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# A Numerical Study on Radiation-conduction Interaction with Steady Streamwise Surface Temperature Variations Over a Vertical Cone

***Md. Kutub Uddin[1], Rabindra Nath Mondal[2*], Md. Sharif Uddin[3] and Sanjit Kumar Paul[4]***

[1]Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh
[2]Department of Mathematics, Jagannath University, Dhaka-1100, Bangladesh
[3]Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh
[4]Department of Basic Sciences and Humanities, University of Asia Pacific, Dhaka-1209, Bangladesh
[*]Email: rnmondal71@yahoo.com

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**Abstract.** Radiation-conduction interaction with steady streamwise surface temperature variation over a vertical cone has been analyzed by using finite difference method. The surface rate of heat transfer eventually alternates in sign with distance from the leading edge, but no separation occurs unless the amplitude of the thermal modulation is sufficiently high. Numerical results are obtained for different values of the physical parameters, the radiation parameter $R_d$, Prandtl number Pr and the surface temperature wave amplitude $a$. It is found that as Pr decreases, the skin friction increases and at $\mathrm{Pr} = 0.01$, the wave amplitude becomes higher than that at Pr = 7.0. It is also found that the rate of heat transfer increases as $R_d$ increases but at a decreasing rate, i.e. when $R_d$ =1 the skin friction and the rate of heat transfer increase significantly in respect of $R_d$ =10. We have also found that as the surface temperature wave amplitude increases, the rate of heat transfer also increases and for decreasing of the wave amplitude it decreases gradually.

***Keywords:*** radiation, skin friction, surface temperature, wave amplitude, heat transfer

***AMS Mathematics Subject Classification (2010):*** 76Dxx, 76D99, 76E09

## 1. Introduction

Radiative convective flows are encountered in many industrial and environmental processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Mathematically the equations for radiative heat transfer with absorption, scattering and emission can be generated by one of two approaches, namely the continuum model or the spectral radiative treatment of a single particle. Details of the derivation of the general equation of radiative heat transfer are provided in the classic monograph by Chandrasekhar (1960).

Little is currently known about the boundary layer flows of radiating fluids. The inclusion of conduction-radiation effects in the energy equation, however, leads to a more highly nonlinear partial differential equation. The majority of studies concerned with the interaction of thermal radiation and natural convection were made by Sparrow and Cess (1962), Arpaci (1972), Cheng and Ozisik (1972), Hasegawa et al.(1972), and Bankston et al. (1977) for the case of a vertical semi-infinite plate. Soundalgekar and Takhar (1993) studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using Cogley-Vincentine-Giles equilibrium model and Hossain and Takhar (1996) analyzed the effect of radiation using the Rosseland diffusion approximation which leads to a nonsimilar mixed convective boundary-layer flow of an optically dense viscous incompressible fluid past a heated vertical plate with a uniform free stream velocity and surface temperature. The boundary layer equations were obtained using a group of transformations and they are valid in both the forced convective and free convective limits. The resulting equations were solved using an implicit finite difference method. The problem of natural convection-radiation interaction on boundary layer flows with the Rosseland diffusion approximation has been studied by Hossain and Alim (1997) and Hossain et al. (1998). Hossain and Rees (1998) investigated the effect of radiation-conduction interaction in the mixed convective flow along a slender impermeable vertical cylinder. Kutubuddin, Hossain and Pop (1999a, 1999b) analyzed the effect of conduction-radiation interaction on the forced, free and mixed convection flow from a horizontal cylinder. Various papers have been published which deal with the effects of surface variations; for example, Yao (1983) and Moulic and Yao (1989a, 1989b) have sought to investigate the effects of streamwise surface undulations of free and mixed convection from vertical surfaces held at uniform temperatures. Rees (1999) has been concerned with the effect of sinusoidal surface temperature variations, although in that case the surface variations were spanwise, thereby giving rise to a three-dimensional flow-field.

