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Note of Nested SuperHyperGraph and Unified SuperHyperGraph

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Abstract. Hypergraphs extend classical graphs by allowing *hyperedges* to connect arbitrary nonempty subsets of vertices, thereby capturing higher-order, group-level interactions. Superhypergraphs further broaden this setting by iteratively applying the powerset construction, yielding layered supervertices and supporting multi-level relational structure.

A *nested hypergraph* makes hierarchy explicit by permitting hyperedges to contain not only vertices but also other hyperedges; a rank function enforces well-foundedness and excludes membership cycles. A *unified (and uniform) hypergraph* integrates three hyperedge types—simple, nesting, and directed—within a single formalism, thus simultaneously modeling group relations, hyperedge-in-hyperedge hierarchy, and oriented source–target interactions.

In this paper, we extend the notion of nested and unified hypergraphs within the SuperHyperGraph framework by introducing *Nested SuperHyperGraphs* and *Unified SuperHyperGraphs*, and we investigate their fundamental structural properties.

Keywords: SuperHyperGraph, HyperGraph, Nested hypergraph, Unified hypergraph

AMS Mathematics Subject Classification (2010): 05C65, 05C75, 05C99

1. Introduction

Networked systems are classically modeled by *graphs*, where objects are represented by vertices and binary relations by edges [1]. Although this abstraction is well-suited to pairwise interactions, it becomes restrictive when the underlying system involves *simultaneous* interactions among three or more entities. *Hypergraphs* address this limitation by allowing each hyperedge to join an arbitrary nonempty subset of vertices, thereby representing higher-order relations directly [2].

Even so, many real-world datasets and engineered systems exhibit relationships that are not only higher- order but also *layered*, *nested*, and inherently *hierarchical*. To capture such multi-level incidence patterns,

Smarandache introduced the notion of a *SuperHyperGraph*. Informally, a SuperHyperGraph is built via iterative powerset-based constructions, so that vertices (*supervertices*) may themselves be set-valued objects and edges can encode connectivity across multiple levels [3, 4]. As a consequence, SuperHyperGraphs have attracted increasing attention in both theory and applications [5–7].

A complementary, explicit approach to hierarchy is provided by *nested hypergraphs*, in which hyperedges may contain not only vertices but also other hyperedges [8]. In a different direction, a *unified (and uniform) hypergraph* integrates three hyperedge types—simple, nesting, and directed—within a single framework, thereby simultaneously modeling group relations, hyperedge-in-hyperedge hierarchy, and oriented source–target interactions [8].

While research on SuperHyperGraphs, nested hypergraphs, and unified (and uniform) hypergraphs is important in its own right, concepts that *combine* these formalisms have received comparatively little systematic study. To bridge this gap, we extend nested and unified hypergraphs within the SuperHyperGraph framework by introducing *Nested SuperHyperGraphs* and *Unified SuperHyperGraphs*, and investigate their fundamental structural properties. These constructions are expected to provide a transparent and flexible language for modelling complex networks with pronounced hierarchical organisation.

2. Preliminaries

This section fixes notation and recalls the basic constructions used throughout the paper. Unless stated otherwise, all graphs and hypergraphs are finite.

2.1. Hypergraphs and SuperHyperGraphs

Standard graph models represent a system by a vertex set together with *binary* edges, which is often adequate when interactions occur only in pairs [1]. However, many datasets naturally involve *higher-order* relations among three or more entities. Hypergraphs accommodate such group interactions by allowing an edge (a *hyperedge*) to connect any nonempty subset of vertices [9, 10]. Accordingly, hypergraph-based methods have been developed in a broad range of areas, including hypergraph neural networks [11–13], multi-criteria decision models [14, 15], and chemical informatics [16, 17].

In addition to being higher-order, real systems frequently exhibit *multi-level* organization. To encode such hierarchical structure, SuperHyperGraphs extend the hypergraph formalism by iterating the powerset construction: vertices may become set-valued *supervertices* drawn from higher powerset layers, and edges then describe incidence among these higher-level objects [3, 7]. We next recall the underlying set-theoretic operators and then state the incidence-form definition used in this paper.

Definition 2.1. (Base set)**.** [18] A *base set* is a nonempty set S chosen as the primitive domain from which all subsequent set-valued constructions are formed.

Definition 2.2. (Powerset)**.** (cf. [19, 20]) For any set S, the *powerset* of S is

$$\mathrm{P}(S) = \{\, A \mid A \subseteq S \,\},$$

the family of all subsets of S (including $\emptyset$ and S).

