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On Delta Nirmala and Multiplicative Delta Nirmala Indices of Certain Nanotubes

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Abstract. In this study, we introduce the delta Nirmala index and its exponential of a graph. Also, we define the multiplicative delta Nirmala index of a graph. Furthermore, we compute these indices for certain nanotubes.

Keywords: delta Nirmala index, multiplicative delta Nirmala index, nanotube.

AMS Mathematics Subject Classification (2010): 05C07, 05C09, 05C92

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Let $\delta(G)$ denote the minimum degree among the vertices of G. We refer [1] for undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology.

The δ vertex degree was introduced in [2] and it is defined as

$$\delta_u = d_G(u) - \delta(G) + 1.$$

We introduce the delta Nirmala index of a graph and it is defined as

$$\delta N(G) = \sum_{uv \in E(G)} \sqrt{\delta_u + \delta_v}.$$

Considering the delta Nirmala index, we define the delta Nirmala exponential of a graph G as

$$\delta N(G, x) = \sum_{uv \in E(G)} x^{\sqrt{\delta_u + \delta_v}}.$$

Recently, some delta Banhatti indices were studied in [3, 4, 5, 6].

We define the multiplicative delta Nirmala index of a graph G as

$$\delta NII(G) = \prod_{uv \in E(G)} \sqrt{\delta_u + \delta_v}.$$

Recently, some Nirmala indices were studied in [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

In this paper, we determine the delta Nirmala index and its corresponding exponential of some nanotubes. Also, we compute the delta multiplicative Nirmala index of some nanotubes.

2. Results for $HC_5C_7[p,q]$ nanotubes

In this section, we focus on the family of nanotubes, denoted by $HC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. Let G be the graph of a nanotube $HC_5C_7[p,q]$.

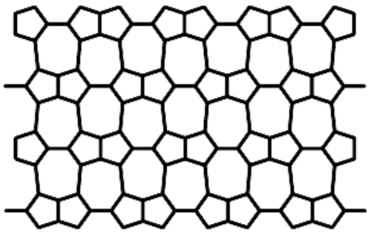

Figure 1: 2-*D* lattice ofnanotubeHC_5C_7 [8, 4]

The 2-D lattice of nanotube $HC_5C_7[p, q]$ is shown in Figure 1..By calculation, we obtain that G has $4pq$ vertices and $6pq – p$ edges. The graph G has two types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_2| = 6pq – 5p$.

Clearly $\delta(G)$=2. Therefore $\delta_u = d_G(u) – \delta(G) + 1 = d_G(u) – 1$. Thus there are two types of δ-edges as given in Table 1.

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 2)	$4p$
(2, 2)	$6pq – 5p$

Table 1: δ-edge partition of $HC_5C_7[p, q]$

Theorem 1. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

(i) $\delta N\left(HC_5C_7[p,q]\right) = 12pq + \left(4\sqrt{3} - 10\right)p.$

(ii) $\delta N\left(HC_5C_7[p,q], x\right) = 4px^{\sqrt{3}} + \left(6pq - 5p\right)x^2.$

Proof: From definitions and by using Table 1, we deduce

(i) $\delta N\left(HC_5C_7[p,q]\right) = \sum_{uv\in E(G)} \sqrt{\delta_u + \delta_v} = 4p\sqrt{1+2} + (6pq - 5p)\sqrt{2+2}$

$= 12pq + \left(4\sqrt{3} - 10\right)p.$

(ii) $\delta N\left(HC_5C_7[p,q], x\right) = \sum_{uv\in E(G)} x^{\sqrt{\delta_u + \delta_v}} = 4px^{\sqrt{1+2}} + \left(6pq - 5p\right)x^{\sqrt{2+2}}$

$$= 4px^{\sqrt{3}} + (6pq - 5p)x^2.$$

Theorem 2. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$\delta NII\left(HC_5C_7[p,q]\right) = 3^{2p} \times 2^{(6pq-5p)}.$$

