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Certain Characteristics of Σα -Compact Spaces

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Abstract. In this paper, we introduce and investigate the concept of Σα-compact spaces within the framework of Σα-shelter. Furthermore, we establish several characterization theorems concerning our proposed perspective in fuzzy topological spaces, fuzzy subspaces, fuzzy continuous mappings, fuzzy open mappings, fuzzy $\mathscr{T}_1$ -spaces, fuzzy $\mathscr{T}_2$ -spaces (Hausdorff spaces), and various miscellaneous characteristics within fuzzy topological domains. Finally, we explore the "good extension" property of the proposed compactness.

Keywords: Fuzzy set, fuzzy topological space, Σα-shelter, Σα-compact.

AMS Mathematics Subject Classification (2020): 54A05, 54A10

1. Introduction

Zadeh [1] initially dealt with fuzzy sets in 1965, introduced the concept of fuzzy sets, and described fuzziness mathematically. Chang [2] first used an open cover to present the idea of fuzzy compactness, and later, Wong [3] extended the theory of fuzzy topological spaces based on Chang's [2] concept and discussed various covering characterizations that define these spaces. Lowen [4, 5] invented the three types of fuzzy compactness: strong fuzzy compactness and ultra-fuzzy compactness, and later, Chadwick [6] expanded on Lowen's compactness concept. Additionally, Lowen [7, 8] discussed compactness in fuzzy real lines as well as compact product fuzzy topological spaces. In fuzzy topological spaces, Ying [9, 10] addressed the concept of fuzzifying compactness; N-compactness was presented by Guojun [11], and it was then generalized by Dongsheng [12]. A new way to define fuzzy compactness in L-fuzzy topological spaces was addressed by Shi [13], and the idea of the notion of degree of fuzzy compactness in L-fuzzy topological spaces was addressed by Li and Shi [14]. Hong [15] first introduced RS-compactness, and Noiri [16] provided several of its properties. Additionally, the features of RS-compactness were discussed by Kudri and Warner [17], and El-Shafei et al. [18]. Using the notion of fuzzy upper limit, Georgiou and Papadopoulos [19] described well-known fuzzy compactness, and Warner [20] discussed the modelling of general topology by fuzzy topology, which is illustrated by fuzzy compactness. A good definition of compactness in an L-fuzzy topological spaces was presented by Warner and McLean [21], and this idea was extended to arbitrary L-

fuzzy subsets by Kudri [22], who further discussed its characteristics. Chen [23] discussed about the fuzzy compactness within the induced I(L)-fuzzy topological spaces. Two notions of intuitionistic fuzzy compactness in intuitionistic fuzzy topological spaces were introduced by Mahbub et al. [24]. Abbas [25] addressed there are three types of intuitionistic compactness, intuitionistic almost compactness, and intuitionistic near compactness and discussed them in the general framework of intuitionistic fuzzy topological spaces. Rachana Navalakhe and Purva Rajwade [26] define and study a new class of Nano fuzzy open sets called Nano fuzzy b-open sets in Nano fuzzy topological spaces. Gantner et al. [27] introduced the concept of α-shading and α-compactness due to L-fuzzy topological spaces, and Mashhour et al. [28] extended this concept. Kim [29] explored about the concepts of α-compactness regarding fuzzy topological spaces and establish a product theorem for an arbitrary product of α-compact fuzzy spaces. A relationship between α-compact fuzzy topological spaces and general topological spaces was demonstrated by Benchalli and Siddapur [30]. Shi [31] addressed a new way form of α-compactness in L-topological spaces, and Aygün [32] provided a good definition of fuzzy α-compactness in L-fuzzy topological spaces. Talukder and Ali [33] introduced partially α-compact (abbreviated as $p\alpha$-compact) fuzzy sets, and together with their various characteristics in fuzzy topological areas were discussed. Also, Talukder and Hossain [34] introduced slightly $\Sigma\alpha$-shelter, slightly $\Sigma\alpha$-compact fuzzy sets and discussed their various characterizations. The primary motivation behind introducing the concept of $\Sigma\alpha$-compact spaces is to address the limitations of existing fuzzy compactness notions, which often restrict the tracking of localized topological coverings. While classical α-shading and variants capture foundational structures [39-42], they lack flexibility when handling cumulative threshold constraints across multiple fuzzy subsets. By deploying the framework of $\Sigma\alpha$-shelters, this paper provides a robust and non-trivial extension of compactness in fuzzy topological spaces. The originality of our work lies in bridging these generalized shelters with fundamental topological elements, including fuzzy subspaces, continuous mappings, and separation axioms such as fuzzy $\mathscr{T}_1$-spaces, fuzzy $\mathscr{T}_2$-spaces (Hausdorff) spaces. Furthermore, demonstrating that $\Sigma\alpha$-compactness satisfies the "good extension" property highlights its theoretical consistency, opening new avenues for applications in mathematical modeling and systems dealing with structured uncertainty. The motive in relation to this paper is that we addressed and discuss the notion of $\Sigma\alpha$-shelter and $\Sigma\alpha$-compact fuzzy spaces with several characteristics.

2. Preliminaries

In this section, we systematically review several essential definitions and algebraic properties that form the baseline of our main results. These conventions are aligned with the established literature referenced across our discourse.

Definition 2.1. [1] Consider the set $\mathscr{G}$ which is not an empty and $I = [0,1]$ is the closed unit interval. A function or mapping $e : \mathscr{G} \to I$, which allocates to every component $g \in \mathscr{G}$, which is a fuzzy set. $e(g)$ represents a grade or degree of the membership of g

. $I^{\mathscr{O}}$ is stands for whole fuzzy sets in $\mathscr{O}$. Every ingredient of $I^{\mathscr{O}}$ may also be treated as a fuzzy subset of $\mathscr{O}$.

Definition 2.2. [1] If a fuzzy set has an identically 0 (zero) grade of membership, it is considered an empty fuzzy set. The number $0_{\mathscr{O}}$ represents it.

Definition 2.3. [1] If a fuzzy set has an identically 1 (one) grade of membership, it is considered a whole fuzzy set. It is symbolized by $1_{\mathscr{O}}$.