A significant number of authors have investigated laminar free convection for two-dimensional axisymmetric flows. Merk and Prins (1953, 1954) developed the general relations for similar solutions on isothermal axisymmetric forms and showed that the vertical cone has such a solution. Approximate boundary layer techniques were utilized to arrive at an expression for the dimensionless heat transfer. Broun et al. (1961) contributed two more isothermal axisymmetric bodies for which similar solutions exist, and used an integral method to provide heat transfer results for these and the cone over a wide range of Prandtl number. Similarity solutions for free convection from the vertical cone have been exhausted by Hering and Grosh (1962). They showed that the similarity solutions to the boundary layer equations for a cone exist when the wall temperature distribution is a power function of distance along a cone ray. In their investigation, they presented the results by numerical integration of the transformed equations for non-isothermal temperature distributions for Prandtl number equals to 0.7. Latter, Hering (1965) extended the analysis to investigate for low Prandtl numbers. In the present paper, we have investigated the combined effects of surface temperature variations and radiation on the steady boundary-layer flow of a Newtonian fluid from a heated vertical cone. It is well known that power-law surface temperature distributions (and also power-law surface heat fluxes) give rise to self-similar boundary layer flows (Ostrach, 1952; Sparrow and Gregg, 1958). But here we are interested in another form of surface variation, namely, sinusoidal variations about a mean temperature, which is held above the ambient temperature of the fluid. As in Rees (1999), this type of surface distribution may be taken as a simplified model of the effects of a periodical array of heaters behind or within the heated surface. An accurate analysis of such a configuration requires a detailed examination of the effects of solid conduction within the heated surface, but the aim of the present work is to simplify the problem by imposing a surface temperature distribution. In this way, we can determine a large amount of information about the resulting flow using both numerical methods.

## 2. Mathematical formulations

A steady two-dimensional laminar free convection flow of the boundary layer induced by a heated semi-infinite surface immersed in an incompressible Newtonian fluid is considered. In particular, the heated surface is maintained at the steady temperature, fluid having temperature, $T$, from a vertical cone. The physical coordinates ($x$, $y$) are chosen such that $x$ is measured from the leading edge in the stream wise direction and $y$ is measured normal to the surface of the cone. The coordinate system, velocity direction and the gravity orientation are shown in Figure 1.

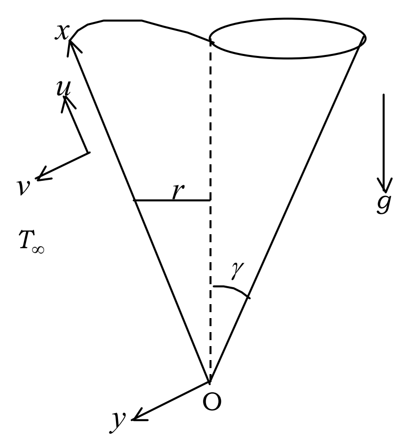


**Figure 1**: Physical model and the co-ordinate system

The boundary layer form of the equations for flow is

$$T = T_{\infty} + \left(T_W - T_{\infty}\right)\left(1 - a\sin(\pi \hat{x} d)\right) \qquad (1)$$

where $T_{\infty}$ is the ambient fluid temperature, $T_w$ is the mean-surface temperature such that $T_w > T_{\infty}$, a is the relative amplitude of the surface temperature variations and 2d is the wavelength of the variations. After a suitable non-dimensionalisation the steady two-dimensional equations of motion are given by

$$\frac{\partial(\hat{r}\hat{u})}{\partial \hat{x}} + \frac{\partial(\hat{r}\hat{v})}{\partial \hat{y}} = 0 \qquad (2)$$

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{x}} + \nu\nabla^2 \hat{u} + g\beta(T - T_{\infty})\cos\varphi \qquad (3)$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \alpha\left[\theta_{yy} + \frac{16\sigma}{3\kappa(a+\sigma_s)}\left\{\theta^3\theta_y\right\}_y\right] \qquad (4)$$

with $\hat{r} = \hat{x}\sin\phi$.

