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Fuzzy Graph Coloring Technique to Classify the Accidental Zone of a Traffic Control

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Abstract. Let G= (V,E,σ,μ) be a simple connected undirected fuzzy graph. In this paper, we use a fuzzy graph model to represent a traffic network of a city and discuss a method to find the different type of accidental zones in a traffic flows. Depending on the possibility of accident, this paper classifies accidental zone of a traffic flow into three type's namely α-strong, β-strong and δ-strong accidental zones. The advantage of this type of classifications is that it helps to minimize the accident. We also discuss two different approaches of fuzzy graph coloring for solving two major problems in traffic control. First approach is based on fuzzy coloring of fuzzy graphs and fuzziness of vertices. Using this approach, we can minimize the total waiting time of a traffic flow. It will help to reduce the traffic jam of a road. Second approach is based on the vertex coloring function of a fuzzy graph (crisp mode). The function is based on α cut of graph Gα=(Vα,Eα),the α cuts of fuzzy graph G. Here α value is used for aptitude level of the driver.

Keywords: Fuzzy graphs, fuzzy graph coloring, traffic lighting

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Fuzzy graphs were introduced by Rosenfeld [2], ten years after Zadeh's landmark paper ''Fuzzy Sets" [8]. Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of information theory, neural network, expert systems, cluster analysis, medical diagnosis, control theory, etc.

Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties [8]. Bhattacharya [11] has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges.

In graph theory, arc analysis is not very important as all arcs are strong in the sense of [4]. But in fuzzy graphs it is very important to identify the nature of arcs and no such analysis on arcs is available in the literature except the division of arcs as strong and non strong in [12]. Depending on the strength of an arc, the authors Sunil Mathew and Sunitha[13] classify strong arcs into two types namely α -strong and β-strong and introduce two other types of arcs in fuzzy graphs which are not strong and are termed as δ and δ^* arcs. Graph coloring is one of the most important concepts in graph theory and is used in many real time applications like job scheduling, aircraft scheduling, computer network security, map coloring and GSM mobile phone networks, automatic channel allocation for small wireless local area networks. The proper coloring of a graph is the coloring of the vertices with minimal number of colors such that no two adjacent vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph. The fuzzy coloring of a fuzzy graph was defined by the authors in Eslahchi and Onagh [9]. Then Pourpasha [10] also introduced different approaches to color the fuzzy graph. Section 2 contains preliminaries and in Section 3 we introduce different type of arc in a fuzzy graph. In Section 4, we discuss different type of vertices in fuzzy graph. In Section 5, we discuss how to represent any traffic flow using fuzzy graph and Section 6, we introduce a method to find different type of accidental zone in a traffic flow depending upon their possibility of accident. In Section 7 and Section 8, we introduce two different approaches to color this fuzzy traffic graph.

2. Preliminary Notes

In this Section, we present some definitions related to fuzzy graph and their properties.

Definition 2.1. Fuzziness occurs in a fuzzy graph in five different ways, introduced by M.Blue [14]. Fuzzy graph is a graph G_F satisfying one of the following types of fuzziness (G_F of the ith type) or any of its combination:

(i) $G_{F1}=\{G_1,G_2,G_3 , . . . , G_F\}$ where fuzziness is on each graph G_i

(ii) $G_{F2}=\{ V, E_F \}$ where the edge set is fuzzy.

(iii) $G_{F3}=\{ V, E(t_F , h_F) \}$ where both the vertex and edge sets are crisp, but the edges have fuzzy heads $h(e_i)$ and fuzzy tails $t(e_{i.})$

(iv) $G_{F4}=\{V_F , E \}$ where the vertex set is fuzzy.

(v) $G_{F5} = \{V, E(w_F) \}$ where both the vertex and edge sets are crisp but the edges have fuzzy weights.

In this paper, we use a fuzzy graph G which is a combination of G_{F2} and G_{F4} . So fuzzy graph G= $G_{F2} \cup G_{F4}$. We can define this fuzzy graph using the membership values of vertices and edges. Let V be a finite nonempty set. The triple G= (V,E, σ, μ) is called a fuzzy graph on V where μ and σ are fuzzy sets on V and E $(V \times V)$ respectively, such that $\mu(\{u, v\}) \le \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

Let us consider a fuzzy graph G : (V,E, σ, μ) be V=(u,v,w,x) E={uv,vw,uw,wx,xu} where $\mu(u,v)=1= \mu(v,w)= \mu(u,w)$, $\mu(x,w)=0.5$,$\mu(u,x)=0.1$ and $\sigma=\{1,1,1,0.7\}$.

Note that a fuzzy graph is a generalization of crisp graph in which

$$\mu(v) = 1 \text{ for all } v \in V \text{ and}$$
$$\rho(i, j) = 1 \text{ if } (i, j) \in E$$
$$= 0 \text{ otherwise}$$

so all the crisp graphs are fuzzy graph but all fuzzy graphs are not crisp graph.