Proof: From definition and by using Table 1, we deduce

$$\delta NII\left(HC_5C_7[p,q]\right) = \prod_{uv\in E(G)} \sqrt{\delta_u + \delta_v} = \left(\sqrt{1+2}\right)^{4p} \times \left(\sqrt{2+2}\right)^{(6pq-5p)}$$
$$= 3^{2p} \times 2^{(6pq-5p)}.$$

3. Results for $SC_5C_7[p,q]$ nanotubes

In this section, we focus on the family of nanotubes, denoted by $SC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[p,q]$ is presented in Figure 2.

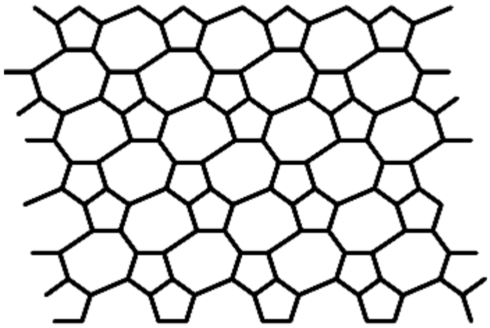

Figure 2: 2-*D* lattice of nanotube $SC_5C_7[p,q]$

Let G be the graph of $SC_5C_7[p,q]$. By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. Also by calculation, we get that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = q$.

$E_2 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6q$.

$E_2 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6pq - p - 7q$.

Clearly $\delta(G)=2$. Thus $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. There are three types of δ-edges as given in Table 2.

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	q
(1, 2)	$6q$
(2, 2)	$6pq - p - 7q$

Table 2: δ-edge partition of $SC_5C_7[p, q]$

Theorem 3. Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then

(i) $\delta N\left(SC_5C_7[p,q]\right)=12pq-2p+\left(\sqrt{2}+6\sqrt{3}-14\right)q.$

(ii) $\delta N\left(SC_5C_7[p,q],x\right)=qx^{\frac{1}{\sqrt{2}}}+6qx^{\frac{1}{\sqrt{3}}}+\left(6pq-p-7q\right)x^{\frac{1}{2}}.$

Proof: From definitions and by using Table 2, we deduce

(i) $\delta N\left(SC_5C_7[p,q]\right)=\sum_{uv\in E(G)}\sqrt{\delta_u+\delta_v}=q\sqrt{1+1}+6q\sqrt{1+2}+(6pq-p-7q)\sqrt{2+2}$

$$=12pq-2p+\left(\sqrt{2}+6\sqrt{3}-14\right)q.$$

(ii) $S\delta B\left(SC_5C_7[p,q],x\right)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{\delta_u+\delta_v}}}=qx^{\frac{1}{\sqrt{1+1}}}+6qx^{\frac{1}{\sqrt{1+2}}}+\left(6pq-p-7q\right)x^{\frac{1}{\sqrt{2+2}}}$

$$=qx^{\frac{1}{\sqrt{2}}}+6qx^{\frac{1}{\sqrt{3}}}+\left(6pq-p-7q\right)x^{\frac{1}{2}}.$$

Theorem 4. Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then

$$\delta NII\left(SC_5C_7[p,q]\right)=3^{2p}\times 2^{(6pq-5p)}.$$

Proof: From definition and by using Table 1, we deduce

$$\delta NII\left(SC_5C_7[p,q]\right)=\prod_{uv\in E(G)}\sqrt{\delta_u+\delta_v}=\left(\sqrt{1+2}\right)^{4p}\times\left(\sqrt{2+2}\right)^{(6pq-5p)}$$

$$=3^{2p}\times 2^{(6pq-5p)}.$$

4. Conclusion

In this paper, we have defined the delta Nirmala and multiplicative delta Nirmala indices of a graph. Also, the delta Nirmala and multiplicative delta Nirmala indices of certain nanotubes are determined.

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Authors' Contributions. This is the authors' sole contribution.

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