Definition 2.3. (n-th powerset and nonempty n-th powerset)**.** [21] Let H be a set. Define iterated powersets recursively by

$$\mathrm{P}^{0}(H) = H, \qquad \mathrm{P}^{k+1}(H) = \mathrm{P}\ \mathrm{P}^{k}(H) \quad (k \geq 0).$$

The *nonempty* iterated powersets are defined by

$$\mathrm{P}_{*}^{0}(H) = H, \qquad \mathrm{P}_{*}^{(k+1)}(H) = \mathrm{P}_{*}\ \mathrm{P}_{*}^{k}(H) \qquad (k \geq 0), \text{ where } \mathrm{P}_{*}(X)$$

$$= \mathcal{P}(X) \setminus \{\emptyset\}.$$

Definition 2.4. (Hypergraph)**.** [22, 23] A (finite) *hypergraph* is a pair $H = (V, E)$ consisting of a nonempty finite vertex set V and a finite family E of hyperedges such that each hyperedge $e \in E$ is a nonempty subset of V.

Definition 2.5. (Level-n SuperHyperGraph (incidence form))**.** (cf. [5]) Fix a finite base set V_0 and an integer $n \geq 0$. Let $V_n \subseteq \mathcal{P}^n(V_0)$ be a finite set; its elements are called *n-supervertices*. A *level-n SuperHyperGraph* is a pair

$$H^{(n)} = (V_n, \mathcal{E}), \qquad \emptyset \neq \mathcal{E} \subseteq \mathcal{P}(V_n) \setminus \{\emptyset\}.$$

Thus each *n-superedge* $E \in \mathcal{E}$ is a nonempty subset of the vertex set V_n. When $n = 0$, this reduces to an ordinary finite hypergraph; if moreover $|E| = 2$ for all $E \in \mathcal{E}$, it is a graph.

Remark (Set-theoretic typing). Supervertices are set-valued objects and may otherwise coincide with set-coded hyperedges. To avoid type ambiguity, we treat vertices and hyperedges as disjoint sorts (e.g., by tagging them via a disjoint union).

2.2. Nested HyperGraph

A nested hypergraph lets hyperedges include vertices and other hyperedges, with a rank function preventing cycles, thereby modelling explicit multilevel hierarchical incidence structures directly, and is finite.

Definition 2.6. (Nested hypergraph)**.** Let V be a finite nonempty set (the *vertex set*). A *nested hypergraph* on V is a pair $H = (V, E)$, where E is a finite set, together with a *rank function* $\rho : V \cup E \to \mathbb{N}$ satisfying:

1. $\rho(v) = 0$ for all $v \in V$;
2. for every $e \in E$, e is a nonempty finite set with
$$e \subseteq V \cup E,$$

and for every $x \in e$ one has $\rho(x) < \rho(e)$.

An element $e \in E$ is called a *nesting hyperedge* if $e \cap E \neq \emptyset$ (i.e., e contains at least one hyperedge as a member). We say that H is *(nontrivially) nested* if it has at least one nesting hyperedge.

Definition 2.7. (Nesting relation)**.** In a nested hypergraph $H = (V, E)$, a hyperedge $f \in E$ is said to be *nested in* $e \in E$ if $f \in e$. The transitive closure of $\in$ on E induces a hierarchy of hyperedges (multi-level nesting).

2.3. Unified (and uniform) hypergraph

A unified (and uniform) hypergraph allows simple, nesting, and directed hyperedges in one framework, representing group relations, hyperedge-in-hyperedge hierarchy, and source–target interactions.

Definition 2.8. (Unified hyperedge types)**.** Let V be a nonempty set (of *nodes*). Define the class $\mathcal{E}(V)$ of *unified hyperedges* as the *least* class closed under the following constructors:

Simple hyperedge. For any nonempty subset $S \subseteq V$, the set S is a hyperedge, written $S \in \mathrm{E}(V)$. Such an S is called a *simple hyperedge* and its *order* is $|S|$.

Nesting hyperedge. For any nonempty finite set $F \subseteq \mathrm{E}(V)$, the set F is a hyperedge, written $F \in \mathrm{E}(V)$. Such an F is called a *nesting hyperedge* (a hyperedge whose members are hyperedges), and its *order* is
$|F|$ (the number of member hyperedges).

Directed hyperedge. For any $e_1, e_2 \in \mathrm{E}(V)$, the ordered pair (e_1, e_2) is a hyperedge, written $(e_1, e_2) \in \mathrm{E}(V)$. Such (e_1, e_2) is called a *directed hyperedge*, where e_1 is the *source* hyperedge and e_2 is the *target* hyperedge.

