*Annals of Pure and Applied Mathematics*
*Vol. 2, No. 1, 2012, 10-17*
*ISSN: 2279-087X (P), 2279-0888(online)*
*Published on 24 November 2012*
*www.researchmathsci.org*

Annals of
**Pure and Applied Mathematics**

# On Semiprime n-Ideals in Lattices

***M. Ayub Ali*[1], *R. M. Hafizur Rahman*[2], *A. S. A. Noor*[3] *and M. M. Rahman*[4]**

[1]Department of Mathematics, Jagannath University, Dhaka, Bangladesh.
Email: ayub_ju@yahoo.com
[2]Department of Mathematics, Begum Rokeya University, Rangpur, Bangladesh.
[3]Department of ECE, East West University, Dhaka, Bangladesh.
Email : noor@ewubd.edu
[4]Department of Mathematics, Satkhira City College, Satkhira, Bangladesh.

*Received 1 November 2012; accepted 19 November 2012*

**Abstract.** Yehuda Rav has given the concept of Semi-prime ideals in a general lattice by generalizing the notion of 0-distributive lattices. In this paper we introduce the concept of semi prime $n$-ideals in lattices when $n$ is a neutral element. For a fixed element $n$ in a lattice $L$, any convex sublattice containing $n$ is called an $n$-ideal. Here we give several characterizations of semi prime $n$-ideals of lattices. We include a Prime Separation Theorem in a general lattice with respect to annihilator $n$-ideal containing a semi prime $n$-ideal.

***Keywords.*** $n$-distributive lattice, Semi-prime $n$-ideal, Annihilator $n$-ideal, Maximal convex sublattice, Prime convex sublattice.

***AMS Mathematics Subject Classifications (2010):*** 06A12, 06A99, 06B10

## 1. Introduction

In generalizing the notion of pseudo complemented lattice, J. C. Varlet [9] introduced the notion of 0-distributive lattices. Then [2] have given several characterizations of these lattices. On the other hand, [7] have studied them in meet semi lattices. A lattice $L$ with $0$ is called a *0-distributive* lattice if for all $a,b,c \in L$ with $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Let $L$ be a lattice and $n \in L$. Any convex sublattice of $L$ containing $n$ is called an $n$-ideal of $L$. An element $n \in L$ is called a standard element if for $a,b \in L$, $a \wedge (b \vee n) = (a \wedge b) \vee (a \wedge n)$, while $n$ is called a neutral element if

(i) it is standard and
(ii) $n \wedge (a \vee b) = (n \wedge a) \vee (n \wedge b)$ for all $a,b \in L$.

Set of all $n$-ideals of a lattice $L$ is denoted by $I_n(L)$ which is an algebraic lattice; where $\{n\}$and $L$ are the smallest and largest elements. For two $n$-ideals $I$ and $J$, $I \cap J$ is the infimu q hm and
$I \vee J = \{x \in L / i_1 \wedge j_1 \leq x \leq i_2 \vee j_2, \textit{for some } i_1, i_2 \in I \textit{ and } j_1, j_2 \in J\}$. The $n$-ideal generated by a finite numbers of elements $a_1, a_2, ..., a_m$ is called a finitely generated $n$-ideal denoted by $< a_1, a_2, ..., a_m >_n$. Moreover, $< a_1, a_2, ..., a_m >_n =$
$\{x \in L / a_1 \wedge a_2 \wedge ... \wedge a_m \wedge n \leq x \leq a_1 \vee a_2 \vee ... \vee a_m \vee n\}$
$= [a_1 \wedge a_2 \wedge .... \wedge a_m \wedge n, a_1 \vee a_2 \vee .... \vee a_m \vee n]$

Thus, every finitely generated $n$-ideal is an interval containing $n$. An $n$-ideal generated by a single element $a \in L$ is called a principal $n$-ideal denoted by $< a >_n$ and $< a >_n = [a \wedge n, a \vee n]$. Moreover $[a,b] \cap [c,d] = [a \vee c, b \wedge d]$ and $[a,b] \vee [c,d] = [a \wedge c, b \vee d]$. If $n$ is a neutral element, then by [6], $< a >_n \cap < b >_n =< m(a,n,b) >_n$, where $m(x,y,z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$. For detailed literature on $n$-ideals we refer the reader to consult [5, 6].

