Annals of Pure and Applied Mathematics
Vol. 2, No. 2, 2012, 185-193
ISSN: 2279-087X (P), 2279-0888(online)
Published on 28 December 2012
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Some Properties of 0-distributive Nearlattice

Md. Zaidur Rahman[1], Md. Bazlar Rahman[1] and A.S.A.Noor[2]

[1]Department of Mathematics
Khulna University of Engineering and Technology
E mail: mzrahman1968@gmail.com

[2]Department of ECE
East West University, Dhaka
E mail: noor@ewubd.edu

Received 3 December 2012; accepted 21 December 2012

Abstract. In this paper we studied different properties of 0-distributive nearlattice. Here we prove that for a filter A of S, $A^0 = \{x \in S \mid x \wedge a = 0, \text{ for some } a \in A\}$ is an ideal if and only if S is 0-distributive. Then we include several characterizations of 0-distributive nearlattice using A^0 where A is a filter. Finally we show that S is 0-distributive if and only if for all $a, b, c \in S$,

$$(a \wedge (b \vee c))^{\perp} = (a \wedge b)^{\perp} \cap (a \wedge c)^{\perp} \text{ provided } b \vee c \text{ exists.}$$

***Keywords**:* 0-distributive nearlattice, Maximal filter, Prime filter, Minimal prime down set, Minimal prime ideal.

***AMS Mathematics Subject Classification (2010)**:* 06A12, 06A99, 06B10

1. Introduction

J.C. Varlet [5] has given the definition of a 0-distributive lattice to generalize the notion of pseudocomplemented lattice. By [5], a lattice with 0 is called a 0-distributive lattice if for all $a, b, c \in L$ with $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Then many authors including [1] and [4] studied the 0-distributive properties in lattices and meet semilattices. Recently [6] have studied the 0-distribuitve property in a nearlattice.

A *nearlattice* is a meet semilattice together with the property that any two elements possessing a common upper bound have a supremum. This property is known as the *upper bound property.*

A nearlattice S is called *distributive* if for all $x, y, z \in S$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, provided $y \vee z$ exists in S. For detailed literature on nearlattices, we refer the reader to consult [2] and [3].

A nearlattice S with 0 is called 0-distributive if for all $x, y, z \in S$ with $x \wedge y = 0 = x \wedge z$ and $y \vee z$ exists imply $x \wedge (y \vee z) = 0$.

It can be easily proved that it has the following alternative definition:
S is 0-distributive if for all $x, y, z, t \in S$ with $x \wedge y = 0 = x \wedge z$ imply $x \wedge ((t \wedge y) \vee (t \wedge z)) = 0$; $(t \wedge y) \vee (t \wedge z)$ exists by the upper bound property of S. Of course, every distributive nearlattice S with 0 is 0-distributive. Figure 1 is an example of a non-modular nearlattice which is 0-distributive, while Figure 2 gives a modular nearlattice which is not 0-distributive.

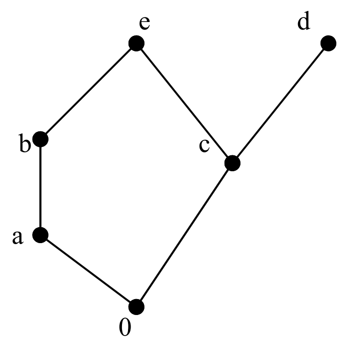

Figure 1

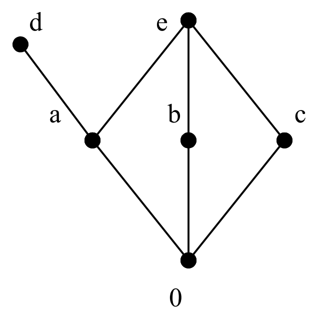

Figure 2

A subset I of a nearlattice S is called a *down set* if $x \in I$ and for $t \in S$ with $t \leq x$ imply $t \in I$. An *ideal(down set)* I in a nearlattice S is a non-empty subset of S such that it is a down set and whenever $a \vee b$ exists for $a, b \in I$, then $a \vee b \in I$. A proper ideal I in S is called a *prime ideal(down set)* if $a \wedge b \in I$ implies that either $a \in I$ or $b \in I$. A non-empty subset F of S is called a filter if $t \geq a$, $a \in F$ implies $t \in F$ and if a, $b \in F$ then $a \wedge b \in F$. A proper filter F in S is called *prime* if $a \vee b$ exists and $a \vee b \in F$ implies either $a \in F$ or $b \in F$. In lemma 1, we prove that F is a filter of S if and only if S-F is a prime down set. Moreover, it is easy to shown that a prime down set P is a prime ideal if and only if $S - P$ is a prime filter.

