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Calculation of Degree Ratio F-Index of Certain Networks

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Abstract. In this study, we introduce the degree ratio F-index and the modified degree ratio F-index of a graph. Furthermore, we compute these degree ratio F-indices of certain networks such as silicate networks, oxide networks and honeycomb networks.

Keywords: Degree ratio F-index index, modified degree ratio F-index index, network.

AMS Mathematics Subject Classification (2010): 05*C*10, 05*C*69.

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

In [3], Kulli defined the degree ratio Nirmala index of a graph as

$$DRN(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u)}{d_G(v)} + \frac{d_G(v)}{d_G(u)}}.$$

Recently, some degree ratio indices were studied in [4, 5, 6, 7, 8, 9] and some Nirmala indices were studied in [10, 11].

In [12], Furtula and Gutman defined the F-index of a graph G as

$$F(G) = \sum_{uv \in E(G)} \left(d_G(u)^2 + d_G(v)^2 \right).$$

We define the degree ratio F-index of a graph G as

$$DRF(G) = \sum_{uv \in E(G)} \left(\left(\frac{d_G(u)}{d_G(v)} \right)^2 + \left(\frac{d_G(v)}{d_G(u)} \right)^2 \right).$$

We define the modified degree ratio F-index of a graph G as

$$^{m}DRF(G) = \sum_{uv \in E(G)} \frac{d_G(u)^2 d_G(v)^2}{d_G(u)^4 + d_G(v)^4}$$

In this paper, the degree ratio F-index and modified degree ratio F-index of silicate networks, oxide networks and honeycomb networks are computed.

2. Results for some standard graphs

Proposition 1. If G is an r-regular graph with n vertices, $r \geq 3$, then

$$DRF(G) = nr.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 3$. Then $|E(G)| = \frac{nr}{2}$. For any vertex u in $V(G)$, $d_G(u) = r$. Then

$$DRF(G) = \sum_{uv \in E(G)} \left(\left(\frac{d_G(u)}{d_G(v)} \right)^2 + \left(\frac{d_G(v)}{d_G(u)} \right)^2 \right)$$

$$= \frac{nr}{2} \left(\left(\frac{r}{r} \right)^2 + \left(\frac{r}{r} \right)^2 \right) = \frac{(1+1)nr}{2} = nr.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then $DRF(C_n) = 2n.$

Corollary 1.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$DRF(K_n) = n(n-1).$$

Proposition 2. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$DRF(K_{m,n}) = mn\left(\left(\frac{m}{n} \right)^2 + \left(\frac{n}{m} \right)^2 \right).$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn *edges* such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges.

$$DRF(G) = \sum_{uv \in E(G)} \left(\left(\frac{d_G(u)}{d_G(v)} \right)^2 + \left(\frac{d_G(v)}{d_G(u)} \right)^2 \right) = mn\left(\left(\frac{m}{n} \right)^2 + \left(\frac{n}{m} \right)^2 \right).$$

Corollary 2.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$DRF(K_{n,n}) = mn\left(\left(\frac{n}{n} \right)^2 + \left(\frac{n}{n} \right)^2 \right) = 2mn.$$

Corollary 2.2. Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$DRF(K_{1,n}) = n\left(\left(\frac{1}{n} \right)^2 + \left(\frac{n}{1} \right)^2 \right) = \frac{1+n^4}{n}.$$

3. Results for silicate networks

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network is symbolised by SLn, where n is the number of hexagons between the centre and the boundary of SL_n. A 2-dimensional silicate network is presented in Figure 1.

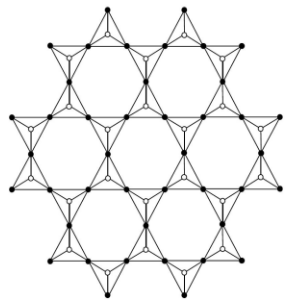

Figure 1: A 2-dimensional silicate network

Let G be the graph of a silicate network SL_n. The graph G has $15n^2 + 3n$ vertices and $36n^2$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6n$.
$E_{36} = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_{36}| = 18n^2 + 6n$.
$E_{66} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{66}| = 18n^2 - 12n$.

We compute the degree ratio F-index index of SL_n.

Theorem 1. The degree ratio F-index of a silicate network SL_n is

$$DRF(SL_n) = \frac{225}{2}n^2 + \frac{27}{2}n.$$

Proof: Let G be the graph of a silicate network SL_n. We obtain

$$DRF(G) = \sum_{uv \in E(G)} \left(\left(\frac{d_G(u)}{d_G(v)} \right)^2 + \left(\frac{d_G(v)}{d_G(u)} \right)^2 \right)$$

$$= 6n\left(\left(\frac{3}{3}\right)^2 + \left(\frac{3}{3}\right)^2 \right) + \left(18n^2 + 6n\right)\left(\left(\frac{3}{6}\right)^2 + \left(\frac{6}{3}\right)^2 \right)$$

$$+ \left(18n^2 - 12n\right)\left(\left(\frac{6}{6}\right)^2 + \left(\frac{6}{6}\right)^2 \right)$$

$$= \frac{225}{2}n^2 + \frac{27}{2}n.$$

We compute the modified degree ratio F-index of SL_n.