A proper convex sublattice $M$ of a lattice $L$ is called a maximal convex sublattice if for any convex sublattice $Q$ with $Q \supseteq M$ implies either $Q = M$ or $Q = L$. A proper convex sublattice $M$ is called a prime convex sublattice if for any $t \in M$, $m(a,t,b) \in M$ implies either $a \in M$ or $b \in M$. Similarly, an $n$-ideal $P$ of $L$ is called a prime $n$-ideal if $m(a,n,b) \in P$ implies either $a \in P$ or $b \in P$. Equivalently, $P$ is prime if and only if $< a >_n \cap < b >_n \subseteq P$ implies either $< a >_n \subseteq P$ or $< b >_n \subseteq P$. Moreover, by [9], we know that every prime convex sublattice $P$ of $L$ is either an $n$- ideal or a filter.

By [1] $L$ is called an $n$-distributive lattice if for all,
$< a >_n \cap < b >_n = \{n\}$ and $< a >_n \cap < c >_n = \{n\}$ imply
$< a >_n \cap [< b >_n \vee < c >_n] = \{n\}$. Equivalently, $L$ is called $n$-distributive if $a \wedge b \leq n \leq a \vee b$ and $a \wedge c \leq n \leq a \vee c$ imply $a \wedge (b \vee c) \leq n \leq a \vee (b \wedge c)$.

Y. Rav [8] an ideal $I$ of a lattice is called a *semi prime ideal* if for all $x, y, z \in L$, $x \wedge y \in I$ and $x \wedge z \in I$ imply $x \wedge (y \vee z) \in I$. Thus, a lattice $L$ with 0, is called *0-distributive* if and only if (0] is a semi prime ideal. Let $n$ be a neutral element of a lattice $L$. An $n$-ideal J of $L$ is called a semi prime $n$-ideal if for all $a, b, c \in L$, $< a >_n \cap < b >_n \subseteq J$ and $< a >_n \cap < c >_n \subseteq J$ imply $< a >_n \cap (< b >_n \vee < c >_n) \subseteq J$. In a distributive lattice every $n$-ideal is semi prime. Moreover, every prime $n$-ideal is semi prime. Lattice itself with an element $n$ is of course a semi prime $n$-ideal. It is easy to see that a lattice with the element $n$ is $n$-distributive if $\{ n \}$ is a semi prime $n$-ideal. In the pentagonal lattice $\{0, a, b, c, n; a < b, a \vee c = b \vee c = n, a \wedge c = b \wedge c = 0\}$, $n$ is neutral. Here

$\{n\}$ and $<0>_n = L$ are semi prime but not prime. Moreover, $<a>_n, <c>_n$ are prime but $<b>_n$ is not even semi prime. Again in $M_3 = \{0, a, b, c, n; a \wedge b = b \wedge c = c \wedge a = 0 = \quad a \vee b = b \vee c = c \vee a = n\}$, $<0>_n = L$ is semi prime. But {n}, $<a>_n$, $<b>_n$, $<c>_n$ are not semi prime.

Throughout the paper we will consider $n$ as a neutral element.

**Lemma 1.** *Intersection of any class of prime (semi prime) n- ideals of a lattice is a semi-prime n- ideal.*

**Proof:** Suppose $\{P_k : k \in T\}$ is a class of prime (semi prime) $n$-ideals of $L$. Let $a, b, c \in L$ and $I = \bigcap\{P_k : k \in T\}$. Clearly $I$ is an $n$-ideal. Let $<a>_n \cap <b>_n \subseteq I$ and $<a>_n \cap <c>_n \subseteq I$. Then $<a>_n \cap <b>_n \subseteq P_k$ and $<a>_n \cap <c>_n \subseteq P_k$ for all $P_k$. Since each $P_k$ is prime (semi prime), so $<a>_n \cap (<b>_n \vee <c>_n) \subseteq P_k$ for all $k$.

