$ vertices. We can make use of the first theorem of graph theory, i.e. sum of degree of all vertices is equal to twice the number of edges.

i.e. $\sum_{i=1}^{n} \deg(v_i) = 2m$

We know from previous results that

- $deg_{\mu^{|}(G)}(v_i) = n-1$
- $deg_{\mu^{|}(G)}\left(v_i^{|}\right) = n - \deg(v_i)$
- $deg_{\mu^{|}(G)}(x) = n$

The partial complement Mycielskian graph has n vertices of degree n-1 and n duplicate vertices of degree $n-\deg(v_i)$, one vertex x has degree n

By the first theorem of graph theory, we have

$n(n-1) + \sum_{i=1}^{n}(n - degv_i) + n = 2E(\mu^{|}(G))$

$n^2 - n + \sum_{i=1}^{n} n - \sum_{i=1}^{n} deg(v_i) + n = 2E(\mu^{|}(G))$

$n^2 + n^2 - 2m = 2E(\mu^{|}(G))$

$\frac{2n^2-2m}{2} = E(\mu^{|}(G))$

$E\left(\mu^{|}(G)\right) = n^2 - m.$

Theorem 4. The partial complement Mycielskian graph is not Eulerian.

Proof: Let G be a graph and $\mu^{|}(G)$ be the partial complement Mycielskian graph of G.

Case 1. If n is odd in G, then vertex x is of odd degree in $\mu^{|}(G)$ (since the degree of x=no of vertices of G)

Hence $\mu^{|}(G)$ is not Eulerian.

Case 2. If n is even, then deg(v_i) in $\mu^{|}(G)$ will be n-1(since each original vertex will be of odd degree in $\mu^{|}(G)$)

Hence $\mu^{|}(G)$ is not Eulerian.

Theorem 5. Vertex Independent and Vertex Covering Number of partial complement Mycielskian graph is n and $n+1$.

Proof: Since the partial complement Mycielskian graph $\mu^{|}(G)$ has n independent vertices, i.e. the duplicate vertices are always independent.

By theorem, we have $\alpha(G) + \beta(G) = n$ and also $\alpha\left(\mu^{|}(G)\right) = n$

$\alpha\left(\mu^{|}(G)\right) + \beta\left(\mu^{|}(G)\right) = 2n + 1$

$n + \beta\left(\mu^{|}(G)\right) = 2n + 1$

$\beta\left(\mu^{|}(G)\right) = 2n + 1 - n$

$\beta\left(\mu^{|}(G)\right) = n + 1$

Hence, $\alpha\left(\mu^{|}(G)\right) = n$ and $\beta\left(\mu^{|}(G)\right) = n + 1$.

Theorem 6. Edge Independent and Edge Covering Number of partial complement Mycielskian graph is n and $n + 1$.

Proof: Suppose the partially complement Mycielskian graph has edge covering number n, then it will cover $2n$ number of vertices since the partially complement Mycelskain graph has $2n + 1$ vertices. One vertex is not covered in the graph. So $n + 1$ edges are required to cover $2n + 1$ vertices. Therefore, the edge covering number is $n + 1$.

By theorem we have $\alpha^{|}(G) + \beta^{|}(G) = n$ and Hence $\beta^{|}\left(\mu^{|}(G)\right) = n + 1$

$\alpha^{|}\left(\mu^{|}(G)\right) + \beta^{|}\left(\mu^{|}(G)\right) = 2n + 1$

$\alpha^{|}\left(\mu^{|}(G)\right) + n + 1 = 2n + 1$

$\alpha^{|}\left(\mu^{|}(G)\right) = 2n + 1 - n - 1$

$\alpha^{|}\left(\mu^{|}(G)\right) = n$

Hence $\alpha^{|}\left(\mu^{|}(G)\right) = n$ and $\beta^{|}\left(\mu^{|}(G)\right) = n + 1$.

Theorem 7. Maximum degree of partial complement Mycielskian graph is n.

Proof: It is very clear that partial complement Mycielskian graph has extra vertex x which is adjacent to all duplicate vertices so the number of vertices partial complement Mycielskian graph is n.

Theorem 8. The minimum degree of partial complement Mycielskain graph is $n - \Delta(G)$.

Proof: Suppose v_i is a vertex with $\Delta(G)$ from the construction of partial complement Mycielskain graph $v_i^{|}$ is adjacent to vertices v_j which are not adjacent to v_i in G, from result 2 we have $deg(v_i^{|}) = n - deg(v_i)$. Hence the minimum degree of partial complement Mycielskain graph is $n - \Delta(G)$.

Theorem 9. For complete graph K_n, the partial complement Mycielskian graph is isomorphic to $K_n \cup K_{1,n}$.

Proof: In a complete graph $K_{n,}$ every vertex is adjacent to n-1 vertices. So, every duplicate vertex will be adjacent to extra vertex x, which forms a star graph $K_{1,n,}$ which is disconnected from the complete graph K_n. Therefore, the partial complement Mycielskain graph of a complete graph K_n is isomorphic to $K_n \cup K_{1,n}$.

Theorem 10. The partial complement Mycielskian graph is connected if and only if G is not isomorphic to $K_n, n \geq 2$.

Proof: Suppose the partial complement Mycielskian graph is connected; then it is very clear from Theorem 9 that it should not be isomorphic to K_n because the partial complement Mycielskian graph of K_n is disconnected.

Conversely, G is not isomorphic to $K_n, n \geq 2$, then the partial complement Mycielskian graph is connected because the partial complement Mycielskian graph is disconnected only when G is K_n.

5. Conclusion and scope

In this article, we have discussed some basic properties of the partial complement Mycielskian graph. This study can be further extended to other structural properties of the partial complement Mycielskian graph.

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