The boundary conditions are:

$$\begin{aligned} &\hat{u} = 0, \quad \hat{v} = 0, \quad T = T_{\infty} + (T_w - T_{\infty})(1 + a\sin\pi \tfrac{\hat{x}}{L}) \text{ at } \hat{y} = 0 \\ &\hat{u} = 0, \quad T = T_{\infty} \text{ as } \hat{y} \to \infty \end{aligned} \qquad (5)$$

In the derivation of equation (2) the Boussinesq approximation has been assumed. We note that the Grashof number *Gr* has been based on *d*, half the dimensional wavelength of the thermal waves. In the equations, *u* and *v* are, respectively, the velocity components in the *x* and *y* directions respectively, *T* is the fluid temperature, $\nu$ is the kinematic viscosity, $\beta$ is the thermal expansion coefficient, $\alpha$ is the thermal diffusivity, $\kappa$ is the thermal conductivity, *a* is the Rosseland mean absorption coefficient, $\sigma$ is the Stephan-Boltzman constant, $\sigma_s$ is the scattering coefficient. When the surface temperature is uniform and the Grashof number is very large, the resulting boundary-layer flow is self-similar. But the presence of sinusoidal surface temperature distributions, such as that given by Eq. (1), renders the boundary-layer flow non-similar.

The boundary-layer equations are obtained by introducing the scaling

$$u = \frac{L}{\nu}Gr^{-\frac{1}{2}}\hat{u}, \quad v = \frac{L}{\nu}Gr^{-\frac{1}{4}}\hat{v}, \quad x = \frac{\hat{x}}{L}, \qquad (6)$$

$$y = \frac{\hat{y}}{L} Gr^{\frac{1}{4}}, \quad p = \frac{L^2}{\rho \nu^2} Gr^{-1} \hat{p}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

into equation (2), formally letting *Gr* become asymptotically large and retaining only the leading order terms. Thus we obtain

$$u_x + v_y = 0 \tag{7}$$

$$uu_x + vu_y = u_{yy} + \theta \tag{8}$$

$$uv_x + vv_y = -p_y + v_{yy} \tag{9}$$

$$u\theta_x + v\theta_y = \alpha Gr^{-1/2}\left[\theta_{yy} + \frac{16\sigma}{3\kappa(a+\sigma_s)}\left\{\theta^3\theta_y\right\}_y\right] \tag{10}$$

where the asterisk superscripts have been omitted for clarity of presentation. Equation (9) serves to define the pressure field in terms of the two velocity components and is decoupled from the other three equations. Therefore, we shall not consider it further. As the equations are two-dimensional we define a stream function $\psi$, in the usual way.

$$u = \frac{1}{r}\psi_y, \quad v = -\frac{1}{r}\psi_x \tag{11}$$

and therefore, Eq. (7) is satisfied automatically. Guided by the familiar self-similar form corresponding to a uniform surface temperature, we use the substitution

$$\psi = x^{3/4} r f(\eta, x), \quad \theta = g(\eta, x), \quad r = x \sin\phi \tag{12}$$

where

$$\eta = y / x^{1/4} \tag{13}$$

is the pseudo-similarity variable. Equations (8) and (9) reduce to

$$f''' + g + \tfrac{7}{4} ff'' - \tfrac{1}{2} f'f' + x(f_x f'' - f_x f') = 0 \tag{14}$$

$$\frac{1}{\Pr}\left[\left\{1 + \frac{4}{3} R_d \left(1 + (\theta_w - 1)g\right)^3\right\} g'\right]' + \tfrac{7}{4} fg' + x(f_x g' - f'g_x) = 0 \tag{15}$$

where $P_r$ and $R_d$ are respectively the Prandtl number and radiation parameter, which are defined as

$$\Pr = \frac{\nu}{\kappa}, \quad R_d = \frac{4}{3}\frac{\sigma T_\infty{}^3}{k(a_R + \sigma_s)}, \quad \theta_w = \frac{T_w}{T_\infty}.$$

The boundary conditions are

$$f = 0, \quad f' = 0, \quad g = 1 + a\sin \pi x \quad \text{at } \eta = 0 \text{ and } f'g \to 0 \text{ as } \eta \to \infty. \tag{16}$$

In equations (14)-(16), primes denote derivatives with respect to $\eta$.