Hence $<a>_n \cap (<b>_n \vee <c>_n) \subseteq I$, and so $I$ is semi-prime. ■

**Corollary 2.** *Intersection of two prime (semi prime) ideals is a semi-prime n- ideal.* ■

**Lemma 3**. *Every convex sublattice disjoint from an n-ideal $I$ is contained in a maximal convex sub lattice disjoint from $I$.*

**Proof:** Let $F$ be a convex sub lattice in $L$ disjoint from $I$. Let F be the set of all convex sub lattices containing $F$ and disjoint from $I$. Then F is nonempty as $F \in$ F. Let $C$ be a chain in F and let $M = \bigcup(X : X \in C)$. We claim that $M$ is a convex sub lattice. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since $C$ is a chain, either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. So $x, y \in Y$. Then $x \wedge y, x \vee y \in Y$ and so $x \wedge y, x \vee y \in M$. Thus $M$ is a sublattice. Now let $x \leq t \leq y$ with $x, y \in M$. Then $x, y \in X$ for some $X \in C$. Thus, by convexity of X, $t \in X$ and so $t \in M$. Therefore $M$ is convex. Moreover, $M \supseteq F$. So $M$ is a maximum element of $C$. Then by Zorn's Lemma, F has a maximal element, say $Q \supseteq F$. ■

**Lemma 4.** *Every maximal convex sublattice disjoint to an $n$-ideal $J$ is either a maximal ideal or a maximal filter.*

**Proof.** Let $F$ be a maximal convex sublattice disjoint to $J$. Since $F = (F] \cap [F)$, so either $(F] \cap J = \varphi$ or $[F) \cap J = \varphi$. If not, let $x \in (F] \cap J$ and $y \in [F) \cap J$. Then $x \in J$ and $x \leq f_1$ for some $f_1 \in F$ and $y \in J$ and $y \geq f_2$ for some $f_2 \in F$. Now, $f_2 \leq x \vee f_2 \leq f_1 \vee f_2$ implies by convexity that $x \vee f_2 \in F$. Also $x \leq x \vee f_2 \leq x \vee y$ implies by convexity that $x \vee f_2 \in J$. It follows that

$x \vee f_2 \in F \cap J$, which is a contradiction. Therefore either $(F] \cap J = \varphi$ or $[F) \cap J = \varphi$. Since $F$ is maximal so, either $F$ =( $F$ ] or $F$ =[ $F$ ). In other words, $F$ must be either an ideal or a filter. ■

**Lemma 5.** *Let $I$ be an n- ideal of a lattice $L$. A convex sublattice $M$ disjoint from $I$ is a maximal convex sublattice disjoint from $I$ if and only if for all $a \notin M$, there exists $b \in M$ such that $m(a,n,b) \in I$.*

**Proof.** Let $M$ be maximal and disjoint from $I$ and $a \notin M$. Let $m(a,n,b) \notin I$ for all $b \in M$ .Consider $M_1 = \{y \in L : y \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq y \vee n; b \in M\}$. It is easy to cheek that $M_1$ is a convex sublattice as $n$ is neutral. Also $M_1 \cap I = \varphi$. If not, let $x \in M_1 \cap I$. Then $x \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq x \vee n$ for some $b \in M$ and $x \in I$.

Thus, $x \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee (a \wedge n) \vee (b \wedge n) \leq (a \wedge b) \vee n \leq x \vee n$ implies $m(a,n,b) \in I$ which gives a contradiction. Now for $b \in M$ $b \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq b \vee n$ implies $b \in M_1$ and so $M \subseteq M_1$. Also $a \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq a \vee n$ implies $a \in M_1$ but $a \notin M$. Hence $M \subset M_1$. Thus we have a contradiction to the maximality of $M$. Hence there exists some $b \in M$ such that $m(a,n,b) \in I$

Conversely, Suppose the given condition holds. If $M$ is not maximal disjoint from $I$, then by Lemma 3, there exists a maximal convex sublattice $N \supset M$ and disjoint with $I$. For any $a \in N - M$, there exists $b \in M$ such that $m(a,n,b) \in I$. Now, $a,b \in N$ implies $a \wedge b, a \vee b \in N$. By Lemma 4, $N$ is either an ideal or a filter. Hence $(a \wedge b) \vee n \in N$ or $(a \vee b) \wedge n \in N$ but not both. For otherwise, $n \in N$ would give a contradiction to $I \cap N = \varphi$. Now any of the above causes will imply $m(a,n,b) \in N$ and so $m(a,n,b) \in I \cap N$ which is again a contradiction. Therefore $M$ must be a maximal convex sublattice disjoint with $I$. ■

Let $L$ be a lattice with neutral element $n$. For $A \subseteq L$, We define $A^{\perp_n} = \{x \in L : m(x,n,a) = n \text{ for all } a \in A\}$. $A^{\perp_n}$ is always a convex subset containing $n$ but it is not necessarily an $n$ - ideal.