A proper filter M of a nearlattice S is called *maximal* if for any filter Q with $Q \supseteq M$ implies either $Q = M$ or $Q = S$. Dually, we define a *minimal prime ideal (down set)*

Let L be a lattice with 0. An element a^* is called the *pseudocomplement* of a if $a \wedge a^* = 0$ and if $a \wedge x = 0$ for some $x \in L$, then $x \leq a^*$. A lattice L with 0 and 1 is called *pseudocomplemented* if its every element has a pseudocomplement. Since a nearlattice S with 1 is a lattice, so pseudocomplementation is not possible in a general nearlattice. A nearlattice S with 0 is called sectionally pseudocomplemented if the interval $[0,x]$ for each $x \in S$ is pseudocomplemented. For $A \subseteq S$, we denote $A^{\perp} = \{x \in S \mid x \wedge a = 0 \text{ for all } a \in A\}$. If S is distributive then clearly $A^{\perp}$ is an ideal of S.

Moreover, $A^{\perp} = \bigcap_{a \in A} \{\{a\}^{\perp}\}$. If A is an ideal, then obviously $A^{\perp}$ is the pseudocomplement of A in $I(S)$. Therefore, for a distributive nearlattice S with 0, I(S) is pseudocomplemented.

2. Some Results

Lemma 1. In a nearlattice S, F is a proper filter if and only if is a prime down set.

Proof: Let F be a proper filter. Let $x \in S - F$ and $t \leq x$. Then $x \notin F$ and so $t \notin F$ as F is a filter. Hence $t \in S - F$ and so $S - F$ is a down set.

Now let $a \wedge b \in S - F$ for some $a, b \in S$. Then $a \wedge b \notin F$. This implies either $a \notin F$ or $b \notin F$ and so either $a \in S - F$ or $b \in S - F$. Therefore $S - F$ is prime.

Conversely, suppose $S - F$ is a prime down set. Let $a \in F$ and $t \geq a$ $(t \in S)$. Then $a \notin S - F$ and so $t \notin S - F$ as it is a down set. Thus $t \in F$ and so F is an up set. Now let $a, b \in F$, then $a \notin S - F$ and $b \notin S - F$. Since $S - F$ is prime, so $a \wedge b \notin S - F$. This implies $a \wedge b \in F$, and so F is a filter. ●

Lemma 2. Every proper filter of a nearlattice with 0 is contained in a maximal filter.

Proof. Let F be a proper filter in S with 0.Let $\mathcal{F}$ be the set of all proper filters containing F. Then $\mathcal{F}$ is non-empty as $F \in \mathcal{F}$. Let C be a chain in $\mathcal{F}$ and let $M = \bigcup\{X \mid X \in C\}$. We claim that M is a filter with $F \subseteq M$. Let $x \in M$ and $y \geq x$. Then $x \in X$ for some $X \in C$. Hence $y \in X$ as X is a filter. Therefore, $y \in M$. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since C is a chain, either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. So $x, y \in Y$. Then $x \wedge y \in Y$ as Y is a filter. Hence $x \wedge y \in M$. Moreover M contains F. So M is maximum element of C. Then by Zorn's lemma $\mathcal{F}$ has a maximal element, say Q with $F \subseteq Q$.●

Following result trivially follows from lemma 2 and lemma 1

Corollary 3. Every prime down set of a nearlattice contains a minimal prime down set.●

Theorem 4. Let S be a nearlattice with 0. Then the following conditions are equivalent.

(i) S is 0-distributive.

(ii) If A is a non-empty subset of S and B is a proper filter intersecting A, then there is a minimal prime ideal containing $A^{\perp}$ and disjoint from B.

(iii) For each non-zero element $a \in S$ and each proper filter B containing a, there is a prime ideal containing $\{a\}^{\perp}$ and disjoint from B.

(iv) For each non-zero element $a \in S$ and each proper filter B containing a, there is a prime filter containing B and disjoint from $\{a\}^{\perp}$.

(v) For each non-zero element $a \in S$ and each prime down set B not containing a, there is a prime filter containing $S - B$ and disjoint from $\{a\}^{\perp}$.