Theorem 2. The modified degree ratio F-index index of a silicate network SL_n is

$$^{m}DRF(SL_n) = \frac{225}{17}n^2 - \frac{27}{17}n.$$

Proof: Let G be the graph of a silicate network SL_n. We obtain

$$^{m}DRF(G)=\sum_{uv\in E(G)}\frac{d_G(u)^2 d_G(v)^2}{d_G(u)^4+d_G(v)^4}$$
$$=6n\frac{3^2\times 3^2}{3^4+3^4}+\left(18n^2+6n\right)\frac{3^2\times 6^2}{3^4+6^4}+\left(18n^2-12n\right)\frac{6^2\times 6^2}{6^4+6^4}$$
$$=\frac{225}{17}n^2-\frac{27}{17}n.$$

4. Results for oxide networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 2.

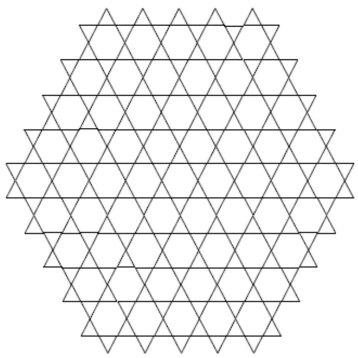

Figure 2: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. By calculation, we obtain that G has $9n^2+3n$ vertices and $18n^2$ edges. In G, by algebraic method, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{24} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $|E_{24}| = 12n$.

$E_{44} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_{44}| = 18n^2 - 12n$.

We compute the degree ratio F-index of OX_n.

Theorem 3. The degree ratio F-index index of an oxide network OX_n is
$$DRF(OX_n)=36n^2+27n.$$

Proof: Let G be the graph of an oxide network OX_n. We obtain

$$DRF(G)=\sum_{uv\in E(G)}\left(\left(\frac{d_G(u)}{d_G(v)}\right)^2+\left(\frac{d_G(v)}{d_G(u)}\right)^2\right)$$
$$=12n\left(\left(\frac{2}{4}\right)^2+\left(\frac{4}{2}\right)^2\right)+\left(18n^2-12n\right)\left(\left(\frac{4}{4}\right)^2+\left(\frac{4}{4}\right)^2\right)$$
$$=36n^2+27n.$$

We compute the modified degree ratio F-index of OX_n.

Theorem 4. The modified degree ratio F-index index of an oxide network OX_n is

$$ {}^{m}DRF\left(OX_n\right) = 9n^2 - \frac{54}{17}n. $$

Proof: Let G be the graph of an oxide network OX_n. We obtain

$$ \begin{aligned} {}^{m}DRF(G) &= \sum_{uv\in E(G)} \frac{d_G(u)^2 d_G(v)^2}{d_G(u)^4 + d_G(v)^4} \\ &= 12n\frac{2^2\times 4^2}{2^4+4^4} + \left(18n^2 - 12n\right)\frac{4^2\times 4^2}{4^4+4^4} \\ &= 9n^2 - \frac{54}{17}n. \end{aligned} $$

5. Results for honeycomb networks

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 3.

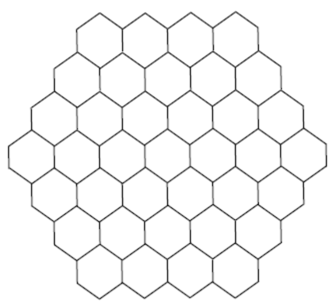

Figure 3: A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2–3n$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_{22}| = 6$.
$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 12n – 12$.
$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 9n^2 – 15n + 6$.

We compute the degree ratio F-index of HC_n.

Theorem 5. The degree ratio F-index index of a honeycomb network HC_n is

$$ DRF\left(HC_n\right) = 18n^2 + \frac{7}{3}n - \frac{25}{3}. $$

Proof: Let G be the graph of a honeycomb network HC_n. We obtain

$$DRF(G)=\sum_{uv\in E(G)}\left(\left(\frac{d_G(u)}{d_G(v)}\right)^2+\left(\frac{d_G(v)}{d_G(u)}\right)^2\right)$$

$$=6\left(\left(\frac{2}{2}\right)^2+\left(\frac{2}{2}\right)^2\right)+\left(12n-12\right)\left(\left(\frac{2}{3}\right)^2+\left(\frac{3}{2}\right)^2\right)$$

$$+\left(9n^2-15n+6\right)\left(\left(\frac{3}{3}\right)^2+\left(\frac{3}{3}\right)^2\right)$$

$$=18n^2+\frac{7}{3}n-\frac{25}{3}.$$

We compute the modified degree ratio F-index of HC_n.

Theorem 6. The modified degree ratio F-index index of an oxide network HC_n is

$$^{m}DRF\left(HC_n\right)=\frac{9}{2}n^2-\frac{591}{194}n-\frac{330}{17}.$$

Proof: Let G be the graph of an oxide network HC_n. We obtain

$$^{m}DRF(G)=\sum_{uv\in E(G)}\frac{d_G(u)^2\, d_G(v)^2}{d_G(u)^4+d_G(v)^4}$$

$$=6\frac{2^2\times 2^2}{2^4+2^4}+\left(12n-12\right)\frac{2^2\times 3^2}{2^4+3^4}+\left(9n^2-15n+6\right)\frac{3^2\times 3^2}{3^4+3^4}$$

$$=\frac{9}{2}n^2-\frac{591}{194}n-\frac{330}{17}.$$

6. Conclusion

In this paper, the degree ratio F-index and modified degree ratio F-index of a graph are defined. Also the degree ratio F-index and modified degree ratio F-index of certain networks are determined.

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Author Contributions: This is a single-author manuscript, and all aspects of the research, including conceptualization, analysis, writing, and revision, were carried out solely by the author. Consequently, full credit for the work is attributed to the author.

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