**Theorem 6.** *Let $L$ be an n-distributive lattice. Then for $A \subseteq L$, $A^{\perp_n} = \{x \in L : m(x,n,a) = n$ for all $a \in A\}$ is a semi-prime n- ideal.*

**Proof.** By [1,Theorem 6] we already know that $A^{\perp_n}$ is an $n$-ideal. This is also equivalent to the condition $I_n(L)$ is pseudo complemented.

Now let $<x>_n \cap <y>_n \subseteq A^{\perp_n}$ and $<x>_n \cap <z>_n \subseteq A^{\perp_n}$. Then for all $a \in A$.
This implies $<x>_n \cap <y>_n \cap <a>_n = \{n\} = <x>_n \cap <z>_n \cap <a>_n$
$<y>_n \subseteq (<x>_n \cap <a>_n)^*, <z>_n \subseteq (<x>_n \cap <a>_n)^*$ and so $<y>_n \vee <z>_n \subseteq (<x>_n \cap <a>_n)^*$ and this implies $<x>_n \cap <a>_n \cap (<y>_n \vee <z>_n) = \{n\}$ for all $a \in L$. Hence $<x>_n \cap (<y>_n \vee <z>_n) \subseteq A^{\perp_n}$ and so $A^{\perp_n}$ is a semi prime $n$-ideal. ■

Let $A \subseteq L$ and $J$ be an $n$-ideal of $L$. We define $A^{\perp^n{}_J} = \{x \in L : m(x,n,a) \in J \text{ for all } a \in A\}$. This is clearly a convex subset containing $J$. In presence of distributivity, this is an $n$- ideal. $A^{\perp^n{}_J}$ is called an $n$-annihilator of $A$ relative to $J$. We denote $I_J(L)$, the set of all $n$-ideals containing $J$. Of course, $I_J(L)$ is a bounded lattice with $J$ and $L$ as the smallest and the largest elements. If $A \in I_J(L)$, and $A^{\perp^n{}_J}$ is an $n$- ideal, then $A^{\perp^n{}_J}$ is called an annihilator $n$- ideal and it is the pseudo complement of $A$ in $I_J(L)$.

Following Theorem gives some nice characterizations semi prime $n$-ideals which is also a generalization of [3, Theorem 7].

**Theorem 7.** *Let $L$ be a lattice and $J$ be an n-ideal of $L$. The following conditions are equivalent.*
(i) *$J$ is semi prime.*
(ii) *$\{a\}^{\perp^n{}_J} = \{x \in L : x \wedge a \in J\}$ is a semi prime n- ideal containing $J$.*
(iii) *$A^{\perp^n{}_J} = \{x \in L : x \wedge a \in J \text{ for all } a \in A\}$ is a semi prime n-ideal containing $J$.*
(iv) *$I_J(L)$ is pseudo complemented*
(v) *$I_J(L)$ is a 0 –distributive lattice.*
(vi) *Every maximal convex sublattice disjoint from $J$ is prime.*

**Proof:** (i)$\Rightarrow$(ii). $\{a\}^{\perp^n{}_J}$ is clearly a convex subset containing $J$. Now let $x, y \in \{a\}^{\perp^n{}_J}$. Then $<x>_n \cap <a>_n \subseteq J$ and $<y>_n \cap <a>_n \subseteq J$. Since $J$ is semi prime, so $<a>_n \wedge (<x>_n \vee <y>_n) \in J$. Now $<x \wedge y>_n \cap <a>_n \subseteq J$ $<x \wedge y>_n \subseteq <x>_n \vee <y>_n = [x \wedge y \wedge n, x \vee y \vee n]$. Also $<x \vee y>_n \subseteq <x>_n \vee <y>_n$. Thus and $<x \vee y>_n \cap <a>_n \subseteq J$. Therefore $x \wedge y, x \vee y \in \{a\}^{\perp^n{}_J}$. This implies $\{a\}^{\perp^n{}_J}$ is an $n$- ideal containing $J$.