Proof. $(i) \Rightarrow (ii)$ Suppose (i) holds. Let A be a non-empty subset of S and B is a proper filter such that $B \cap A \neq \phi$. By Lemma 1, $S - B$ is a prime down set and so by corollary 3, $S - B$ contains a minimal prime down set N. Clearly $N \cap B = \phi$. Also $S - B \not\supseteq A$ and so $N \not\supseteq A$. Then there exists $p \in A$ such that $p \notin N$. Now suppose $x \in A^{\perp}$. Then $x \wedge a = 0$ for all $a \in A$. Thus $x \wedge p = 0 \in N$. Since N is prime and $p \notin N$, so $x \in N$ and so $A^{\perp} \subset N$. Since S is 0-distributive so by [6, theorem 9], N is a minimal ideal.

$(ii) \Rightarrow (iii)$. Suppose (ii) holds. Now $\{a\} \subset S$ and suppose B is a proper filter containing a. Then $B \cap \{a\} \neq \phi$. Thus by (ii) there is a prime ideal containing $\{a\}^{\perp}$ and disjoint from B.

$(iii) \Rightarrow (iv)$ This is trivial as P is an ideal of a nearlattice S if and only if $S - P$ is a prime filter.

$(iv) \Rightarrow (v)$ This is trivial by lemma 1.

$(v) \Rightarrow (i)$ Suppose (v) holds and a be a non-zero element of S. Then by lemma 1, $B = S - [a)$ is a prime down set and $a \notin B$. Then by (v) there is a prime filter Q containing $S - B$ and disjoint from $\{a\}^{\perp}$. Then $a \in Q$ and so by [6, theorem 9], S is 0-distributive.●

For a subset A of a nearlattice S, we define
$\mathrm{A}^0 = \{\mathrm{x} \in \mathrm{S} \mid \mathrm{x} \wedge \mathrm{a} = 0 \text{ for some } \mathrm{a} \in \mathrm{A}\}$. It is easy to see that A^0 is a down set. Moreover, $\{\mathrm{a}\}^0 = \{\mathrm{a}\}^{\perp}$. Now we have the following result.

Theorem 5. Let S be a nearlattice with 0. Then the following conditions are equivalent.

(i) S is 0-distributive.

(ii) A^0 is an ideal for every filter A of S.

(iii) $\{a\}^0$ is an ideal.

Proof. $(i) \Rightarrow (ii)$ Let S be a 0-distributive. We already know that A^0 is a down set. Now let $x, y \in A^0$ and $x \vee y$ exists. Then $x \wedge a = 0 = y \wedge b$ for some $a, b \in A$. Hence $x \wedge a \wedge b = 0 = y \wedge a \wedge b$, $a \wedge b \in A$ as it is a filter. Thus $a \wedge b \wedge (x \vee y) = 0$ as S is 0-distributive. Therefore $x \vee y \in A^0$ and so A^0 is an ideal.

$(ii) \Rightarrow (iii)$ This is trivial as $\{a\}^0 = \{a\}^\perp$.

$(iii) \Rightarrow (i)$ Suppose (ii) holds. Let $a, b, c \in S$ with $a \wedge b = 0 = a \wedge c$ and $b \vee c$ exists. Consider $[a)$. Then $b, c \in [a)^0$. Since $[a)^0$ is an ideal so $b \vee c \in [a)^0$. Thus $a \wedge (b \vee c) = 0$, and hence S is 0-distributive.●

Lemma 6. Let A and B be filter of a nearlattice S with 0 such that $A \cap B^0 = \phi$. Then there is a minimal prime down set N containing B^0 and $N \cap A = \phi$.

Proof. Since $A \cap B^0 = \phi$, so $0 \notin A \vee B$. So $A \vee B$ is a proper filter of S. Then by lemma 2, $A \vee B \subseteq M$ for some maximal filter. Now $B \subseteq M$ and consequently $M \cap B^0 = \phi$. By lemma 1, $S - M = N$ is a minimal prime down set. Clearly $B^0 \subseteq N$ and $N \cap A = \phi$.●

Theorem 7. Let S be a nearlattice with 0. Then the following conditions are equivalent.

(i) S is 0-distributive.

(ii) If A and B are filter of S such that A and B^0 are disjoint, there is a minimal prime ideal containing B^0 and disjoint from A.

(iii) If A is a filter of S and B is a prime down set containing A^0, there is a minimal prime ideal containing A^0 and contained in B.

(iv) If A is a filter of S and B is a prime down set containing A^0, there is a prime filter containing and disjoint from A^0.