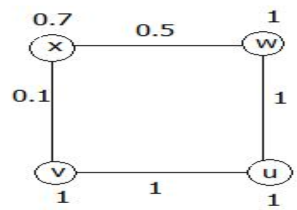


Fig 2.1:

Definition 2.2. The α cut of fuzzy graph defined as $G_\alpha = (V_\alpha, E_\alpha, \sigma, \mu)$ where $V_\alpha = \{v \in V | \sigma \geq \alpha\}$ and $E_\alpha = \{e \in E | \mu \geq \alpha\}$.

Definition 2.3. A path P of length n is a sequence of distinct nodes $u_1, u_2, u_3, \ldots u_n$ such that $\mu(u_{i-1} u_i) > 0$; i=1, 2, . . ., n and the degree of membership of a weakest arc is defined as its strength. The strength of the path is defined as $\min(\mu(u_{i-1} u_i))$.

Example 2.1. xuv be a path of fuzzy graph G in Fig 2.1.The path xuv consist of two edges xu and uv. The membership value of xu and uv are 0.1 and 1. The strength of the path xuv is $\min(\mu(xu), \mu(uv))=0.1$.

Definition 2.4. A fuzzy graph $G=(V,E,\sigma, \mu)$ is called strong if $\mu(u,v)= \min(\sigma(u), \sigma(v))$ for all(u,v)in μ^* and is complete if $\mu(u,v)= \min(\sigma(u), \sigma(v))$ for all (u,v) in σ^*.

Definition 2.5. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$. An x- y path P is called a strongest x- y path if its strength equals to $CONN_G(x,y)$.

Example 2.2. In the Fig 2.1, the paths between x and u nodes are xu, xwu, xwvu. Membership value of path xu is 0.1 and membership value of path the xwu is the minimum value between two edges xw and wu and that is 0.5. Similarly membership value of the path xwuv is also 0.5. The $CONN_G(xu)$ is maximum of the strengths of all paths between x and u. So the $CONN_G(xu)=Max(xu,xwu,xwvu)=Max(0.1,0.5,0.5)=0.5$

3. Types of arcs in a fuzzy graph

The notion of strength of connectedness plays a significant role in the structure of fuzzy graph. Depending on the value $CONN_G(x,y)$ of an arc (x,y) in a fuzzy graph G we define the following three different types of arcs. Note that $CONN_{G-(x,y)}(x,y)$ is the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x,y).

Definition 3.1. An arc (x,y) of G is called α-strong if $\mu(xy) > CONN_{G-(x,y)}(x,y)$.

Definition 3.2. An arc (x,y) of G is called β-strong if $\mu(xy)=CONN_{G-(x,y)}(x,y)$.

Definition 3.3. An arc (x,y) of G is called δ-strong if $\mu(xy)<CONN_{G-(x,y)}(x,y)$.

Definition 3.4. An arc (x,y) of G is called δ^*-strong if $\mu(xy)>\mu(uv)$ where (u,v) is the weakest arc in fuzzy graph

Definition 3.5. An fuzzy graph G=(V,E, ,σ, μ) is called an α-strong path if all its arcs are α strong and is called a β -strong path if all its arcs are β-strong.

Example 3.1. Let G : (V,E, σ, μ) where V=(u,v,w,x) E={uv,vw,uw,wx,xu} where, μ(u,v)=1= μ(w,v), μ(u,w)=0.4 μ(u,x)=0.1 and σ={1,1,1,0.5}. Now we will classify all the arc as α-strong, β-strong and δ-strong arc.
For the arc uv, μ(uv)= 1,
$CONN_G(uv)$=Max{uv,uwv,uxwv}
= Max{uv,min(uw,wv),min(ux,xw,wv)}.
After deleting the arc uv the value of $CONN_{G-(u,v)}$ is
$CONN_{G-(u,v)}$ =Max{min(uw,wv),min(ux,xw,wv)}
= Max{min(0.4,1),min(0.1,0.3,1)}
=Max{0.4,0.1}=0.4.

The value of μ(uv) is greater than $CONN_{G-(u,v)}$. so uv is a α-strong arc. Similarly, we can show that (v,w) and (w,x) are α strong arc. For the arc (w,u), μ(uw)=0.4. After deleting the arc uw the $CONN_{G-(u,w)}$ value will be 1. Here $CONN_{G-(u,w)} > \mu(uw)$.So it is a δ strong arc. Similarly (u,x) is also δ strong arc.

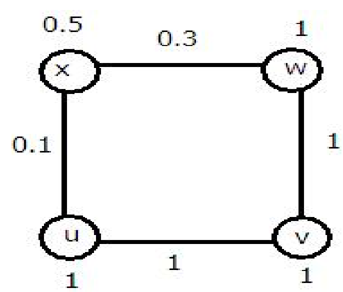


Fig 3.1:

5. Representing the traffic lights problem using fuzzy graph

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In many real world problems, we get partial information about that problem. So there is vagueness in the description of the objects or in its relationships or in both. To describe this type of relation, we need to design graph model with fission of type 1 fuzzy set. This fission of fuzzy set with graph is known as fuzzy graph.

