

Some Star and Bistar Related Divisor Cordial Graphs

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Abstract. A divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \dots, |V|\}$ such that an edge uv is assigned the label 1 if either $f(u) | f(v)$ or $f(v) | f(u)$ and the label 0 if $f(u) \nmid f(v)$, then number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a divisor cordial labeling is called a divisor cordial graph. In this paper we prove that splitting graphs of star $K_{1,n}$ and bistar $B_{n,n}$ are divisor cordial graphs. Moreover we show that degree splitting graph of $B_{n,n}$, shadow graph of $B_{n,n}$ and square graph of $B_{n,n}$ admit divisor cordial labeling..

Keywords: Divisor cordial labeling, star, bistar.

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1. Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For standard terminology and notations related to graph theory we refer to Gross and Yellen [1] while for number theory we refer to Burton [2]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

Definition 1.1. If the vertices are assigned values subject to certain condition(s) then it is known as *graph labeling*.

Any graph labeling will have the following three common characteristics:

1. A set of numbers from which vertex labels are chosen;
2. A rule that assigns a value to each edge;

3. A condition that this value has to satisfy.

According to Beineke and Hegde [3] graph labeling serves as a frontier between number theory and structure of graphs. Graph labelings have many applications within mathematics as well as to several areas of computer science and communication networks. According to Graham and Sloane [4] the harmonious labelings are closely related to problems in error correcting codes while odd harmonious labeling is useful to solve undetermined equations as described by Liang and Bai [5]. The optimal linear arrangement concern to wiring network problems in electrical engineering and placement problems in production engineering can be formalised as a graph labeling problem as stated by Yegnanaryanan and Vaidhyathan [6]. For a dynamic survey of various graph labeling problems along with an extensive bibliography we refer to Gallian [7].

Definition 1.2. A mapping $f : V(G) \rightarrow \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v of G under f .

Notation 1.3. If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Then

$v_f(i)$ = number of vertices of G having label i under f

$e_f(i)$ = number of vertices of G having label i under f^*

Definition 1.4. A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *cordial* if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit [8]. This concept is explored by many researchers like Andar *et al.* [9,10], Vaidya and Dani [11] and Nasreen [12]. Motivated through the concept of cordial labeling Babujee and Shobana [13] introduced the concepts of cordial languages and cordial numbers. Some labeling schemes are also introduced with minor variations in cordial theme. Product cordial labeling, total product cordial labeling and prime cordial labeling are among mention a few. The present work is focused on divisor cordial labeling.

Definition 1.5. A *prime cordial labeling* of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \rightarrow \{1, 2, 3, \dots, |V(G)|\}$ and the induced function $f^* : E(G) \rightarrow \{0,1\}$ is defined by $f^*(e = uv) = 1$, if $\gcd(f(u), f(v)) = 1$;
 $= 0$, otherwise.

which satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. A graph which admits prime cordial labeling is called a *prime cordial graph*.

The concept of prime cordial labeling was introduced by Sundaram *et al.* [14] and in the same paper they have investigated several results on prime cordial labeling. Vaidya and Vihol [15,16] as well as Vaidya and Shah [17,18] have proved many results on prime cordial labeling.

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Motivated by the concept of prime cordial labeling, Varatharajan *et al.* [19] introduced a new concept called divisor cordial labeling by combining the divisibility concept in Number theory and Cordial labeling concept in Graph labeling. This is defined as follows.

Definition 1.6. Let $G = (V(G), E(G))$ be a simple graph and $f : \{1, 2, \dots, |V(G)|\} \rightarrow V(G)$ be a bijection. For each edge uv , assign the label 1 if either $f(u) | f(v)$ or $f(v) | f(u)$ and the label 0 if $f(u) \nmid f(v)$. f is called a *divisor cordial labeling* if $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a *divisor cordial graph*.

In the same paper [19] they have proved that path, cycle, wheel, star, $K_{2,n}$ and $K_{3,n}$ are divisor cordial graphs while K_n is not divisor cordial for $n \geq 7$. Same authors in [20] have discussed divisor cordial labeling of full binary tree as well as some star related graphs.

It is important to note that prime cordial labeling and divisor cordial labeling are two independent concepts. A graph may possess one or both of these properties or neither as exhibited below.

- i) P_n ($n \geq 6$) is both prime cordial as proved in [14] and divisor cordial as proved in [19].
- ii) C_3 is not prime cordial as proved in [14], but it is divisor cordial as proved in [19].
- iii) We found that a 7-regular graph with 12 vertices admits prime cordial labeling, but does not admit divisor cordial labeling.
- iv) Complete graph K_7 is not a prime cordial as stated in Gallian [7] and not divisor cordial as proved in [19].