Now let $<x>_n \cap <y>_n \subseteq \{a\}^{\perp^n J}$ and $<x>_n \cap <z>_n \subseteq \{a\}^{\perp^n J}$. Then $<x>_n \cap <y>_n \cap <a>_n \subseteq J$ and $<x>_n \cap <z>_n \cap <a>_n \subseteq J$. Thus, $(<x>_n \cap <a>_n) \cap <y>_n \subseteq J$ and $(<x>_n \cap <a>_n) \cap <z>_n \subseteq J$. Then $(<x>_n \cap <a>_n) \cap (<y>_n \vee <z>_n) \subseteq J$, as $J$ is semi prime . This implies $<x>_n \cap (<y>_n \vee <z>_n) \subseteq \{a\}^{\perp^n J}$ and so $\{a\}^{\perp^n J}$ is semi prime.

(ii)$\Rightarrow$(iii). This is trivial by Lemma 1, as $A^{\perp^n J} = \bigcap(\{a\}^{\perp^n J}; a \in A)$.

(iii)$\Rightarrow$(iv). Since for any $A^{\perp^n J}$ is an $n$ - ideal, it is the pseudo complement of $A$ in $I_J(L)$, so $I_J(L)$ is pseudo complemented.

(iv)$\Rightarrow$(v). This is trivial as every pseudo complemented lattice is 0-distributive.

(v)$\Rightarrow$(vi). Let $I_J(L)$ be 0-distributive. Suppose $F$ is a maximal convex sublattice disjoint from $J$. Suppose $x, y \notin F$. Then by Lemma 5, there exist $a \in F$, $b \in F$ such that $m(x,n,a) \in J, m(y,n,b) \in J$. Thus $<x>_n \cap <a>_n \subseteq J, <y>_n \cap <b>_n \subseteq J$ and so $<x>_n \cap <a>_n \cap <b>_n \subseteq J, <y>_n \cap <b>_n \cap <a>_n \subseteq J$. Thus, $<x>_n \cap <m(a,n,b)>_n \subseteq J$, $<y>_n \cap <m(a,n,b)>_n \subseteq J$. Since $I_J(L)$ is 0-distributive, so, $<m(a,n,b)>_n \cap (<x>_n \vee <y>_n) \subseteq J$. By a routine calculation, we have $[(a \vee b \vee (x \wedge y)) \wedge n, (a \wedge b \wedge (x \vee y)) \vee n] \subseteq J$. This implies $(a \vee b \vee (x \wedge y)) \wedge n \in J$ and $(a \wedge b \wedge (x \vee y)) \vee n \in J$. By Lemma 4, $F$ is either an ideal or a filter. Suppose $F$ is filter. If $x \vee y \in F$, Then $(a \wedge b \wedge (x \vee y)) \vee n \subseteq F \cap J$, which is a contradiction. Thus, $x \vee y \notin F$. Similarly by considering $F$ as an ideal, if $x \wedge y \in F$, we have $(a \vee b \vee (x \wedge y)) \wedge n \subseteq F \cap J$, which also gives a contradiction. Thus $x \wedge y \notin F$. Therefore, $F$ must be prime.

(vi)$\Rightarrow$(i). Let $a, b, c \in L$ with $<a>_n \cap <b>_n \subseteq J$, $<a>_n \cap <c>_n \subseteq J$. Then $[(a \vee b) \wedge n, (a \wedge b) \vee n] \subseteq J$ and $[(a \vee c) \wedge n, (a \wedge c) \vee n] \subseteq J$. Thus $(a \vee b) \wedge n, (a \wedge b) \vee n \in J$ and $(a \vee c) \wedge n, (a \wedge c) \vee n \in J$. Now $<a>_n \cap (<b>_n \cap <c>_n) = [a \wedge n, a \vee n] \cap [b \wedge c \wedge n, b \vee c \vee n]$ $= [(a \vee (b \wedge c)) \wedge n, (a \wedge (b \vee c)) \vee n]$. If $<a>_n \cap (<b>_n \cap <c>_n) \square J$, then either $(a \vee (b \wedge c)) \wedge n \notin J$ or $(a \wedge (b \vee c)) \vee n \notin J$. Without loss of generality, suppose $(a \wedge (b \vee c)) \vee n \notin J$. Let $F = [(a \wedge (b \vee c)) \vee n)$. Then