**3. Numerical solutions**

The parabolic system of equations (14)-(15) together with boundary conditions (16) is non-similar and its numerical solution must be obtained using a marching method. The results presented here were obtained by using the Keller-box method. After reducing the equations (14) and (15) to first-order form in $\eta$, the subsequent second-order accurate discretisation based halfway between the grid points in both the $\eta$- and $x$-directions yields a set of nonlinear difference equations which are solved using a multi-dimensional Newton-Raphson iteration scheme. The results presented in Figure 2 to Figure 15 are based on uniform grids in both coordinate directions. There were 101 grid-points lying between $\eta$ = 0 and $\eta$ = 10 and 201 between $x$ = 0 and $x$ = 10. We restrict the presentation of our results to the three values of the Prandtl number, Pr = 0.01 (liquid metal) Pr = 0.7 (air) and Pr = 7.0 (water).

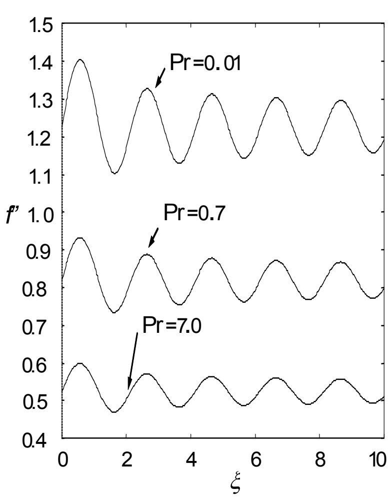


**Figure 2**: Skin friction against ξ for $R_d$ = 0.0, a = 0.2 and Pr = 7.0, 0.7, 0.01.

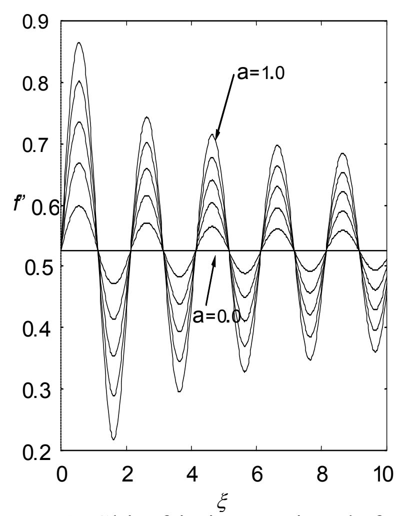


**Figure 3**: Skin friction against ξ for $R_d$ = 0.0, a = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 at Pr = 7.0

Figure 2 shows the evolution of $f''(\eta = 0)$ with $x$, a scaled surface shear stress, for constant values of the temperature wave amplitude, $a$, and in absence of the radiation parameter $R_d$ for various values of Pr. In this figure, we observe that as

Pr is decreasing, the skin friction is increasing. One point may be mentioned here that when Pr = 0.01, the wave amplitude is higher than that at Pr = 7.0. It is found that as $x$ increases, the amplitude of oscillation of the shear stress curves decays slowly. In Figure 3- Figure 5, we show the results of the evolution with $x$ of surface shear stress for various values of the temperature wave amplitude, $a$, and the constant radiation parameter $R_d$ for different values of Pr. In these figures, we observe that as Pr is decreasing, the skin friction is increasing. We also observe that as the surface temperature wave amplitude is increasing, shear stress is also increasing and for decreasing of the wave amplitude it is decreasing gradually. Some aspects of the overall behavior of these curves may be explained by observing that the boundary layer is thinner when the surface temperature is relatively high and thicker when it is low. Thus, we should expect high shear stresses and rates of heat transfer at, or perhaps just beyond, where the surface temperature attains its maximum values. There is an obvious qualitative difference between the curves shown in Figure 2 and those in Figure 5. It is found that as $x$ increases, the amplitude of oscillation of the shear stress curves decays slowly with $x$.