Generally there are three types of problems that can be considered in the area of graph labeling.

1. How a particular labeling is affected under various graph operations;
2. To investigate new graph families which admit particular graph labeling;
3. Given a graph theoretic property P, characterize the class/classes of graphs with property P that admit particular graph labeling.

The problems of second type are largely discussed while the problems of first and third types are not so often but they are of great importance. The present work is aimed to discuss the problems of first kind in the context of divisor cordial labeling.

Definition 1.7. For a graph G the *splitting graph* $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 1.8. [21] Let $G = (V(G), E(G))$ be a graph with $V = S_1 \cup S_2 \cup S_3 \cup \dots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V \setminus \cup S_i$. The *degree splitting graph* of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, w_3, \dots, w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

Definition 1.9. The *shadow graph* $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G'' . Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G'' .

Definition 1.10. For a simple connected graph G the *square of graph* G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G .

2. Main Results

Theorem 2.1. $S'(K_{1,n})$ is a divisor cordial graph.

Proof : Let $v_1, v_2, v_3, \dots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and $u, u_1, u_2, u_3, \dots, u_n$ are added vertices corresponding to $v, v_1, v_2, v_3, \dots, v_n$ to obtain $S'(K_{1,n})$. Let G be the graph $S'(K_{1,n})$ then $|V(G)| = 2n + 2$ and $|E(G)| = 3n$. To define $f : V(G) \rightarrow \{1, 2, \dots, 2n + 2\}$ we consider following three cases.

Case 1: $n=2$ to 8

For $n=2$, $f(v)=4, f(u)=1, f(v_1)=3, f(v_2)=2$, and $f(u_1)=5, f(u_2)=6$. Then $e_f(0) = 3 = e_f(1)$.

For $n=3$, $f(v)=3, f(u)=2, f(v_1)=5, f(v_2)=6, f(v_3)=7$ and $f(u_1)=1, f(u_2)=9, f(u_3)=4$. Then $e_f(0) = 5, e_f(1) = 4$.

For $n=4$, $f(v)=3, f(u)=2, f(v_1)=5, f(v_2)=6, f(v_3)=8, f(v_4)=10$ and $f(u_1)=1, f(u_2)=4, f(u_3)=7, f(u_4)=9$. Then $e_f(0) = 6 = e_f(1)$.

For $n=5$, $f(v)=3, f(u)=2, f(v_1)=5, f(v_2)=6, f(v_3)=8, f(v_4)=10, f(v_5)=12$ and $f(u_1)=1, f(u_2)=4, f(u_3)=7, f(u_4)=9, f(u_5)=11$. Then $e_f(0) = 7, e_f(1) = 8$.

For $n=6$, $f(v)=3, f(u)=2, f(v_1)=5, f(v_2)=6, f(v_3)=8, f(v_4)=10, f(v_5)=12, f(v_6)=14$ and $f(u_1)=1, f(u_2)=4, f(u_3)=7, f(u_4)=9, f(u_5)=11, f(u_6)=13$. Then $e_f(0) = 9 = e_f(1)$.

For $n=7$, $f(v)=3, f(u)=2, f(v_1)=5, f(v_2)=6, f(v_3)=8, f(v_4)=10, f(v_5)=12, f(v_6)=14, f(v_7)=16$ and $f(u_1)=1, f(u_2)=4, f(u_3)=7, f(u_4)=9, f(u_5)=11, f(u_6)=13, f(u_7)=15$. Then $e_f(0) = 10, e_f(1) = 11$.

For $n=8$, $f(v)=3, f(u)=2, f(v_1)=1, f(v_2)=6, f(v_3)=12, f(v_4)=5, f(v_5)=7, f(v_6)=11, f(v_7)=13, f(v_8)=17$ and $f(u_1)=4, f(u_2)=8, f(u_3)=10, f(u_4)=14, f(u_5)=16, f(u_6)=18, f(u_7)=9, f(u_8)=15$. Then $e_f(0) = 12 = e_f(1)$.