Definition 2.4. [1] Suppose $\mathscr{O}$ is not an empty set and $\mathscr{A} \subseteq \mathscr{O}$. Following that, the characteristic function $1_{\mathscr{A}} : \mathscr{O} \to \{0, 1\}$ is described by $1_{\mathscr{A}}(g) = \begin{cases} 1 & \text{if } g \in \mathscr{A} \\ 0 & \text{if } g \notin \mathscr{A} \end{cases}$

As a result, we declare that $\mathscr{A}$ is a fuzzy set in $\mathscr{O}$ and that $1_{\mathscr{A}}$ represents this fuzzy set. Hence, any subset of a set $\mathscr{O}$ can be thought of as a fuzzy set with a range of $\{0, 1\}$.

Definition 2.5. [1] Suppose e and z are fuzzy sets in $\mathscr{O}$. We ascertain

(1) $e = z$ iff $e(g) = z(g)$ for whole $g \in \mathscr{O}$

(2) $e \subseteq z$ iff $e(g) \leq z(g)$ for whole $g \in \mathscr{O}$

(3) $\Omega = e \cup z$ iff $\Omega(g) = (e \cup z)(g) = \max[e(g), z(g)]$ for whole $g \in \mathscr{O}$

(4) $\xi = e \cap z$ iff $\xi(g) = (e \cap z)(g) = \min[e(g), z(g)]$ for whole $g \in \mathscr{O}$

(5) $\beta = e^c$ iff $\beta(g) = 1 - e(g)$ for whole $g \in \mathscr{O}$ and we identify that e^c is the complement of e .

Note: It should be observed that the fuzzy sets e and z are disjoint iff $e \cap z = 0$

De-Morgan's laws 2.6. [1] The De-Morgan's Laws are tangible for fuzzy sets in $\mathscr{O}$ i.e. for any fuzzy sets e and z are in $\mathscr{O}$, after that

(1) $1 - (e \cup z) = (1 - e) \cap (1 - z)$

(2) $1 - (e \cap z) = (1 - e) \cup (1 - z)$

For each fuzzy set in e in $\mathscr{O}$, $e \cap (1 - e)$ not compulsory be zero and $e \cup (1 - e)$ not compulsory be one.

Distributive laws 2.7. [1] The distributive laws for fuzzy sets in $\mathscr{O}$ are nevertheless concrete i.e. for any fuzzy sets e , z and $\hbar$ are in $\mathscr{O}$, then

(1) $e \cup (z \cap \hbar) = (e \cup z) \cap (e \cup \hbar)$

(2) $e \cap (z \cup \hbar) = (e \cap z) \cup (e \cap \hbar)$

Definition 2.8. [35] Presume ξ is a fuzzy set in $\mathscr{O}$. Next to that, ξ is treated as a quasi-coincident (shortly q-coincident) with a fuzzy set Ω in $\mathscr{O}$, identified by $\xi q \Omega$ iff ξ_o for some $g \in \mathscr{O}$.

Definition 2.9. [2] Presume $p : \mathscr{O} \to \mathscr{B}$ is a mapping and e is a fuzzy set in $\mathscr{O}$. After that, the image of e, symbolized by $p(e)$, is a fuzzy set in $\mathscr{B}$ whose membership function is provided by

$$p(e)(d) = \begin{cases} \sup\{e(g) : g \in p^{-1}(d)\} & \text{when } p^{-1}(d) \neq \phi \\ 0 & \text{when } p^{-1}(d) \neq \phi \end{cases}.$$

Definition 2.10. [2] Presume $p : \mathscr{O} \to \mathscr{B}$ is a mapping and z is a fuzzy set in $\mathscr{B}$. After that, the inverse of z, symbolized by $p^{-1}(z)$, is a fuzzy set in $\mathscr{O}$ whose membership function is described by $p^{-1}(z)(g) = z(p(g))$ for whole $g \in \mathscr{O}$.

Definition 2.11. [36] Consider $\mathscr{O}$ is a not an empty set and $\mathscr{V}$ is a collecting of subsets of $\mathscr{O}$. Next to that, $\mathscr{V}$ is viewed as topology on $\mathscr{O}$ when

(1) ϕ, $\mathscr{O} \in \mathscr{V}$

(2) $\mathscr{G}_i \in \mathscr{V}$ for each $i \in \mathscr{J}$, after that $\bigcup_{i \in \mathscr{J}} \mathscr{G}_i \in \mathscr{V}$

(3) $\mathscr{G}$, $\mathscr{H} \in \mathscr{V} \Rightarrow \mathscr{G} \cap \mathscr{H} \in \mathscr{V}$

The pair $(\mathscr{O}, \mathscr{V})$ is treated as a topological space; any ingredient $\mathscr{D} \in \mathscr{V}$ is treated as an open set in topology $\mathscr{V}$.

Definition 2.12. [36] A topological space $(\mathscr{O}, \mathscr{V})$ has a subset $\mathscr{M}$ that is compact iff there is a finite subcover for each of its open covers.

Definition 2.13. [2] Consider $\mathscr{O}$ is not an empty set and in $\mathscr{O}$, ρ is a collecting of fuzzy sets. Next to that, ρ is viewed as a fuzzy topology on $\mathscr{O}$, when

(1) $0, 1 \in \rho$

(2) $e_i \in \rho$ for each $i \in \mathscr{J}$, after that $\bigcup_{i \in \mathscr{J}} e_i \in \rho$

(3) e, $z \in \rho$, after that $e \cap z \in \rho$

The twins $(\mathscr{O}, \rho)$ is viewed as a fuzzy topological space and, shortened to fts. Each members in ρ are treated as ρ-open fuzzy sets. Any fuzzy set is treated as ρ-closed iff its complement is ρ-open. When there won't be any confusion, we'll call it a ρ-open (ρ-closed) fuzzy set simply an open (closed) fuzzy set.

Definition 2.14. [2] Presume $(\mathscr{S}, \rho)$ and $(\mathscr{B}, \zeta)$ are fuzzy topological spaces. A mapping $p: (\mathscr{S}, \rho) \to (\mathscr{B}, \zeta)$ is viewed as fuzzy continuous iff inverse for each ζ -open fuzzy set is ρ-open or likewise inverse for each ζ -closed fuzzy set is ρ-closed.

Definition 2.15. [3] Presume $(\mathscr{S}, \rho)$ and $(\mathscr{B}, \zeta)$ are fuzzy topological spaces. Consider $p: (\mathscr{S}, \rho) \to (\mathscr{B}, \zeta)$ is a charting from an fts $(\mathscr{S}, \rho)$ to another fts $(\mathscr{B}, \zeta)$. After that, p is viewed as fuzzy open charting iff $p(e) \in \zeta$ for each $e \in \rho$.