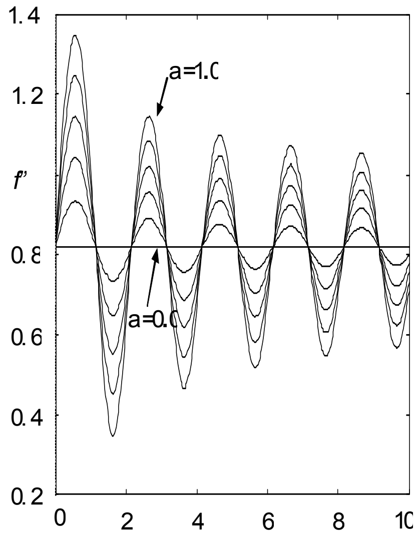


**Figure 4**: Skin friction against ξ for $R_d$ = 0.0, a = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 at Pr = 0.7

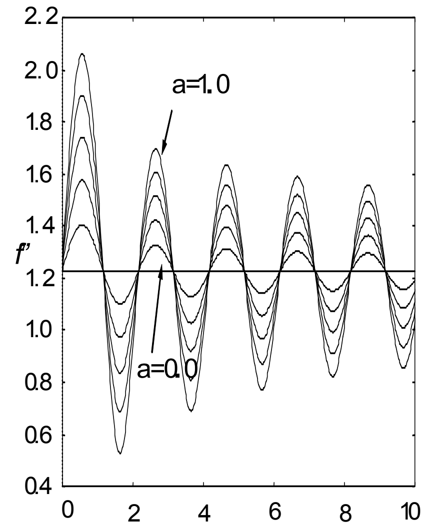


**Figure 5**: Skin friction against ξ for $R_d$ = 0.0, a = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 at Pr = 0.01

Now we pay our attention to the figures presented in Figure 6 to Figure 8, where the evolution with $x$ of surface shear stress for constant values of the temperature wave amplitude, $a$, and the various radiation parameter $R_d$ for different values of Pr are shown. An interesting finding of the present study is that, when the radiation parameter $R_d$ is increasing, the skin friction is also increasing but when $R_d$ = 0, the result of skin friction is exactly the same as found by Rees (1999). In this study, we found that the skin friction is increasing as $R_d$ is increasing but at a decreasing rate. That is, when $R_d$ = 1.0 then skin friction increases significantly in respect of $R_d$ =10.