(v) For each non-zero element $a \in S$ and each prime down set B containing $\{a\}^0$, there is a prime filter containing $S - B$ and disjoint from $\{a\}^0$.

Proof. $(i) \Rightarrow (ii)$ Suppose (i) holds. Let A and B be filter of S such that $A \cap B^0 = \phi$. Then by lemma 2, there is a minimal prime down set N such that $N \supseteq B^0$ and $N \cap A = \phi$. Since S is 0-distributive it follows from [6,theorem 9] that N is a minimal prime ideal.

$(ii) \Rightarrow (iii)$ Suppose (ii) holds. Let A be a filter of S and B is a prime down set containing A^0. Then by lemma 1, $S - B$ is a filter such that $(S - B) \cap A^0 = \phi$. Then by (ii) there is a minimal prime ideal containing A^0 and disjoint from $S - B$, that is contained in B.

$(iii) \Rightarrow (iv)$ This is trivial by lemma 1.

$(iv) \Rightarrow (v)$ Let a be a non-zero element of S and B be a prime down set containing $\{a\}^0$. Let $A = [a)$. Then $B \supset \{a\}^0 = [a)^0 = A^0$. Then by (iv), there is a prime filter containing $S - B$ and disjoint from $\{a\}^0$.

$(v) \Rightarrow (i)$ Suppose (v) holds and let a be any non-zero element of S. By lemma 1, $S - [a)$ is a prime down set not containing a. since $(a] \cap \{a\}^0 = (0] \subset S - [a)$, it follows that $S - [a)$ contains $\{a\}^0$ as $S - [a)$ is prime. Then by (v) , there is a prime filter B containing $[a) = S - (S - [a))$ and disjoint from $\{a\}^0$. Clearly $a \in B$. Hence by [6, theorem 9], S is 0-distributive. ●

We conclude the paper with the following characterizations of 0-distributive nearlattices. To prove this we need the following lemma.

Lemma 8. Let S be a nearlattice with 0. Suppose $A, B \in I(S)$ and $a, b \in S$, then we have the following:

(i) If $A \cap B = (0]$, then $B \subseteq A^{\perp}$.

(ii) $A \cap A^{\perp} = (0]$.

(iii) $a \le b$ implies $\{b\}^{\perp} \subseteq \{a\}^{\perp}$ and $\{a\}^{\perp\perp} \subseteq \{b\}^{\perp\perp}$.

(iv) $\{a\}^{\perp} \cap \{a\}^{\perp\perp} = (0]$

(v) $\{a \wedge b\}^{\perp\perp} = \{a\}^{\perp\perp} \cap \{b\}^{\perp\perp}$.

(vi) $\{a\} \subseteq \{a\}^{\perp\perp}$.

(vii) $\{a\}^{\perp} = \{a\}^{\perp\perp\perp}$.

(viii) If $a \vee b$ exists and S is 0-distributive, then $\{a \vee b\}^{\perp} = \{a\}^{\perp} \cap \{b\}^{\perp}$.

Proof. (i) Let $b \in B$. Then $a \wedge b = 0$ for all $a \in A$ as $A \cap B = (0]$. Therefore, $b \in A^{\perp}$ and so $B \subseteq A^{\perp}$.

(ii) Let $x \in A \cap A^{\perp}$. Then $x \in A$ and $x \wedge a = 0$ for all $a \in A$. Thus in particular $x = x \wedge x = 0$. Hence $A \cap A^{\perp} = (0]$.

(iii) Let $a \le b$. Suppose $x \in \{b\}^{\perp}$. Then $x \wedge b = 0$ and $x \wedge a \le x \wedge b = 0$ implies $x \wedge a = 0$. Thus $x \in \{a\}^{\perp}$. Therefore, $\{b\}^{\perp} \subseteq \{a\}^{\perp}$. Now let $x \in \{a\}^{\perp\perp}$. Then $x \wedge p = 0$ for all $p \in \{a\}^{\perp}$. Let $q \in \{b\}^{\perp}$. Since $\{b\}^{\perp} \subseteq \{a\}^{\perp}$, so $q \in \{a\}^{\perp}$. Hence $x \wedge q = 0$, which implies $x \in \{b\}^{\perp\perp}$. Therefore $\{a\}^{\perp\perp} \subseteq \{b\}^{\perp\perp}$.