Now for the remaining two cases let,

$$s = \left\lfloor \frac{n+1}{3} \right\rfloor, m = \left\lfloor \frac{2n+2}{3} \right\rfloor - 1 - s, t = \left\lceil \frac{3n}{2} \right\rceil - (n + s + 2).$$

$$x_1 = s + t + 1, x_2 = n - x_1, x_3 = n - s + m - t, x_4 = n - x_3.$$

Case 2: $n=9, 10, 11, 12$ ($t=0$)

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$$f(v) = 2, \quad f(u) = 3,$$

$$f(v_1) = 1,$$

$$f(v_{1+i}) = 6i; \quad 1 \leq i \leq s$$

$$f(v_{x_1+1+2i}) = 5 + 6i; \quad 0 \leq i < \left\lfloor \frac{x_2}{2} \right\rfloor$$

$$f(v_{x_1+2+2i}) = 7 + 6i; \quad 0 \leq i < \left\lfloor \frac{x_2}{2} \right\rfloor$$

$$f(u_{1+2i}) = 4 + 6i; \quad 0 \leq i < \left\lfloor \frac{n-s}{2} \right\rfloor$$

$$f(u_{2+2i}) = 8 + 6i; \quad 0 \leq i < \left\lfloor \frac{n-s}{2} \right\rfloor$$

$$f(u_{n-s+i}) = 9 + 6(i-1); \quad 1 \leq i \leq m$$

For the vertices $f(u_{x_3+1}), f(u_{x_3+2}), \dots, f(u_n)$ assign distinct remaining odd numbers.

This assigns all the vertex labels for case 2.

Case 3: $n \geq 13$ ($t \geq 1$)

$$f(v) = 2, \quad f(u) = 3,$$

$$f(v_1) = 1,$$

$$f(v_{1+i}) = 6i; \quad 1 \leq i \leq s$$

$$f(v_{s+1+i}) = 9 + 6(i-1); \quad 1 \leq i \leq t$$

$$f(v_{x_1+1+2i}) = 5 + 6i; \quad 0 \leq i < \left\lfloor \frac{x_2}{2} \right\rfloor$$

$$f(v_{x_1+2+2i}) = 7 + 6i; \quad 0 \leq i < \left\lfloor \frac{x_2}{2} \right\rfloor$$

$$f(u_{1+2i}) = 4 + 6i; \quad 0 \leq i < \left\lfloor \frac{n-s}{2} \right\rfloor$$

$$f(u_{2+2i}) = 8 + 6i; \quad 0 \leq i < \left\lfloor \frac{n-s}{2} \right\rfloor$$

$$f(u_{n-s+i}) = f(v_{s+t+1}) + 6i; \quad 1 \leq i \leq m-t$$

For the vertices $f(u_{x_3+1}), f(u_{x_3+2}), \dots, f(u_n)$ assign distinct remaining odd numbers. This assigns all the vertex labels for case 3.

In view of the above labeling pattern,

$$f(v) | f(u_1), \dots, f(v) | f(u_{n-s}) \text{ and } f(v_1) | f(v), f(v_1) | f(u). \text{ Moreover}$$

$$f(v) | f(v_2), f(v) | f(v_3), \dots, f(v) | f(v_{1+s}), \quad f(u) | f(v_2), f(u) | f(v_3), \dots, \quad f(u) | f(v_{1+s})$$

$$\text{and } f(u) | f(v_{s+2}), f(u) | f(v_{s+3}), \dots, f(u) | f(v_{1+s+t}).$$

Hence, $e_f(1) = n - s + 2 + s + s + t = n + s + 2 + \left\lceil \frac{3n}{2} \right\rceil - (n + S + 2)$.

Therefore, in last two case $e_f(1) = \left\lceil \frac{3n}{2} \right\rceil$ and $e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor$.

Thus, in all the cases we have $|e_f(0) - e_f(1)| \leq 1$.

Hence, $S'(K_{1,n})$ is a divisor cordial graph.

Example 2.2. Let $G = S'(K_{1,13})$, $|V(G)| = 28$ and $|E(G)| = 39$. In accordance with Theorem 2.1 we have $s = 4, m = 4, t = 1, x_1 = 6, x_2 = 7, x_3 = 12, x_4 = 1$ and using the labeling pattern described in case 2. The corresponding divisor cordial labeling is shown in Figure 1. It is easy to visualize that $e_f(0) = 19$ and $e_f(1) = 20$.

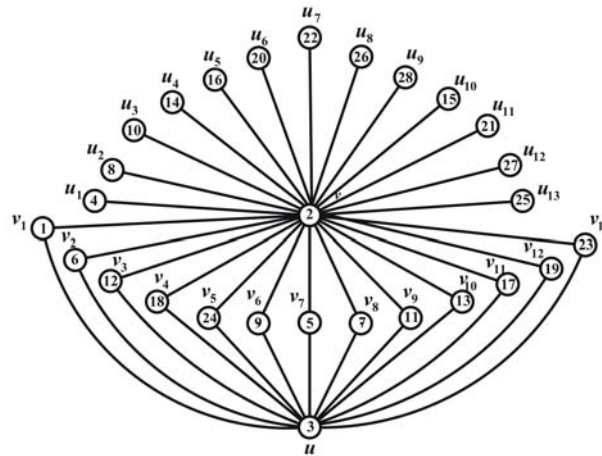


Figure 1: Divisor cordial labeling of $G = S'(K_{1,13})$

Theorem 2.3. $S'(B_{n,n})$ is a divisor cordial graph.