Definition 2.16. [35] Assume $(\mathscr{S}, \rho)$ is an fts and $\mathscr{H} \subseteq \mathscr{S}$. Next to that the gathering $\rho_{\mathscr{H}} = \{e / \mathscr{H} : e \in \rho\}$ is also fuzzy topology on $\mathscr{H}$, treated as subspace fuzzy topology on $\mathscr{H}$ and after that the twins $(\mathscr{H}, \rho_{\mathscr{H}})$ is treated as fuzzy subspace of $(\mathscr{S}, \rho)$.

Definition 2.17. [37] Consider $(\mathscr{S}, \rho)$ is an fts. Next to that $(\mathscr{S}, \rho)$ is described fuzzy $\mathscr{T}_1$ -space iff for each g , $d \in \mathscr{S}$, $g \neq d$, there exist e , $z \in \rho$ such that $e(g) > 0$, $e(d) = 0$ and $z(g) = 0$, $z(d) > 0$.

Definition 2.18. [37] Assume $(\mathscr{S}, \rho)$ is an fts. After that $(\mathscr{S}, \rho)$ is described fuzzy $\mathscr{T}_2$ -space (Hausdorff space) iff for each individual g , $d \in \mathscr{S}$, $g \neq d$, there exist e , z $\in \rho$ such that $e(g) > 0$, $z(d) > 0$ and $e \cap d = 0$.

Definition 2.19. [38] The space $(\mathscr{S}, \mathscr{V})$ is assumed to be topological. A mapping $p :$ $\mathscr{S} \to \mathscr{R}$ (with usual topology) is treated as lower semi-continuous, abbreviated as l.s.c. when for every $a \in \mathscr{R}$, the set $p^{-1}(a, \infty) \in \mathscr{V}$. Assume a set $\mathscr{S}$ and a topology $\mathscr{T}$, consider $\omega(\mathscr{V})$ is the set of whole l. s. c. functions from $(\mathscr{S}, \mathscr{V})$ to I (with usual topology); in this way $\omega(\mathscr{V}) = \{e \in I^{\mathscr{S}} : e^{-1}(a, 1] \in \mathscr{V},\ a \in I_1\}$. It is going to be demonstrated that the $\omega(\mathscr{V})$, which is a fuzzy topology on $\mathscr{S}$. Fuzzy topology is analous to FP of the property P, which is assumed to be a property of topological spaces. As a result FP is viewed as a 'good extension' of P "iff the announcement $(\mathscr{S}, \mathscr{V})$ has P iff $(\mathscr{S}, \omega(\mathscr{V}))$ has FP" maintains good for every topological space $(\mathscr{S}, \mathscr{V})$. As a result, such characteristic functions are l.s.c.

Definition 2.20. [34] Consider $\mathscr{S}$ is a not an empty set, $0 < \alpha \leq 1$ and $\{e_i : i \in \mathscr{J}\}$ is a ingathering of fuzzy sets in $\mathscr{S}$. Next to that, $\{e_i : i \in \mathscr{J}\}$ is handled as a slightly $\Sigma\alpha$ -

shelter, abbreviated as $s\Sigma\alpha$-shelter of a fuzzy set ξ in $\mathscr{O}$ iff $\sum_i e_i(g) \geq \alpha$ for all of $g \in \xi_o$ $(\xi_o \neq \mathscr{O})$. A sub collection of $s\Sigma\alpha$-shelter of ξ, it is likewise a $s\Sigma\alpha$-shelter of ξ is addressed as a $s\Sigma\alpha$-sub shelter of ξ.

3. Principal outcomes

In this portion, we addressed the conception of $\Sigma\alpha$-compact spaces and provided some characterization techniques of $\Sigma\alpha$-compact spaces.

Definition 3.1. Consider $\mathscr{O}$ is not an empty set, $0 < \alpha \leq 1$, and $\{e_i : i \in \mathscr{J}\}$ is a ingathering of fuzzy sets in $\mathscr{O}$. Next to that, $\{e_i : i \in \mathscr{J}\}$ is viewed as a $\Sigma\alpha$-shelter of $\mathscr{O}$ iff $\sum_i e_i(g) \geq \alpha$ for the whole $g \in \mathscr{O}$. A subcollection of $\Sigma\alpha$-shelter of $\mathscr{O}$ which is also a $\Sigma\alpha$-shelter of $\mathscr{O}$ is treated as a $\Sigma\alpha$-subshelter of $\mathscr{O}$.

For the instance, assume $\mathscr{O} = \{a, b, c\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Consider e_1, $e_2 \in I^{\mathscr{O}}$ is defined with $e_1(a) = 0.02$, $e_1(b) = 0.04$, $e_1(c) = 0.03$ and $e_2(a) = 0.04$, $e_2(b) = 0.03$, $e_2(c) = 0.02$. Choose $\alpha = 0.05$. It is clear that $e_1(a) + e_2(a) > \alpha$, $e_1(b) + e_2(b) > \alpha$, $e_1(c) + e_2(c) \geq \alpha$, where a, b, $c \in \mathscr{O}$. Therefore $\{e_1, e_2\}$ is a $\Sigma\alpha$-shelter of $\mathscr{O}$.

Definition 3.2. Presume $(\mathscr{O}, \rho)$ is an fts and $0 < \alpha \leq 1$. Then $(\mathscr{O}, \rho)$ is treated as $\Sigma\alpha$-compact iff each open $\Sigma\alpha$-shelter of $\mathscr{O}$ has a finite $\Sigma\alpha$-subshelter i.e. there exist e_{i_1}, e_{i_2}, ….., $e_{i_n} \in \{e_i\}$ so as to $\sum_i e_{i_k}(g) \geq \alpha$ for all of $g \in \mathscr{O}$ and with $(k \in \mathscr{J}_n)$.

Theorem 3.3. Presume $(\mathscr{O}, \rho)$ is an fts and $\mathscr{H} \subset \mathscr{O}$. After that, $1_{\mathscr{H}}$ is $\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$ iff $1_{\mathscr{H}}$ is $\Sigma\alpha$-compact in $(\mathscr{H}, \rho_{\mathscr{H}})$.