Write $\mathrm{E}_S(V)$, $\mathrm{E}_N(V)$, and $\mathrm{E}_D(V)$ for the subclasses of simple, nesting, and directed hyperedges, respectively.

Definition 2.9. (Unified (and uniform) hypergraph)**.** A *unified hypergraph* is a tuple

$$G = (V, E, X, U),$$

where V is a finite node set, E is a finite set of hyperedges with $E \subseteq \mathrm{E}(V)$, and X and U are (optional) node- and hyperedge-feature data, respectively. If one does not use features, one may write simply $G = (V, E)$.

Definition 2.10. (Nested/directed/simple (as subclasses))**.** Let $G = (V, E, X, U)$ be a unified hypergraph.

1. G is called *nested* if $E \cap \mathrm{E}_N(V) \neq \emptyset$ (i.e., G contains at least one nesting hyperedge).
2. G is called *directed* if $E \cap \mathrm{E}_D(V) \neq \emptyset$ (i.e., G contains at least one directed hyperedge).
3. If $E \subseteq \mathrm{E}_S(V)$, then G is a *simple* (unnested, undirected) hypergraph.

3. Main results

In this section, we present the main results of this paper.

3.1. Nested SuperHyperGraph

A nested superhypergraph has supervertices at level n and superhyperedges that may contain other superhyperedges, forming well-founded hierarchical nesting beyond ordinary hypergraphs for multilevel interactions.

Definition 3.1. (Nested level-n SuperHyperGraph)**.** Fix a finite nonempty base set V_0 and an integer $n \geq 0$. Let $V_n \subseteq \mathrm{P}^n(V_0)$ be a finite set; its elements are called *n-supervertices*. A *nested level-n SuperHyperGraph* is a triple

$$\mathrm{H}^{(n)} = (V_n, \mathrm{E}_n, \rho),$$

where E_n is a finite set (of *superhyperedges*) and $\rho : V_n \cup \mathrm{E}_n \to \mathrm{N}$ is a *rank function* such that:

(i) $\rho(v) = 0$ for all $v \in V_n$;
(ii) for every $e \in \mathrm{E}_n$, e is a nonempty finite set with

$$e \subseteq V_n \cup \mathrm{E}_n,$$

and for every $x \in e$ one has $\rho(x) < \rho(e)$.

A superhyperedge $e \in \mathrm{E}_n$ is called *simple* if $e \subseteq V_n$, and it is called *nesting* if $e \cap \mathrm{E}_n \neq \emptyset$ (i.e., e contains at least one superhyperedge as a member).

Remark 3.2. (Two independent hierarchies)**.** In Definition 3.1, the vertex layer $V_n \subseteq \mathrm{P}^n(V_0)$ encodes *set-theoretic hierarchy inside supervertices*, whereas the rank function ρ encodes *membership-based hierarchy among superhyperedges* (edge-in-edge nesting). The condition $\rho(x) < \rho(e)$ rules out membership cycles and makes the nesting well-founded.

Theorem 3.3 (Generalization of nested hypergraphs and level-n SuperHyperGraphs)**.** *Let V_0 be a finite nonempty set and $n \geq 0$.*

(a) ***(Nested hypergraphs are the** $n = 0$ **case.)*** *If $n = 0$ and $V_0 = V_0 \subseteq \mathrm{P}^0(V_0)$, then a nested level-0 SuperHyperGraph $\mathrm{H}^{(0)} = (V_0, \mathrm{E}_0, \rho)$ is exactly a nested hypergraph on the vertex set V_0 in the sense of Definition 2.6. Conversely, every nested hypergraph (V, E, ρ) is a nested level-0 SuperHyperGraph by taking $V_0 := V$, $\mathrm{E}_0 := E$, and the same rank function ρ.*

(b) ***(Level-n SuperHyperGraphs are the "non-nesting" subclass.)*** *A level-n SuperHyperGraph $\mathrm{H}^{(n)} = (V_n, \mathrm{E})$ in the sense of Definition 2.5 canonically determines a nested level-n SuperHyperGraph $\mathrm{H}^{(n)} = (V_n, \mathrm{E}, \rho)$ by setting*

$$\rho(v) = 0 \ (v \in V_n), \qquad \rho(e) = 1 \ (e \in \mathrm{E}).$$

Conversely, if $\mathrm{H}^{(n)} = (V_n, \mathrm{E}_n, \rho)$ is a nested level-n SuperHyperGraph such that every $e \in \mathrm{E}_n$ is simple (equivalently, E_n contains no nesting superhyperedges), then (V_n, E_n) is a level-n SuperHyperGraph.