(iv) Let $x \in \{a\}^{\perp} \cap \{a\}^{\perp\perp}$. Then $x \in \{a\}^{\perp}$ and $x \in \{a\}^{\perp\perp}$, and so $x \wedge p = 0$ for all $p \in \{a\}^{\perp}$. Thus in particular $x = x \wedge x = 0$, implies $\{a\}^{\perp} \cap \{a\}^{\perp\perp} = (0]$.

(v) Let and $y \in \{a \wedge b\}^{\perp}$. Then we have $(y \wedge a) \wedge b = 0$, which implies $y \wedge a \in \{b\}^{\perp}$. Since $x \in \{b\}^{\perp\perp}$, we get $(x \wedge y) \wedge a = x \wedge (y \wedge a) = 0$. This implies $x \wedge y \in \{a\}^{\perp}$. Since $x \in \{a\}^{\perp\perp}$, we have $x \wedge y \in \{a\}^{\perp\perp}$ as $\{a\}^{\perp\perp}$ is a down set. Thus $x \wedge y \in \{a\}^{\perp\perp} \cap \{b\}^{\perp\perp} = (0]$. Hence $x \wedge y = 0$ for all $y \in \{a \wedge b\}^{\perp}$, which implies $x \in \{a \wedge b\}^{\perp\perp}$. Therefore, $\{a\}^{\perp\perp} \cap \{b\}^{\perp\perp} \subseteq \{a \wedge b\}^{\perp\perp}$.

Conversely, since $a \wedge b \le a, b$, so by (iii) $\{a \wedge b\}^{\perp\perp} \subseteq \{a\}^{\perp\perp}$ and $\{a \wedge b\}^{\perp\perp} \subseteq \{b\}^{\perp\perp}$. Hence $\{a \wedge b\}^{\perp\perp} = \{a\}^{\perp\perp} \cap \{b\}^{\perp\perp}$.

(vi) $x \in \{a\}^{\perp\perp}$ implies $x \wedge p = 0$ for all $p \in \{a\}^{\perp}$. Now $p \in \{a\}^{\perp}$ implies $p \wedge a = 0$. Thus we have $a \wedge p = 0$ for all $p \in \{a\}^{\perp}$, which implies $a \in \{a\}^{\perp\perp}$.

(vii) Since $\{a\}^{\perp} \cap \{a\}^{\perp\perp} = (0]$ so by (i) $\{a\}^{\perp} \subseteq \{a\}^{\perp\perp\perp}$.

Conversely, let $x \in \{a\}^{\perp\perp\perp}$. Then $x \wedge p = 0$ for all $p \in \{a\}^{\perp\perp}$. But by (vi) we have $a \in \{a\}^{\perp\perp}$. Therefore $x \wedge a = 0$ and so $x \in \{a\}^{\perp}$. Thus $\{a\}^{\perp\perp\perp} \subseteq \{a\}^{\perp}$, and so $\{a\}^{\perp} = \{a\}^{\perp\perp\perp}$.

(viii) Suppose S is 0-distributive and $a \vee b$ exists. Since $a, b \leq a \vee b$ so by (iii) $\{a \vee b\}^{\perp} \subseteq \{a\}^{\perp} \cap \{b\}^{\perp}$.

Conversely, let $x \in \{a\}^{\perp} \cap \{b\}^{\perp}$. Then $x \wedge a = 0 = x \wedge b$ implies $x \wedge (a \vee b) = 0$ as S is 0-distributive. Thus $x \in \{a \vee b\}^{\perp}$. Therefore $\{a\}^{\perp} \cap \{b\}^{\perp} \subseteq \{a \vee b\}^{\perp}$, and so $\{a \vee b\}^{\perp} = \{a\}^{\perp} \cap \{b\}^{\perp}$.●

Theorem 9. Let S be a nearlattice with 0. Then the following conditions are equivalent.

(i) S is 0-distributive.

(ii) For any non-empty subset A of S, $A^{\perp}$ is the intersection of all the minimal prime ideal not containing A.

(iii) For any ideal A of S and any family of ideals $\{A_i \mid i \in I\}$ of S, $\left(A \cap \left(\underset{i \in I}{\vee} A_i\right)\right)^{\perp} = \underset{i \in I}{\cap}(A \cap A_i)^{\perp}$

(iv) For any three ideals A, B, C of S, $(A \cap (B \vee C))^{\perp} = (A \cap B)^{\perp} \cap (A \cap C)^{\perp}$.

(v) For all $a, b, c \in S$, $(a \wedge (b \vee c))^{\perp} = (a \wedge b)^{\perp} \cap (a \wedge c)^{\perp}$ provided $b \vee c$ exists.