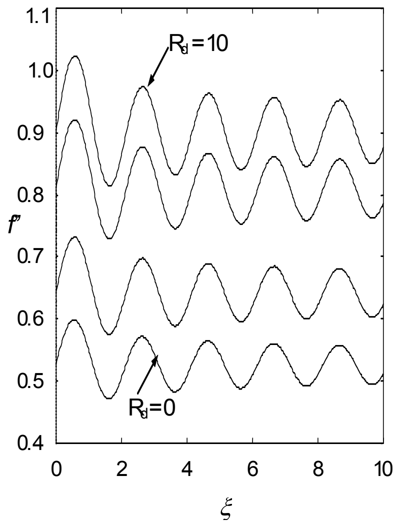


**Figure 6**: Skin friction against ξ for $R_d$ = 0.0,1.0, 5.0, 10.0 and a = 0.2 at Pr = 7.0

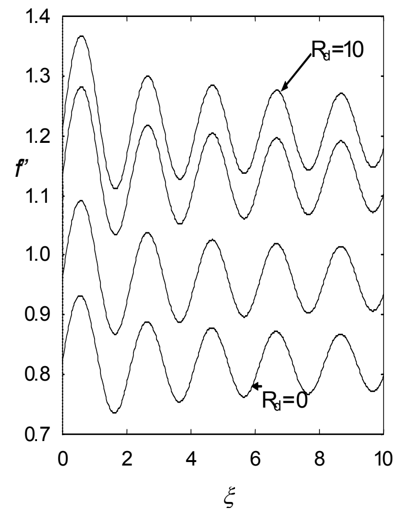


**Figure 7**: Skin friction against ξ for $R_d$ = 0.0, 1.0, 5.0, 10.0 and a = 0.2, at Pr = 0.7

Now we analyze the curves represented in Figure 9 to Figure 15. Figure 9 shows the evolution with *x* of surface rate of heat transfer for constant values of the temperature wave amplitude, *a*, and the constant radiation parameter $R_d$ for various values of Pr. In this figure, we observe that as Pr decreases, the rate of heat transfer increases. It should be mentioned here that when Pr = 7.0, the wave amplitude is higher than that at Pr = 0.01. It is found that as *x* increases, the amplitude of oscillation of the rate of heat transfer increases significantly.

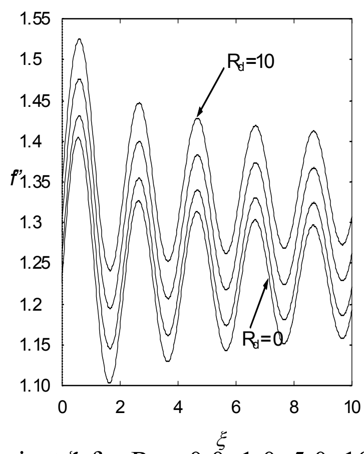


**Figure 8**: Skin friction against ξ for $R_d$ = 0.0, 1.0, 5.0, 10.0 a = 0.2 at Pr = 0.01.

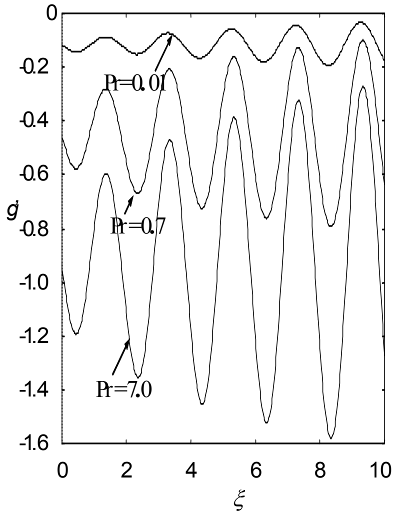


**Figure 9**: Rate of heat transfer against ξ for $R_d = 0.0$, a = 0.2, Pr = 7.0, 0.7, 0.01

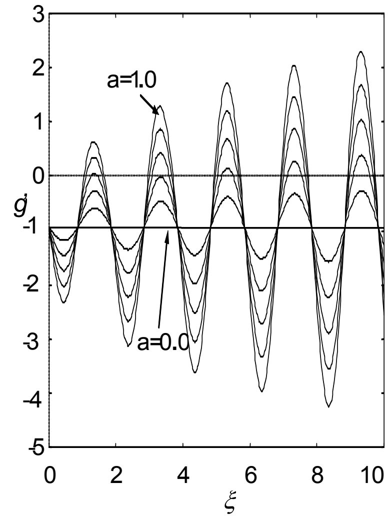


**Figure 10**: Rate of heat transfer against ξ for $R_d = 0.0$, a = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 at Pr = 7.0