Proof: Regard $1_{\mathscr{H}}$ is $\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$. Presume $\mathscr{E} = \{e_i : i \in \mathscr{J}\}$ is an open $\Sigma\alpha$-shelter of $1_{\mathscr{H}}$ in $(\mathscr{H}, \rho_{\mathscr{H}})$. So there exists $z_i \in \rho$ such that $e_i = z_i \mid \mathscr{H} \subseteq z_i$. Thus $\{z_i : i \in \mathscr{J}\}$ is an open $\Sigma\alpha$-shelter of $1_{\mathscr{H}}$ in $(\mathscr{O}, \rho)$. But $1_{\mathscr{H}}$ is $\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$, so $\{z_i : i \in \mathscr{J}\}$ has a finite $\Sigma\alpha$-sub shelter, namely $\{z_{i_r} : r \in \mathscr{J}_n\}$ such that $\sum_i z_{i_r}(g) \geq \alpha$ for all $g \in \mathscr{O}$ and $r \in \mathscr{J}_n$. For if, $g \in \mathscr{O}$, afterward there exist $z_{i_{r_o}}$ such that $\sum_i z_{i_{r_o}}(g) \geq \alpha$ $(r \in \mathscr{J}_n)$ which implies that $\sum_i (z_{i_{r_o}} \mid \mathscr{H})(g) \geq \alpha$ $(r \in \mathscr{J}_n)$ and consequently $\sum_i e_{i_{r_o}}(g) \geq \alpha$ $(r \in \mathscr{J}_n)$, as $\mathscr{H} \subset \mathscr{O}$. Hence $e_{i_o} \in \mathscr{E}$

and thus $\{e_{i_r} : r \in \mathscr{J}_n\}$ is a finite $\Sigma\alpha$ -subshelter of $\mathscr{E}$. Thus $1_{\mathscr{Y}}$ is $\Sigma\alpha$ -compact in $(\mathscr{Y}, \rho_{\mathscr{Y}})$.

Conversely, presume $1_{\mathscr{Y}}$ is $\Sigma\alpha$ -compact in $(\mathscr{Y}, \rho_{\mathscr{Y}})$. Assume $\mathscr{W} = \{z_i : i \in \mathscr{J}\}$ is an open $\Sigma\alpha$ -shelter of $1_{\mathscr{Y}}$ in $(\mathscr{G}, \rho)$. Set $e_i = z_i \mid \mathscr{Y}$. To show this, presume $g \in \mathscr{G}$. When $g \in \mathscr{Y}$, after that, there exists $z_{i_o} \in \mathscr{W}$ such that $e_{i_o} = z_{i_o} \mid \mathscr{Y}$. On the other hand, $e_{i_o} \in \rho_{\mathscr{Y}}$, so $\sum_i e_{i_o}(g) \geq \alpha$ for whole $g \in \mathscr{G}$. Thus $\{e_i : i \in \mathscr{J}\}$ is an open $\Sigma\alpha$ -shelter of $1_{\mathscr{Y}}$ in $(\mathscr{Y}, \rho_{\mathscr{Y}})$. But $1_{\mathscr{Y}}$ is $\Sigma\alpha$ -compact in $(\mathscr{Y}, \rho_{\mathscr{Y}})$, so $\{e_i : i \in \mathscr{J}\}$ has a finite $\Sigma\alpha$ -subshelter, namely $\{e_{i_r} : r \in \mathscr{J}_n\}$ such that $\sum_i e_{i_r}(g) \geq \alpha$ for all $g \in \mathscr{G}$. For if $g \in \mathscr{G}$, then here exists $e_{i_{r_o}}$ such that $\sum_i e_{i_{r_o}}(g) \geq \alpha$ implies that $\sum_i (z_{i_{r_o}} \mid \mathscr{Y})(g) \geq \alpha$ implies that $\sum_i z_{i_{r_o}}(g) \geq \alpha$, as $\mathscr{Y} \subset \mathscr{G}$. So $\{z_{i_r} : r \in \mathscr{J}_n\}$ is a finite $\Sigma\alpha$ -subshelter of $\mathscr{W}$. Therefore $1_{\mathscr{Y}}$ is $\Sigma\alpha$ -compact in $(\mathscr{G}, \rho)$.

Theorem 3.4. Consider $(\mathscr{G}, \rho)$ is an fts and $0 < \alpha \leq 1$. If each collection of closed fuzzy sets in $(\mathscr{G}, \rho)$ which has vacant intersection has a finite subcollection with vacant intersection, after that $(\mathscr{G}, \rho)$ is $\Sigma\alpha$ -compact. On the contrary, it is not always true everywhere.

Proof: Presume $\{e_i : i \in \mathscr{J}\}$ is an open $\Sigma\alpha$ -shelter of $(\mathscr{G}, \rho)$ i.e. $\sum_i e_i(g) \geq \alpha$ for whole $g \in \mathscr{G}$. For the first context of the theorem, we get $\bigcap_{i \in \mathscr{J}} e_i^c = 0_{\mathscr{G}}$, so we write $\bigcup_{i \in \mathscr{J}} e_i = 1_{\mathscr{G}}$ and hence clearly $\sum_i e_i(g) \geq \alpha$ for whole $g \in \mathscr{G}$. Again, for the second adherence of the theorem, we have $\bigcap_{k \in \mathscr{J}_n} e_{i_k}^c = 0_{\mathscr{G}}$, so we are able to write $\bigcup_{k \in \mathscr{J}_n} e_{i_k} = 1_{\mathscr{G}}$ and therefore it is clearly shows that $\sum_i e_{i_k}(g) \geq \alpha$ for all $g \in \mathscr{G}$ $(k \in \mathscr{J}_n)$ and hence it is clearly shows that $\{e_k : k \in \mathscr{J}_n\}$ is a finite $\Sigma\alpha$ -subshelter of $\{e_i : i \in \mathscr{J}\}$. As a result, $(\mathscr{G}, \rho)$ is $\Sigma\alpha$ -compact.

On the contrary, we can take into account the following instance:

Presume $\mathscr{G} = \{a, b, c\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Presume e_1, e_2, e_3, $e_4 \in I^{\mathscr{G}}$ is described by $e_1(a) = 0.03$, $e_1(b) = 0.04$, $e_1(c) = 0.02$; $e_2(a) = 0.04$, $e_2(b) = 0.08$, $e_2(c) = 0.01$; $e_3(a) = 0.04$, $e_3(b) = 0.08$, $e_3(c) = 0.02$ and $e_4(a) = 0.03$, $e_4(b) = 0.04$, $e_4(c) = 0.01$. Choose $\rho = \{0, e_1, e_2, e_3, e_4, 1\}$, then $(\mathscr{G}, \rho)$ is an fts. Take $\alpha = 0.06$. After that, clearly $(\mathscr{G}, \rho)$ is $\Sigma\alpha$ -compact. Now, closed fuzzy sets are $e_1^c(a)=0.97$, $e_1^c(b)=0.96$, $e_1^c(c)=0.98$; $e_2^c(a)=0.96$, $e_2^c(b)=0.92$, $e_2^c(c)=0.99$; $e_3^c(a)=0.96$, $e_3^c(b)=0.92$, $e_3^c(c)=0.98$ and $e_4^c(a)=0.97$, $e_4^c(b)=0.96$, $e_4^c(c)=0.99$. Hence we see that $e_1^c \cap e_2^c \cap e_3^c \cap e_4^c \neq 0_{\mathscr{A}}$. Therefore, the opposite of the theorem does not always hold.