$F \cap J = \phi$. If not, let $y \in F \cap J$. Then $y \geq (a \wedge (b \vee c)) \vee n$, $y \in J$. Thus $n \leq (a \wedge (b \vee c)) \vee n \leq y$ implies $(a \wedge (b \vee c)) \vee n \in J$, which is a contradiction. Then by Lemma 3, there exists a maximal filter $F \supseteq [a \wedge (b \vee c))$ and disjoint from $J$. But a convex sublattice containing a filter is itself a filter. By (vi), $M$ is a prime filter. Now $a \vee n \in M$ and $b \vee c \vee n \in M$. Since $M$ is a prime filter and $n \notin M$, so $a \in M$ and $b$ *or* $c \in M$. Thus either $a \wedge b \in M$ or $a \wedge c \in M$. Hence $(a \wedge b) \vee n \in M \cap J$ or $(a \wedge c) \vee n \in M \cap J$ which is again a contradiction. Therefore, $< a >_n \cap (< b >_n \cap < c >_n) \subseteq J$ and so $J$ is a semi prime $n$-ideal. ■

**Corollary 8**: *In a lattice $L$, every convex sublattice disjoint to a semi-prime n- ideal $J$ is contained in a prime convex sublttice.*
**Proof:** This immediately follows from Lemma 3 and Theorem 7. ■

**Theorem 9:** I*f $J$ is a semi-prime $n$-ideal of a lattice $L$ and $J \neq A = \bigcap \{J_\lambda : J_\lambda$ is an n-ideal containing $J\}$, Then $A^{\perp^n J} = \{x \in L : \{x\}^{\perp^n J} \neq J\}$.*

**Proof:** Let $x \in A^{\perp^n J}$. Then $m(x,n,a) \in J$ for all $a \in A$. So $a \in \{x\}^{\perp^n J}$ for all $a \in A$. Then $A \subseteq \{x\}^{\perp^n J}$ and so $\{x\}^{\perp^n J} \neq J$. Conversely, let $x \in L$ such that $\{x\}^{\perp^n J} \neq J$. Since $J$ is semi-prime, so $\{x\}^{\perp^n J}$ is an $n$- ideal containing $J$. Then $A \subseteq \{x\}^{\perp^n J}$, and so $A^{\perp^n J} \supseteq \{x\}^{\perp^n J \perp^n J}$. This implies $x \in A^{\perp^n J}$, which completes the proof. ■

In [2] a series of characterizations of $n$-distributive lattices are provided. Here we give some results on semi prime $n$-ideals related to their results.

We conclude the paper with the following characterizations of semi-prime $n$- ideals with the help of annihilator $n$-ideals. This is also a generalization of Prime Separation Theorem for $n$-ideals.

**Theorem 10:** *Let $J$ be an n-ideal in a lattice $L$. $J$ is semi- prime if and only if for all convex sublattice $F$ disjoint to $\{x\}^{\perp^n J}$, there is a prime convex sublattice containing F disjoint to $\{x\}^{\perp^n J}$.*

**Proof:** Using Zorn's Lemma we can easily find a maximal convex sublattice $Q$ containing $F$ and disjoint to $\{x\}^{\perp^n J}$. Then either $Q$ is an ideal or a filter. Without loss of generality, suppose $Q$ is a filter. We claim that $x \in Q$. If not, then $Q \vee [x) \supset Q$. By maximality of $Q$, $(Q \vee [x)) \cap \{x\}^{\perp^n j} \neq \varphi$. If $(Q \vee [x)) \cap \{x\}^{\perp^n j} \neq \varphi$ then $t \geq q \wedge x$ for some $q \in Q$ and $m(t,n,x) \in J$. Thus $m(t,n,x) \vee n = (t \wedge x) \vee n \in J$ as $n \in J$. Then $n \leq (q \wedge x) \vee n \leq (t \wedge x) \vee n$

implies $(q \wedge x) \vee n \in J \Rightarrow m(q \vee n, n, x) \in J$. Thus $q \vee n \in \{x\}^{\perp^{n}_{J}}$. Therefore, $x \in Q$ gives a contradiction to the fact that $Q \cap \{x\}^{\perp^{n}_{j}} = \varphi$.