Proof: (a) Assume $n = 0$. Then $\mathrm{P}^0(V_0) = V_0$ and hence $V_n = V_0$ is an ordinary vertex set. By Definition 3.1, each $e \in \mathrm{E}_0$ is a nonempty finite set with $e \subseteq V_0 \cup \mathrm{E}_0$ and $\rho(x) < \rho(e)$ for all $x \in e$, while $\rho(v) = 0$ for all $v \in V_0$. This is precisely the data and conditions in Definition 2.6 (with $V := V_0$ and $E := \mathrm{E}_0$). The converse direction is immediate by the indicated identification.

(b) Let $\mathrm{H}^{(n)} = (V_n, \mathrm{E})$ be a level-n SuperHyperGraph. By Definition 2.5, each $e \in \mathrm{E}$ is a nonempty subset of V_n. Define $\rho(v) = 0$ for $v \in V_n$ and $\rho(e) = 1$ for $e \in \mathrm{E}$. Then for any $x \in e$ we have $x \in V_n$, hence $\rho(x) = 0 < 1 = \rho(e)$, so Definition 3.1(ii) holds.
Thus $\mathrm{H}^{(n)} = (V_n, \mathrm{E}, \rho)$ is a nested level-n SuperHyperGraph, and it has no nesting superhyperedges because every $e \in \mathrm{E}$ satisfies $e \subseteq V_n$.

Conversely, suppose $\mathrm{H}^{(n)} = (V_n, \mathrm{E}_n, \rho)$ is nested and every $e \in \mathrm{E}_n$ is simple, i.e., $e \subseteq V_n$. Then $\emptyset \neq e \subseteq V_n$ for all $e \in \mathrm{E}_n$, so $\mathrm{E}_n \subseteq \mathrm{P}(V_n) \setminus \{\emptyset\}$ and therefore (V_n, E_n) is a level-n SuperHyperGraph by Definition 2.5.

3.2. Unified SuperHyperGraph

A unified superhypergraph is a level-n superhypergraph whose hyperedges may be simple, nested, or directed, enabling hierarchical and oriented higher-order relations among supervertices.

Definition 3.4. (Unified superhyperedge universe)**.** Let V_n be a finite nonempty set. Define $\mathrm{E}^{\mathrm{uni}}(V_n)$ to be the *least* class of objects (called *unified hyperedges*) satisfying the following closure rules:

(i) (**Simple**) If $\emptyset \neq S \subseteq V_n$, then $S \in \mathrm{E}^{\mathrm{uni}}(V_n)$.
(ii) (**Nesting**) If $\emptyset \neq F \subseteq \mathrm{E}^{\mathrm{uni}}(V_n)$ is finite, then $F \in \mathrm{E}^{\mathrm{uni}}(V_n)$.
(iii) (**Directed**) If $e_1, e_2 \in \mathrm{E}^{\mathrm{uni}}(V_n)$, then $(e_1, e_2) \in \mathrm{E}^{\mathrm{uni}}(V_n)$.

Elements arising from (i), (ii), and (iii) are called *simple*, *nesting*, and *directed* hyperedges, respectively. In particular, a nesting hyperedge is a *set of hyperedges* and a directed hyperedge is an *ordered pair* (e_1, e_2) with e_1 as the source and e_2 as the target.

Remark 3.5. (Well-foundedness)**.** The inductive "least-class" construction in Definition 3.4 ensures that every hyperedge has finite construction depth. Hence membership cycles among hyperedges do not arise (in particular, nesting is well-founded).

Definition 3.6. (Unified level-n SuperHyperGraph)**.** Fix a finite nonempty base set V_0 and an integer $n \geq 0$. Let $V_n \subseteq \mathrm{P}^n(V_0)$ be a finite set, whose elements are called *n-supervertices*. A *unified level-n SuperHyperGraph* is a quadruple

$$\mathrm{G}^{(n)} = (V_n, E_n, X, U),$$

where E_n is a finite set of hyperedges with

$$E_n \subseteq \mathrm{E}^{\mathrm{uni}}(V_n),$$

and $X \in \mathrm{R}^{|V_n| \times d}$ and $U \in \mathrm{R}^{|E_n| \times d'}$ are (optional) feature matrices for supervertices and hyperedges, respectively.