Theorem 3.5. Presume $p:\ (\mathscr{A},\rho)\to(\mathscr{B},\ell)$ is a fuzzy continuous and onto mapping, where $(\mathscr{A},\rho)$ and $(\mathscr{B},\ell)$ are fuzzy topological spaces with $(\mathscr{A},\rho)$ is $\Sigma\alpha$ -compact. After that, $(\mathscr{B},\ell)$ is $\Sigma\alpha$ -compact.

Proof: Guess $\{e_i : e_i \in \ell\}$ is an open $\Sigma\alpha$ -shelter of $(\mathscr{B},\ell)$ for $i \in \mathscr{J}$ i.e. $\sum_i e_i(d) \geq \alpha$ for all of $d \in \mathscr{B}$. As p is fuzzy continuous, so $p^{-1}(e_i) \in \rho$ and as a result $\{p^{-1}(e_i) : e_i \in \ell\}$ which is an open $\Sigma\alpha$ -shelter of $(\mathscr{A},\rho)$ i.e. $\sum_i p^{-1}(e_i)(g) \geq \alpha$ for all of $g \in \mathscr{A}$. Since $(\mathscr{A},\rho)$ is $\Sigma\alpha$ -compact, so $\{p^{-1}(e_i) : e_i \in \ell\}$ has a finite $\Sigma\alpha$ -subshlter, say $\{p^{-1}(e_{i_k}) : k \in \mathscr{J}_n\}$. Presently, if $d \in \mathscr{B}$, after that $d = p(g)$ for some $g \in \mathscr{A}$. So, there exist $e_{i_k} \in \{e_i\}$ such that $\sum_i p^{-1}(e_{i_k})(g) \geq \alpha \Rightarrow \sum_i (e_{i_k})(p(g)) \geq \alpha\ (k \in \mathscr{J}_n)$ or $\sum_i (e_{i_k})(d) \geq \alpha$ i.e. $\{e_{i_k}\}$ $(k \in \mathscr{J}_n)$ is a finite $\Sigma\alpha$ -subshelter of $\{e_i : e_i \in \ell\}$. Thus $(\mathscr{B},\ell)$ is $\Sigma\alpha$ -compact.

Theorem 3.6. Presume $p:\ (\mathscr{A},\rho)\to(\mathscr{B},\ell)$ is a fuzzy open and bijective mapping, where $(\mathscr{A},\rho)$ and $(\mathscr{B},\ell)$ are fuzzy topological spaces with $(\mathscr{B},\ell)$ is $\Sigma\alpha$ -compact. After that, $(\mathscr{A},\rho)$ is $\Sigma\alpha$ -compact.

Proof: Suppose $\{e_i : e_i \in \rho\}$, which is an open $\Sigma\alpha$ -shelter of $(\mathscr{A},\rho)$ for $i \in \mathscr{J}$ i.e. $\sum_i e_i(g) \geq \alpha$ when whole $g \in \mathscr{A}$. But p is a fuzzy open, thus $p(e_i) \in \ell$ and hence $\{p(e_i) : e_i \in \rho\}$, which is an open $\Sigma\alpha$ -shelter of $(\mathscr{B},\ell)$. For, when $d \in \mathscr{B}$, then $p^{-1}(d) \in p^{-1}(\mathscr{B})$ and therefore, there exists $e_{i_o} \in \{e_i\}$ so as to $\sum_i e_{i_o}\left(p^{-1}(d)\right) \geq \alpha$ $\Rightarrow \sum_i p(e_{i_o})(d) \geq \alpha$. Since $(\mathscr{B},\ell)$ is $\Sigma\alpha$ -compact, so $\{p(e_i) : e_i \in \rho\}$ has a finite $\Sigma\alpha$ -subshlter, say $\{p(e_{i_k}) : k \in \mathscr{J}_n\}$ so as to $\sum_i p(e_{i_k})(d) \geq \alpha\ \left(k \in \mathscr{J}_n\right)$ for each $d \in \mathscr{B}$. At the moment, if $g \in p^{-1}(\mathscr{B})$, then $g = p^{-1}(d)$ for some of $d \in \mathscr{B}$. Thus there exist $e_{i_k} \in \{e_i\}$ so as to $\sum_i p(e_{i_k})(d) \geq \alpha \ \Rightarrow \sum_i e_{i_k}\left(p^{-1}(d)\right) \geq \alpha \Rightarrow$

$\sum_i e_{i_k}(g) \geq \alpha \ (k \in \mathscr{J}_n)$ i.e. $\{e_{i_k}\}\ (k \in \mathscr{J}_n)$ is a finite $\Sigma\alpha$ -subshelter of $\{e_i\}$. Thus $(\mathscr{S}, \rho)$ is $s\Sigma\alpha$ -compact.

Theorem 3.7. Presume $(\mathscr{S}, \rho)$ is a fuzzy $\mathscr{T}_1$ -space and $\mathscr{A} \subset \mathscr{S}$. If $1_{\mathscr{A}}$ is $\Sigma\alpha$ -compact subset in $(\mathscr{S}, \rho)$ and $g \notin \mathscr{A}$, after that there exist $e, z \in \rho$ such that $e(g) > 0$ and $\mathscr{A} \subseteq z^{-1}(0, 1]$. But the converse is not true in everywhere.