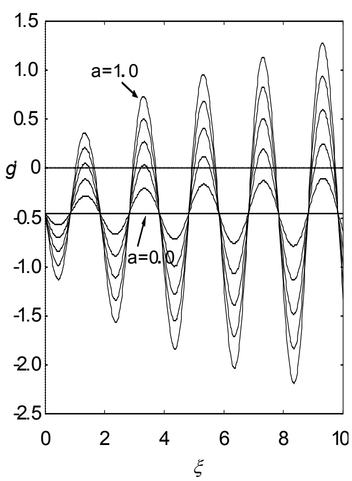


**Figure 11**: Rate of heat transfer against ξ for $R_d = 0.0$, a = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 at Pr = 0.7

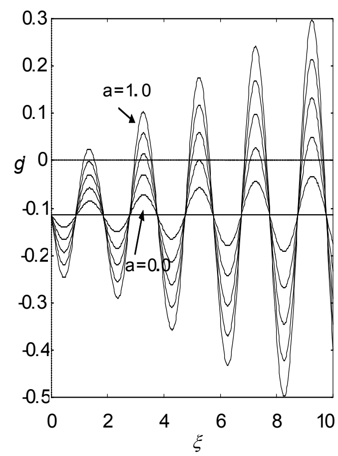


**Figure 12**: Rate of heat transfer against ξ for $R_d = 0.0$, a = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 at Pr = 0.01

In Figure 10 to Figure 12, we observe the evolution with *x* of surface rate of heat transfer for various values of the temperature wave amplitude, *a*, and the constant radiation parameter $R_d$ for different values of Pr. In these figures, we observe that as Pr is decreasing, the rate of heat transfer is increasing. We also observe that as the surface temperature wave amplitude is increasing, the rate of heat transfer is also increasing, and for decreasing the wave amplitude it is decreasing gradually. An important aspect of the overall behavior of these curves may be explained by the fact that the boundary layer is thinner when the surface temperature is relatively high and thicker when it is low. This arises because relatively high surface temperatures induce relatively large upward fluid velocities with the consequent increase in the rate of entertainment into the boundary layer. There is an obvious qualitative difference among the curves shown in Figure 9 to Figure 12. It is found that as *x* increases, the amplitude of oscillation of the rate of heat transfer curves increases gradually with *x*. Indeed, the curves in Figure 10 to Figure 12, we may suggest that, whatever the value of *a* is, there will always be a value of *x* beyond which some part of the rate of the heat transfer curve between successive surface temperature maxima will be positive. This somewhat unusual phenomenon for boundary layer flows which may be explained by noting that when relatively hot fluid encounters a relatively cold part of the heated surface, the overall heat transfer will be from the fluid into the surface rather than the other way around.