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i, 1 \leq i \leq n\}$ where u_i, v_i are pendant vertices. In order to obtain $S'(B_{n,n})$, add u', v', u'_i, v'_i vertices corresponding to u, v, u_i, v_i , where $1 \leq i \leq n$. If $G = S'(B_{n,n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 6n+3$. We define vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, 4(n+1)\}$ as follows. Let p_1 be the highest prime number $< 4(n+1)$.

$$\begin{aligned} f(u) &= 2, & f(u') &= 1, \\ f(v) &= 4, & f(v') &= p_1, \\ f(u_i) &= 6 + 2(i-1); & 1 \leq i \leq n \\ f(u'_i) &= f(u_i) + 2i; & 1 \leq i \leq n \end{aligned}$$

For the vertices v_1, v_2, \dots, v_n and v'_1, v'_2, \dots, v'_n we assign distinct odd numbers (except p_1).

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In view of the above labeling pattern we have, $e_f(0) = 3n + 1, e_f(1) = 3n + 2$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $S'(B_{n,n})$ is a divisor cordial graph.

Illustration 2.4. Divisor cordial labeling of the graph $S'(B_{6,6})$ is shown in Figure 2.

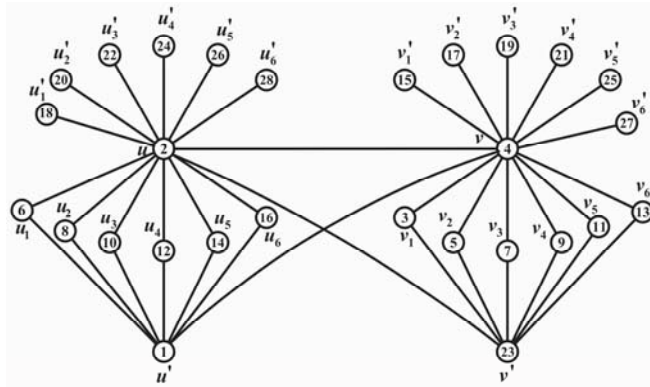


Figure 2: Divisor cordial labeling of the graph $S'(B_{6,6})$

Theorem 2.5. $DS(B_{n,n})$ is a divisor cordial graph.

Proof: Consider $B_{n,n}$ with $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$, where u_i, v_i are pendant vertices. Here $V(B_{n,n}) = V_1 \cup V_2$, where $V_1 = \{u_i, v_i : 1 \leq i \leq n\}$ and $V_2 = \{u, v\}$. Now in order to obtain $DS(B_{n,n})$ from G , we add w_1, w_2 corresponding to V_1, V_2 . Then $|V(DS(B_{n,n}))| = 2n + 4$ and

$E(DS(B_{n,n})) = \{uv, uw_2, vw_2\} \cup \{uu_i, vv_i, w_1u_i, w_1v_i : 1 \leq i \leq n\}$ so $|E(DS(B_{n,n}))| = 4n + 3$.

We define vertex labeling $f : V(DS(B_{n,n})) \rightarrow \{1, 2, \dots, 2n + 4\}$ as follows.

$$f(u) = 4, \quad f(v) = 2n + 3,$$

$$f(w_1) = 1, \quad f(w_2) = 2,$$

$$f(u_i) = 3 + 2(i - 1); \quad 1 \leq i \leq n$$

$$f(v_i) = 6 + 2(i - 1); \quad 1 \leq i \leq n$$

In view of the above defined labeling pattern we have, $e_f(0) = 2n + 2, e_f(1) = 2n + 1$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $DS(B_{n,n})$ is a divisor cordial graph.

Illustration 2.6. Divisor cordial labeling of the graph $DS(B_{5,5})$ is shown in Figure 3.

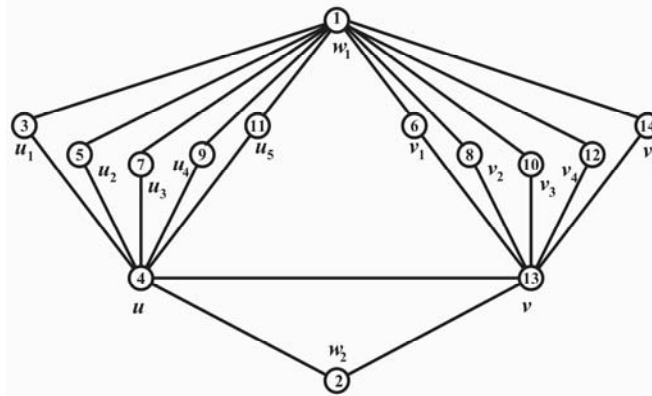


Figure 3: Divisor cordial labeling of the graph $DS(B_{5,5})$

Theorem 2.7. $D_2(B_{n,n})$ is a divisor cordial graph.