Proof: Consider $d \in \mathscr{A}$. Since $g \notin \mathscr{A}$, so it is clear that $g \neq d$. But $(\mathscr{S}, \rho)$ is fuzzy $\mathscr{T}_1$ -space, so there exist $e_d,\ z_d \in \rho$ such that $e_d(g) > 0$, $e_d(d) = 0$ and $z_d(g) = 0, z_d(d) > 0$. Since $1_{\mathscr{A}}$ is $\Sigma\alpha$ -compact in fuzzy $\mathscr{T}_1$ -space $(\mathscr{S}, \rho)$, so it is clearly shows that $\sum_y v_y(y) \geq \alpha$ for all $d \in \mathscr{A}$ i.e. $\{z_d : d \in \mathscr{A}\}$ is an open $\Sigma\alpha$ -shelter of $1_{\mathscr{A}}$ and hence $\{z_d : d \in \mathscr{A}\}$ has a finite $\Sigma\alpha$ -subshelter, namely $\{z_{d_k} : d \in \mathscr{A}\}\ (k \in \mathscr{J}_n)$ such that $\sum_d z_{d_k}(d) \geq \alpha$ for all $d \in \mathscr{A}$. Presently, consider $z = z_{d_1} \cup z_{d_2} \cup \ldots\ldots \cup z_{d_n}$ and $e = e_{d_1} \cap e_{d_2} \cap \ldots\ldots \cap e_{d_n}$. Hence we observe that $z, e \in \rho$. Moreover, $\mathscr{A} \subseteq z^{-1}(0, 1]$ and $e(g) > 0$, as $e_{d_k}(g) > 0$ for all k.

To illustrate, consider the following instance:

Presume $\mathscr{S} = \{a, b\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Presume e_1, e_2, $e_3 \in I^{\mathscr{S}}$ is defined with $e_1(a) = 0.01$, $e_1(b) = 0$; $e_2(a) = 0$, $e_2(b) = 0.02$ and $e_3(a) = 0.01$, $e_3(b) = 0.02$. Put $\rho = \{0, e_1, e_2, e_3, 1\}$, then $(\mathscr{S}, \rho)$ is a fuzzy $\mathscr{T}_1$ -space. Once more, presume $1_{\mathscr{A}} \in I^{\mathscr{S}}$ is described by $1_{\mathscr{A}}(a) = 1$, $1_{\mathscr{A}}(b) = 0$, after that $\mathscr{A} = \{a\}$ and $b \notin \mathscr{A}$. Now, e_1, $e_2 \in \rho$, where $e_2(b) > 0$ and $e_1^{-1}(0, 1] = \{a\}$. Therefore, $\mathscr{A} \subseteq e_1^{-1}(0, 1]$. Choose $\alpha = 0.06$. Then we see that $1_{\mathscr{A}}$ is not $\Sigma\alpha$ -compact in $(\mathscr{S}, \rho)$, as $\sum_k e_k(a) < \alpha$, where $a \in \mathscr{A}$ and $k = 1, 2, 3$. Thus the theoretical discussion is not generally true everywhere.

Theorem 3.8. Presume $(\mathscr{S}, \rho)$ is a fuzzy $\mathscr{T}_1$ -space and $\mathscr{A}$, $\mathscr{C} \subset \mathscr{S}$. If $1_{\mathscr{A}}$, $1_{\mathscr{C}}$ are disjoint $\Sigma\alpha$ -compact subsets in $(\mathscr{S}, \rho)$, after that there exist $e, z \in \rho$ such that $\mathscr{A} \subseteq e^{-1}(0, 1]$ and $\mathscr{C} \subseteq z^{-1}(0, 1]$. Contrarily is not true everywhere.

Proof: Suppoe $d \in \mathscr{A}$. So $d \notin \mathscr{C}$, as $1_{\mathscr{A}}$ and $1_{\mathscr{C}}$ are disjoint. As $1_{\mathscr{C}}$ is $\Sigma\alpha$ -compact in $(\mathscr{S}, \rho)$, so by theorem (3.7), there exist $e_d, z_d \in \rho$ such that $e_d(d) > 0$ and $\mathscr{C} \subseteq z_d^{-1}(0, 1]$. But $1_{\mathscr{A}}$ is $\Sigma\alpha$ -compact in $(\mathscr{S}, \rho)$, so it is evident that

$\sum_{d} e_d(d) \geq \alpha$ for all of $d \in \mathscr{A}$ and as a result $\{e_d : d \in \mathscr{A}\}$ is an open $\Sigma\alpha$-shelter of $1_{\mathscr{A}}$ and so $\{e_d : d \in \mathscr{A}\}$ has a finite $\Sigma\alpha$-subshelter, say $\{e_{d_k} : d \in \mathscr{A}\}$ $(k \in \mathscr{I}_n)$ so as to $\sum_{d} e_{d_k}(d) \geq \alpha$ $(k \in \mathscr{I}_n)$ for all $d \in \mathscr{A}$. Again, since $1_{\mathscr{C}}$ is $\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, so $\{z_d : g \in \mathscr{C}\}$ has a finite $\Sigma\alpha$-subshelter, say $\{z_{d_k} : g \in \mathscr{C}\}$ $(k \in \mathscr{I}_n)$ such that $\sum_{d} z_{d_k}(g) \geq \alpha$ for all $g \in \mathscr{C}$, as $\mathscr{C} \subseteq z^{-1}_{d_k}(0, 1]$ for each k. Now, put $e = e_{d_1} \cup e_{d_2} \cup \ldots\ldots \cup e_{d_n}$ and $z = z_{d_1} \cap z_{d_2} \cap \ldots\ldots \cap z_{d_n}$. Hence we see that $\mathscr{A} \subseteq e^{-1}(0, 1]$ and $\mathscr{C} \subseteq z^{-1}(0, 1]$. Thus $e, z \in \rho$, as e and z are the union and finite intersection of open fuzzy sets respectively.

For the contrary, take into consideration the fuzzy $\mathscr{T}_1$-space in the illustration of the theorem (3.7). Afresh, presume $1_{\mathscr{A}}$, $1_{\mathscr{C}} \in I^{\mathscr{S}}$ defined with $1_{\mathscr{A}}(a) = 0$, $1_{\mathscr{A}}(b) = 0.09$ and $1_{\mathscr{C}}(a) = 0.06$, $1_{\mathscr{C}}(b) = 0$, so $\mathscr{A} = \{b\}$, $\mathscr{C} = \{a\}$. Now, e_1, $e_2 \in \rho$, where $e_1^{-1}(0, 1] = \{a\}$, $e_2^{-1}(0, 1] = \{b\}$. As a result, we can see that $\mathscr{A} \subseteq e_2^{-1}(0, 1]$ and $\mathscr{C} \subseteq e_1^{-1}(0, 1]$, where $1_{\mathscr{A}}$ and $1_{\mathscr{C}}$ are disjoint. Choose $\alpha = 0.07$. Consequently, we observe that $1_{\mathscr{A}}$ and $1_{\mathscr{C}}$ are not $\Sigma\alpha$-compacts in $(\mathscr{S}, \rho)$, because of $\sum_{k} e_k(b) < \alpha$, whence $b \in \mathscr{A}$ and $\sum_{k} e_k(a) < \alpha$, whence $a \in \mathscr{C}$ for $k = 1, 2, 3$. As a result, the theoretical discussion does not hold true in all context.