Now let $z \notin Q$. Then $(Q \vee [z)) \cap \{x\}^{\perp^{n}_{J}} \neq \varphi$. Suppose $y \in (Q \vee [z)) \cap \{x\}^{\perp^{n}_{J}}$ then $y \geq q_1 \wedge z$ for $q_1 \in Q$ and $m(y, n, x) \in J$. Then $(y \wedge x) \vee n \in J$ and so $(q_1 \wedge z \wedge x) \vee n \leq (y \wedge x) \vee n$ implies $(q_1 \wedge z \wedge x) \vee n \in J$. This implies $m(z, n, (q_1 \wedge z \wedge x) \vee n) \in J$. Thus by Lemma 5, $Q$ is a maximal filter disjoint to $J$. Hence by Theorem 7, $Q$ is prime.

Conversely, let $< x >_n \cap < y >_n \in J$, $< x >_n \cap < z >_n \in J$. Suppose $< x >_n \cap (< y >_n \cap < z >_n) \square J$.

Then $[(x \vee (y \wedge z)) \wedge n, (x \wedge (y \vee z)) \vee n] \square J$ This implies either $(x \vee (y \wedge z)) \wedge n \notin J$ or $(x \wedge (y \vee z)) \vee n \notin J$. Suppose $(x \wedge (y \vee z)) \vee n \notin J$. Then $[y \vee z \vee n) \cap \{x\}^{\perp^{n}_{J}} = \varphi$. For otherwise $t \in [y \vee z \vee n) \cap \{x\}^{\perp^{n}_{J}}$ implies $t \geq y \vee z \vee n$ and $m(t, n, x) \in J$. Then $m(t, n, x) \vee n = (t \wedge x) \vee n \in J$. Then $n \leq (x \wedge (y \vee z)) \vee n \leq (t \wedge x) \vee n$ implies $(x \wedge (y \vee z)) \vee n \in J$ gives a contradiction. Similarly $(x \vee (y \wedge z)) \wedge n \notin J$ would imply another contradiction. Therefore, $< x >_n \cap (< y >_n \cap < z >_n) \subseteq J$ and so $J$ is semi prime. ■

## REFERENCES


1. M. Ayub Ali, A. S. A. Noor and Sompa Rani Podder, *n-distributive lattice*, to appear in *Journal of Physical Sciences*, 16 (2012).
2. P. Balasubramani and P. V. Venkatanarasimhan, *Characterizations of the 0-Distributive Lattices, Indian J. Pure Appl. Math.*, 32(3) (2001) 315-324.
3. R. M. Hafizur Rahaman, M. Ayub Ali And A. S. A. nor, On Semiprime Ideals of lattice, to appear in *Institute of Mechanics of Continua and Mathematical Sciences.*
4. R. M. Hafizur Rahman, *A Study on Convex Sublattices of a lattice*. Ph. D Thesis, Rajhsahi University, 2002.
5. M. A. Latif and A. S. A. Noor, A generalization of Stone's representation theorem, *The Rajshahi University Studies (Part B)* 31(2003) 83-87.
6. A. S. A. Noor and M. A. Latif, Finitely generated n-ideals of a lattice, *SEA Bull. Math.,* 22 (1998) 72-79.
7. Y. S. Powar and N. K. Thakare, 0-Distributive semilattices, *Canad. Math. Bull.* 21(4) (1978) 469-475.
8. Y. Rav, Semi prime ideals in general lattices, *Journal of Pure and Applied Algebra,* 56 (1989) 105-118.
9. J. C. Varlet, A generalization of the notion of pseudo-complementedness, *Bull. Soc. Sci. Liege*, 37(1968) 149-158.