Proof: Consider two copies of $B_{n,n}$. Let $\{u, v, u_i, v_i, 1 \leq i \leq n\}$ and $\{u', v', u'_i, v'_i, 1 \leq i \leq n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$. Let G be the graph $D_2(B_{n,n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 4(2n+1)$. We define vertex labeling $f: V(G) \rightarrow \{1, 2, \dots, 4(n+1)\}$ as follows.

Let p_1 be the highest prime number and p_2 be the second highest prime number such that $p_2 < p_1 < 4(n+1)$.

$$\begin{aligned} f(u) &= 2, & f(u') &= 1, \\ f(v) &= p_1, & f(v') &= p_2, \\ f(u_i) &= 6 + 2(i-1); & 1 \leq i \leq n \\ f(u'_i) &= f(u_n) + 2i; & 1 \leq i \leq n \\ f(v_1) &= 4, \end{aligned}$$

For the vertices v_2, v_3, \dots, v_n and v'_1, v'_2, \dots, v'_n we assign distinct odd numbers (except p_1 and p_2).

In view of the above defined labeling pattern we have, $e_f(0) = 4n + 2 = e_f(1)$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $D_2(B_{n,n})$ is a divisor cordial graph.

Illustration 2.8. Divisor cordial labeling of graph $D_2(B_{5,5})$ is shown in Figure 4.

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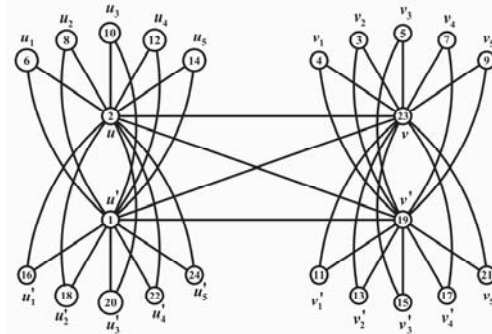


Figure 4: Divisor cordial labeling of graph $D_2(B_{5,5})$

Theorem 2.9. $B_{n,n}^2$ is a divisor cordial graph.

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i, 1 \leq i \leq n\}$ where u_i, v_i are pendant vertices. Let G be the graph $B_{n,n}^2$ then $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$.

We define vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, 2n + 2\}$ as follows.

Let p_1 be the highest prime number $< 2n + 2$.

$$f(u) = 1, \quad f(v) = p_1,$$

$$f(u_1) = 2;$$

$$f(v_i) = 4 + 2(i - 1); \quad 1 \leq i \leq n$$

For the vertices u_2, u_3, \dots, u_n we assign distinct odd numbers (except p_1).

In view of the above defined labeling pattern we have, $e_f(0) = 2n, e_f(1) = 2n + 1$

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $B_{n,n}^2$ is a divisor cordial graph

Illustration 2.10. Divisor cordial labeling of the graph $B_{7,7}^2$ is shown in Figure 5.

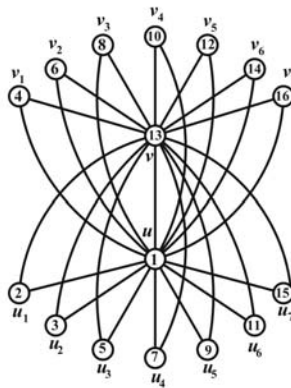


Figure 5: Divisor cordial labeling of the graph $B_{7,7}^2$

3. Concluding Remarks

As all the graphs are not divisor cordial graphs it is very interesting and challenging as well to investigate divisor cordial labeling for the graph or graph families which admit divisor cordial labeling. Here we have contributed some new results by investigating divisor cordial labeling for some star and bistar related graphs.

Varatharajan *et al.* [19] have proved that $K_{1,n}$ and $B_{n,n}$ are divisor cordial graphs while we prove that the splitting graphs of star $K_{1,n}$ and bistar $B_{n,n}$ are also divisor cordial graphs. Thus divisor cordiality remains invariant for the splitting graphs of $K_{1,n}$ and $B_{n,n}$. It is also invariant for degree splitting graph of $B_{n,n}$, shadow graph of $B_{n,n}$ and square graph of $B_{n,n}$.

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