Theorem 3.9. Presume $(\mathscr{S}, \rho)$ is a fuzzy $\mathscr{T}_2$-space (Hausdorff) and $\mathscr{A} \subset \mathscr{S}$. If $1_{\mathscr{A}}$ is $\Sigma\alpha$-compact subset in $(\mathscr{S}, \rho)$ and $g \notin \mathscr{A}$, after that there exist $e, z \in \rho$ such that $e(g) > 0$ and $\mathscr{A} \subseteq z^{-1}(0, 1]$ and $e \cap z = 0$. But the converse is not true everywhere.

Proof: Suppose $d \in \mathscr{A}$. Since $g \notin \mathscr{A}$, then it is clear that $g \neq d$. But $(\mathscr{S}, \rho)$ is fuzzy Hausdorff, then there exist $e_d, z_d \in \rho$ such that $e_d(g) > 0$, $z_d(d) > 0$ and $e_d \cap z_d = 0$. Since $1_{\mathscr{A}}$ is $\Sigma\alpha$-compact in fuzzy Hausdorff space $(\mathscr{S}, \rho)$, then it amply demonstrates that $\sum_{d} z_d(d) \geq \alpha$ for all $d \in \mathscr{A}$ i.e. $\{z_d : d \in \mathscr{A}\}$, which is an open $\Sigma\alpha$-shelter of $1_{\mathscr{A}}$ in $(\mathscr{S}, \rho)$ and consequently $\{z_d : d \in \mathscr{A}\}$ has a finite $\Sigma\alpha$-subshelter, declare $\{z_{d_k} : d \in \mathscr{A}\}$ $(k \in \mathscr{I}_n)$ in a way that $\sum_{d} z_{d_k}(d) \geq \alpha$ for all $d \in \mathscr{A}$. Now, let $e = e_{d_1} \cap e_{d_2} \cap \ldots\ldots \cap e_{d_n}$ and $z = z_{d_1} \cup z_{d_2} \cup \ldots\ldots \cup z_{d_n}$. Hence we observe that $e, z \in \rho$, as e and z are the finite intersection and union of open fuzzy sets as well as. Moreover, $e(g) > 0$, as $e_{d_k}(g) > 0$ for each k and $\mathscr{X} \subseteq z^{-1}(0, 1]$.

Lastly, we must demonstrate that $e \cap z = 0$. Since $e_{d_k} \cap z_{d_k} = 0 \Rightarrow e \cap z_{d_k} = 0$, distributive law allows us to $e \cap z = e \cap (z_{d_1} \cup z_{d_2} \cup \ldots\ldots \cup z_{d_n}) = 0$.
Regarding the converse, take into consideration the fuzzy topology ρ in the instance of the theorem (3.7), after that, $(\mathscr{S}, \rho)$ is also fuzzy Hausdorff space. Presume $1_{\mathscr{X}} \in I^{\mathscr{S}}$ is described by $1_{\mathscr{X}}(a) = 0.06$, $1_{\mathscr{X}}(b) = 0$, then $\mathscr{X} = \{a\}$ and $b \notin \mathscr{X}$. Here e_1, e_2 $\in \rho$, where $e_2(b) > 0$ and $e_1^{-1}(0, 1] = \{a\}$. Therefore, $\mathscr{X} \subseteq e_1^{-1}(0, 1]$. Take $\alpha = 0.04$. Thus $1_{\mathscr{X}}$ is not $\Sigma\alpha$ -compact in $(\mathscr{S}, \rho)$, as $\sum_k e_k(a) < \alpha$, whence $a \in \mathscr{X}$, for $k = 1, 2, 3$. As a result, the theoretical discussion is not always applicable.

Theorem 3.10. Consider $(\mathscr{S}, \rho)$ is a fuzzy Hausdorff space and $\mathscr{X}$, $\mathscr{C} \subset \mathscr{S}$. If $1_{\mathscr{X}}$, $1_{\mathscr{C}}$ are disjoint $\Sigma\alpha$ -compact subsets in $(\mathscr{S}, \rho)$, after that there exist $e, z \in \rho$ so as to $\mathscr{X} \subseteq e^{-1}(0, 1]$ and $\mathscr{C} \subseteq z^{-1}(0, 1]$ and $e \cap z = 0$. On the other hand, it is not always the case.
Proof: Suppose $d \in \mathscr{X}$. Thus, it is evident that $d \notin \mathscr{C}$, as $1_{\mathscr{X}}$ and $1_{\mathscr{C}}$ are disjoint. Since $1_{\mathscr{C}}$ is $\Sigma\alpha$ -compact in fuzzy Hausdorff space $(\mathscr{S}, \rho)$, then by theorem (3.9), there exist e_d, $z_d \in \rho$ such that $e_d(d) > 0$ and $\mathscr{C} \subseteq z_d^{-1}(0, 1]$ and $e_d \cap z_d = 0$. Since $e_d(d) > 0$ and $1_{\mathscr{X}}$ is $\Sigma\alpha$ -compact in $(\mathscr{S}, \rho)$, therefore it is evident that $\sum_d e_d(d) \geq \alpha$ for all $d \in \mathscr{X}$ and consequently $\{e_d : d \in \mathscr{X}\}$, which is an open $\Sigma\alpha$ -shelter of $1_{\mathscr{X}}$ and thus $\{e_d : d \in \mathscr{X}\}$ has a finite $\Sigma\alpha$ -subshelter, say $\{e_{d_k} : d \in \mathscr{X}\}$ $(k \in \mathscr{J}_n)$ so as to $\sum_d e_{d_k}(d) \geq \alpha$ $(k \in \mathscr{J}_n)$ for all $d \in \mathscr{X}$. Afresh, given that $1_{\mathscr{C}}$ is $\Sigma\alpha$ -compact in $(\mathscr{S}, \rho)$, so $\{z_d : g \in \mathscr{C}\}$, which has a finite $\Sigma\alpha$ -subshelter, like as $\{z_{d_k} : g \in \mathscr{C}\}$ $(k \in \mathscr{J}_n)$ so as to $\sum_d z_{d_k}(g) \geq \alpha$ $(k \in \mathscr{J}_n)$ for all $g \in \mathscr{C}$, as $\mathscr{C} \subseteq z_{d_k}^{-1}(0, 1]$ for each k. Presently, consider $e = e_{d_1} \cup e_{d_2} \cup \ldots\ldots \cup e_{d_n}$ and $z = z_{d_1} \cap z_{d_2} \cap \ldots\ldots \cap z_{d_n}$. Hence we observe that $\mathscr{X} \subseteq e^{-1}(0, 1]$ and $\mathscr{C} \subseteq z^{-1}(0, 1]$. Thus $e, z \in \rho$, as e and z are the union and finite intersection of open fuzzy sets respectively.