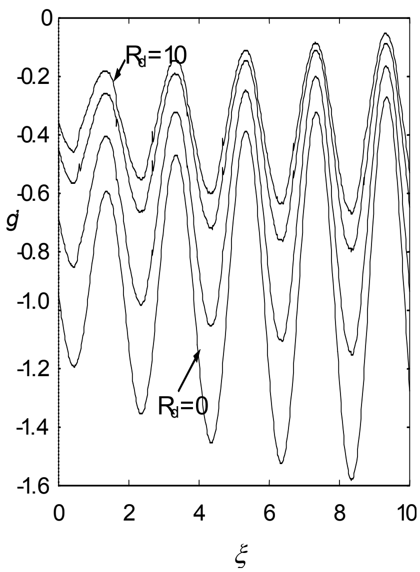


**Figure 13**: Rate of heat transfer against $\xi$ for $R_d$ = 0.0, 1.0, 5.0, 10.0, a = 0.2 at Pr = 7.0

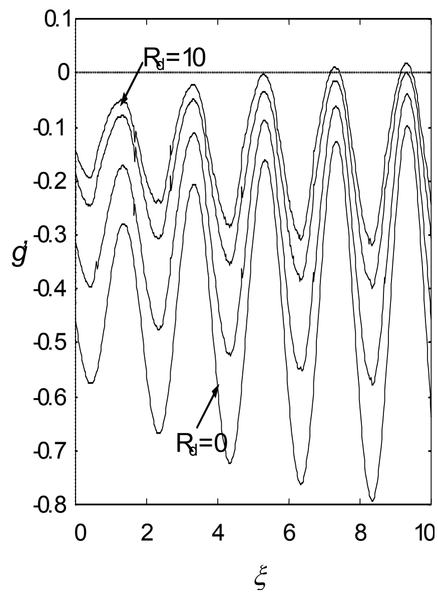


**Figure 14**: Rate of heat transfer against $\xi$ for $R_d$ = 0.0, 1.0, 5.0, 10.0, a = 0.2 at Pr = 0.7

Figure 13 to Figure 15 show the evolution with *x* of surface rate of heat transfer for constant values of the temperature wave amplitude, *a*, and the various radiation parameter $R_d$ for different values of Pr. The most interesting part of this analysis is that, when radiation parameter $R_d$ is increasing, the rate of heat transfer is also increasing but when $R_d = 0$, the result of rate of heat transfer is exactly the same as obtained by Rees (1999). In this study, we found that the rate of heat transfer is increasing as $R_d$ is increasing but at a decreasing rate i.e. when $R_d$ =1 the rate of heat transfer increases more in respect of $R_d$ =10.

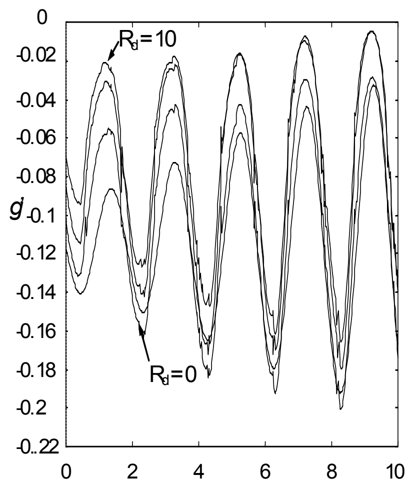


**Figure 15**: Rate of heat transfer against $\xi$ for $R_d = 0.0, 1.0, 5.0, 10.0$, $a = 0.2$ at $Pr = 0.01$.

**4. Conclusions**

A numerical study on the effect of radiation-conduction interaction with steady streamwise surface temperature variation over a vertical cone has been investigated numerically by using a finite difference method. The effect of variations in the Plank number, the surface temperature wave amplitude, and the Prandtl number on the shear stress and rate of surface heat transfer have been presented graphically. We restrict the presentation of our results for three values of the Prandtl number, $\mathrm{Pr} = 0.01$ (Liquid Metal) Pr = 0.7 (air) and Pr. = 7.0 (water). In this study, we observed that as Pr decreases, the skin friction increases. It is found that when $\mathrm{Pr} = 0.01$, the wave amplitude becomes higher than that at Pr = 7.0, and as $x$ increases, the amplitude of oscillation of the shear stress curves decays slowly. An interesting finding is that, when the radiation parameter $R_d$ increases, the skin friction also increases but when $R_d = 0$, the skin friction is exactly the same as that obtained by Rees (1999). In the present study, however, we have found that the skin friction increases as $R_d$ increases but at a decreasing rate, i.e. when $R_d = 1$ the skin friction increases more in respect of $R_d = 10$. We observe that as Pr decreases, the rate of heat transfer increases and at Pr = 7.0, the wave amplitude becomes higher than that at Pr = 0.01. It is found that as $x$ increases, the amplitude of oscillation of the rate of heat transfer is increasing significantly. For the case of surface rate of heat transfer, for various values of the temperature wave amplitude, $a$, and the constant radiation parameter $R_d$ for different values of Pr, we observe that as Pr decreases, the rate of heat transfer increases. We also observe that as the surface temperature wave amplitude increases, the rate of heat transfer also increases, and for decreasing of wave amplitude it decreases gradually. It is found that the rate of heat transfer increases as $R_d$ increases but at a decreasing rate and when $R_d = 1$, the rate of heat transfer increases significantly in respect of $R_d = 10$.

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