If features are not used, we write $\mathrm{G}^{(n)} = (V_n, E_n)$.

Theorem 3.7 (Unified SuperHyperGraphs generalize unified hypergraphs and level-n SuperHyperGraphs)**.** *Let*

V_0 be a finite nonempty set and $n \geq 0$.

(a) ***(Generalization of unified hypergraphs.)*** *For $n = 0$ (so $V_n = V_0$), every unified level-0 SuperHyper- Graph $\mathrm{G}^{(0)} = (V_0, E_0, X, U)$ is precisely a* unified (and uniform) hypergraph *in the sense that hyperedges are of three types (simple, nesting, directed) and the ambient object is $G = (V, E, X, U)$. Conversely, every unified hypergraph $G = (V, E, X, U)$ yields a unified level-0 SuperHyperGraph by taking $V_0 := V$ and $E_0 := E$.*

(b) ***(Generalization of level-n SuperHyperGraphs.)*** *Let $\mathrm{H}^{(n)} = (V_n, \mathrm{E})$ be a level-n SuperHyperGraph in the sense of Definition 2.5. Then $\mathrm{E} \subseteq \mathrm{E}^{\mathrm{uni}}(V_n)$ via the inclusion*

$$\mathrm{P}(V_n) \setminus \{\emptyset\} \hookrightarrow \mathrm{E}^{\mathrm{uni}}(V_n), \qquad e \longmapsto e$$

(viewing each nonempty subset $e \subseteq V_n$ as a simple *unified hyperedge). Hence (V_n, E, X, U) is a unified level-n SuperHyperGraph for any choice of features X, U.*

Conversely, if $\mathrm{G}^{(n)} = (V_n, E_n, X, U)$ is a unified level-n SuperHyperGraph such that every hyperedge in E_n is simple *(equivalently, $E_n \subseteq \mathrm{P}(V_n) \setminus \{\emptyset\}$), then (V_n, E_n) is a level-n SuperHyperGraph.*

Proof: (a) Assume $n = 0$. Then $V_n = V_0$ is an ordinary node/vertex set. By Definition 3.4, a hyperedge is either a nonempty subset of V_0 (simple), a nonempty finite set of hyperedges (nesting), or an ordered pair of hyperedges (directed), with the intended source/target interpretation. This matches the unified-and-uniform hyperedge typing and the ambient tuple form $G = (V, E, X, U)$ stated in the reference.

Conversely, given any unified hypergraph $G = (V, E, X, U)$, set $V_0 := V$ and observe that its hyperedges E are, by definition, elements of $\mathrm{E}^{\mathrm{uni}}(V_0)$; thus it is a unified level-0 SuperHyperGraph.

(b) Let $\mathrm{H}^{(n)} = (V_n, \mathrm{E})$ be a level-n SuperHyperGraph. By Definition 2.5, each $e \in \mathrm{E}$ is a nonempty subset of V_n, hence $e \in \mathrm{E}^{\mathrm{uni}}(V_n)$ by the simple rule in Definition 3.4(i). Therefore

$G^{(n)} = (V_n, E, X, U)$ is a unified level-n SuperHyperGraph.

Conversely, if every hyperedge in E_n is simple, then $E_n \subseteq P(V_n) \setminus \{\emptyset\}$. Thus (V_n, E_n) satisfies Definition 2.5 and is a level-n SuperHyperGraph.

4. Conclusion

In this paper, we extended nested and unified hypergraphs within the SuperHyperGraph framework by introducing *Nested SuperHyperGraphs* and *Unified SuperHyperGraphs*, and we investigated their fundamental structural properties. In future work, we anticipate further developments that incorporate uncertainty-aware formalisms, including Fuzzy Graphs [24, 25], Neutrosophic Graphs [26–28], and Plithogenic Graphs [29].

Data Availability

This paper is theoretical and did not generate or analyze any empirical data. We welcome future studies that apply and test these concepts in practical settings.

Research Integrity

The author confirms that this manuscript is original, has not been published elsewhere, and is not under consideration by any other journal.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and we do not employ them in any way that violates ethical standards.

Disclaimer

The ideas presented here are theoretical and have not yet been empirically tested. While we have strived for accuracy and proper citation, inadvertent errors may remain. Readers should verify any referenced material independently. The opinions expressed are those of the authors and do not necessarily reflect the views of their institutions. This paper was converted from a LaTeX source into a Word document. The author has reviewed the content as carefully as possible; however, formatting issues or layout distortions may still occur. I appreciate your understanding.

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