Lastly, we demonstrate that $e \cap z = 0$. For starts, we note that $e_{d_k} \cap z_{d_k} = 0$ for each $k \Rightarrow e_{d_k} \cap z = 0$, distributive law allows us to $e \cap z = (e_{d_1} \cup e_{d_2} \cup \ldots\ldots \cup e_{d_n})$ $\cap z = 0$.

Regarding the converse, take into consideration the fuzzy topology ρ in the instance of the theorem (3.7); after that, $(\mathcal{S}, \rho)$ is also a fuzzy Hausdorff space. Presume $1_{\mathcal{A}}$, $1_{\mathcal{C}} \in I^{\mathcal{S}}$ described with $1_{\mathcal{A}}(a) = 0$, $1_{\mathcal{A}}(b) = 0.09$ and $1_{\mathcal{C}}(a) = 0.08$, $1_{\mathcal{C}}(b) = 0$, then $\mathcal{A} = \{b\}$, $\mathcal{C} = \{a\}$. Now, e_1, $e_2 \in \rho$, where $e_1^{-1}(0, 1] = \{a\}$ and $e_2^{-1}(0, 1] = \{b\}$. So, it is evident that $\mathcal{A} \subseteq e_2^{-1}(0, 1]$, $\mathcal{C} \subseteq e_1^{-1}(0, 1]$ and $e_1 \cap e_2 = 0$, where $1_{\mathcal{A}}$ and $1_{\mathcal{C}}$ are disjoint. Make a choice $\alpha = 0.07$. Thus we can see that $1_{\mathcal{A}}$ and $1_{\mathcal{C}}$ are not $\Sigma\alpha$ -compacts in $(\mathcal{S}, \rho)$, whereas $\sum_k e_k(b) < \alpha$, whence $b \in \mathcal{A}$ and $\sum_k e_k(a) < \alpha$, whence $a \in \mathcal{C}$ for $k = 1, 2, 3$. Consequently, not all places have the converse of theorem; it is not true everywhere.

Theorem 3.11. Consider $(\mathcal{S}, \mathcal{V})$ is a topological space and $(\mathcal{S}, \omega(\mathcal{V}))$ is an fts, then $(\mathcal{S}, \mathcal{V})$ is compact iff $(\mathcal{S}, \omega(\mathcal{V}))$ is $\Sigma\alpha$ -compact.

Proof: Pretend $(\mathcal{S}, \mathcal{V})$ is compact. Presume $\{e_i : i \in \mathcal{J}\}$, which is an open $\Sigma\alpha$ -shelter of $(\mathcal{S}, \omega(\mathcal{V}))$ i.e. $\sum_i e_i(g) \geq \alpha$ for whole $g \in \mathcal{S}$. Then $e_i^{-1}(a, 1] \in \mathcal{V}$ and hence $\{e_i^{-1}(a, 1] : e_i^{-1}(a, 1] \in \mathcal{V}\}$ is an open cover of $(\mathcal{S}, \mathcal{V})$. Whereas $(\mathcal{S}, \mathcal{V})$ is compact, so $\{e_i^{-1}(a, 1] : e_i^{-1}(a, 1] \in \mathcal{V}\}$, which has a finite subcover, i.e. there exists $e_{i_k}^{-1}(a, 1] \in \mathcal{V}$ $(k \in \mathcal{J}_n)$ so as to $\mathcal{S} = e_{i_1}^{-1}(a, 1] \cup e_{i_2}^{-1}(a, 1] \cup \ldots\ldots \cup e_{i_n}^{-1}(a, 1]$. Presently, as we can see, there exist $e_{i_k} \in \{e_i\}$ $(k \in \mathcal{J}_n)$ so as to $\sum_i e_{i_k}(g) \geq \alpha$ for whole $g \in \mathcal{S}$ and it demonstrates that $\{e_{i_k}\}$ $(k \in \mathcal{J}_n)$ form a finite $\Sigma\alpha$ -subshelter of $\{e_i : i \in \mathcal{J}\}$. In this way, $(\mathcal{S}, \omega(\mathcal{V}))$ is $\Sigma\alpha$ -compact.

On the contrary, let us assume that $(\mathcal{S}, \omega(\mathcal{V}))$ is $\Sigma\alpha$ -compact. Suppose $\{\mathcal{U}_j : j \in \mathcal{J}\}$ is an open cover of $(\mathcal{S}, \mathcal{V})$ i.e. $\mathcal{S} = \bigcup_{j \in \mathcal{J}} \{\mathcal{U}_j : \mathcal{U}_j \in \mathcal{V}\}$. Since $1_{\mathcal{U}_j}$ is l.s.c. then $1_{\mathcal{U}_j} \in \omega(\mathcal{V})$ and $\{1_{\mathcal{U}_j} : 1_{\mathcal{U}_j} \in \omega(\mathcal{V})\}$ is an open $\Sigma\alpha$ -shelter of $(\mathcal{S}, \omega(\mathcal{V}))$. But $(\mathcal{S}, \omega(\mathcal{V}))$ is $\Sigma\alpha$ -compact, so $\{1_{\mathcal{U}_j} : 1_{\mathcal{U}_j} \in \omega(\mathcal{V})\}$ has a finite $\Sigma\alpha$ -subshelter, say $\{1_{\mathcal{U}_{j_k}} : 1_{\mathcal{U}_{j_k}} \in \omega(\mathcal{V})\}$ $(k \in \mathcal{J}_n)$ such that $\sum_i 1_{\mathcal{U}_j}(g) \geq \alpha$ for whole $g \in \mathcal{S}$. Therefore, we can write $\mathcal{S} = \mathcal{U}_{j_1} \cup \mathcal{U}_{j_2} \cup \ldots\ldots \cup \mathcal{U}_{j_n}$, and it is evident that, $\{\mathscr{U}_{j_k}\}$ $(k \in \mathscr{I}_n)$ is a finite subcover of $(\mathscr{S}, \mathscr{V})$. As a result, $(\mathscr{S}, \mathscr{V})$ is compact.

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