Proof. (i) $\Rightarrow$ (ii) Let N be a minimal prime down set not containing A. Then there exists $t \in A$ such that $t \notin N$. Suppose $x \in A^{\perp}$. Then $x \wedge a = 0$ for all $a \in A$. Thus $x \wedge t = 0 \in N$. Since N is prime, so $x \in N$. Hence $A^{\perp} \subseteq N$. Thus, $A^{\perp} \subseteq \cap\{\text{all minimal prime down set not containing } A\} = X$ (say).

Suppose $A^{\perp} \subset X$. Then there exists $x \in X$ such that $x \notin A^{\perp}$. Then for some $b \in A$, $x \wedge b \neq 0$. Thus $F = [x \wedge b)$ is a proper filter. Hence by lemma 2. there is a maximal filter $M \supseteq F$. Then $S - M$ is a minimal prime down set such that $(S - M) \cap F = \phi$. Since $b \in F \subseteq M$, so $b \notin S - M$ and so $S - M \not\supseteq A$. Also $x \in F \subseteq M$ implies $x \notin S - M$. Thus $x \notin X$, which is a contradiction. Therefore, $A^{\perp} = X = \cap\{\text{all minimal prime down set not containing } A\}$. Since S is 0-distributive, so by [6, theorem 9], the result follows.

(ii) $\Rightarrow$ (iii) Suppose (ii) holds. Let $A \subseteq I(S)$ and $\{A_i \mid i \in I\} \subseteq I(S)$. If Q is any minimal prime ideal of S such that $Q \not\supseteq A \cap \left(\underset{i \in I}{\vee} A_i\right)$, then $Q \not\supseteq A \cap A_j$ for some

$j \in I$. By (ii) it follows that $\left\{A \cap \left(\underset{i \in I}{\vee} A_i\right)\right\}^{\perp} \supseteq (A \cap A_i)^{\perp}$. On the other hand, $A \cap A_i \subseteq A \cap \left(\underset{i \in I}{\vee} A_i\right)$. Then by lemma 8 $\{A \cap A_i\}^{\perp} \supseteq \left\{A \cap \left(\underset{i \in I}{\vee} A_i\right)\right\}^{\perp}$. Therefore (iii) holds.

$(iii) \Rightarrow (iv)$ is obvious.

$(iv) \Rightarrow (v)$ Let $A = (a]$, $B = (b]$, $C = (c]$. Then by (iv),
$\{(a] \cap ((b] \vee (c])\}^{\perp} = ((a] \cap (b])^{\perp} \cap ((a] \cap (c])^{\perp}$. Thus
$(a \wedge (b \vee c)]^{\perp} = (a \wedge b]^{\perp} \cap (a \wedge c]^{\perp}$, and so
$\{a \wedge (b \vee c)\}^{\perp} = (a \wedge b)^{\perp} \cap (a \wedge c)^{\perp}$.

$(v) \Rightarrow (i)$ Suppose (v) holds. Let $a, b, c \in S$ with $a \wedge b = 0 = a \wedge c$ such that $b \vee c$ exists. Then $(a \wedge b)^{\perp} = S = (a \wedge c)^{\perp}$. So by (v), $\{a \wedge (b \vee c)\}^{\perp} = S$. Thus, $\{a \wedge (b \vee c)\}^{\perp\perp} = S^{\perp} = (0]$. Hence by lemma 8, $a \wedge (b \vee c) = 0$, and so S is 0-distributive.●

REFERENCES

1. P.Balasubramani and P.V. Venkatanarasimhan, Characterizations of the 0-Distributive Lattice, *Indian J. Pure Appl. Math.*, 32(3) (2001), 315-324.
2. M. B. Rahman, A study on distributive nearlattices, Ph.D Thesis, Rajshahi University, Bangladesh (1994).
3. W.H. Cornish and A.S.A. Noor, Standard elements in a nearlattice, *Bull. Austral. Math. Soc.,* 26(2) (1982) , 185-213.
4. Y. S Pawar and N. K. Thakare, 0-Distributive semilattices, *Canad . Math. Bull.,* 21(4) (1978) , 469-475.
5. J. C. Varlet, A generalization of the notion of pseudo-complementedness, *Bull.Soc. Sci. Liege,* 37 (1968), 149-158.
6. Md. Zaidur Rahman, Md. Bazlar Rahman and A. S. A. Noor, 0-distributive nearlattice, *Annals of Pure and App. Math.,* 2(2) (2012), 177-184.