In this paper, we represent the traffic flows as a fuzzy graph problem. Let we consider a traffic flow shown in Fig 5. 1. Each arrow in Fig 5. 1 indicates the vehicles will go from one direction to another direction. But numbers of vehicles in all paths are not always equal. Due to this reason, we consider it as fuzzy set whose membership value depends upon on vehicles number. If the number of vehicles in any path is high then its membership value will be high and if the number of vehicles in any path is low then its membership value will be low.

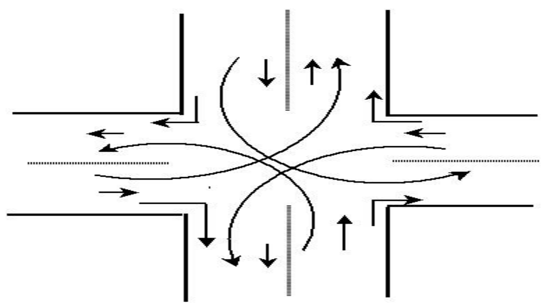


Fig 5.1:

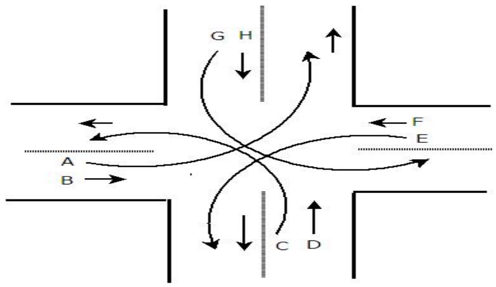
Since the four right turns do not interfere with the other traffic flows, they can safely be dropped from our discussion. The remaining traffic directions are shown in figure 5.2 and are labeled A through H and their membership values are depicted in table 1.If the number of vehicles in any path is greater than 10000 per hour than we consider the membership value of that path is high. If the number of vehicles in any path is greater than or equal to 5000 per hour than we consider the membership value of that path is medium. If the number of vehicles in any path is less than 5000 per hour than we consider the membership value of that path is low. Membership values are represented by symbolic name H for high, M for medium, L for low respectively.

Vertex	A	B	C	D	E	F	G	H
Σ	M	H	M	L	M	H	M	L

Table 1: Membership values of the vertices

We need to develop a traffic pattern so that vehicles can pass through the intersection without interfering with other traffic flows.

In this problem, we represent each traffic flow using the vertices of the fuzzy graph and their membership value depends upon the number of the vehicle of that road. Two vertices are adjacent if the corresponding traffic flows cross each other. For instance, direction C and H intersect, so vertices C and H are adjacent. If two vertices are adjacent then there is a possibility of accident. The possibility of accident depends on the adjacent vertices membership value which represents number of vehicles on the road. If membership values of the adjacent two vertices are high (H) then we considers the



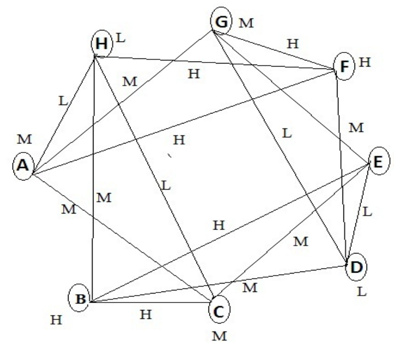


Fig 5.2: **Fig 5.3:**

membership value of that arc is high (H). We consider the arc membership value is low (L) if both the vertices have low (L) membership or one adjacent vertex has low (L) membership value and another node has medium(M) membership value. In another condition, if the membership value of one adjacent vertex is high (H) and another has low (L) membership value or both node has medium (M) membership value then we consider the membership value of that arc is medium (M).

In this paper, we represent each possibility of accident with an edge and their membership value. Membership values of edges are given below in Table 2.

Edge	AH	AG	AF	AC	BC	BD	BH	BE
μ	L	M	H	M	H	M	M	H
Edge	CH	DE	DF	DG	GE	FG	FH	CE
μ	L	L	M	L	M	H	H	M

Table 2: Membership values of the edges

6. Types of accidental zone in a traffic network

Depending upon the membership values of the edges and $CONN_G(x,y)$ of an arc (x,y) in a fuzzy graph G , we can classify the different type of accident zones in traffic flow. We define three different type of accidental zone in the traffic flow. Consider a fuzzy graph G=(V,E σ,μ). Using the fuzzy graph (G), we represent a traffic flow of a city. Let u,v be two routes in the traffic flows and two vertices are adjacent if the corresponding traffic flows cross each other.

Let e=(u,v) be an arc in graph G such that μ(u,v)=x>0. Then do the following step:

1. Obtain G- e
2. Find the value of $CONN_{G-e.}$
3. (a) If x > $CONN_{G-e}$ then e is α-strong accidental zone.
   (b) If $CONN_{G-e}$ =x then e is β-strong accidental zone.
   (c) If $CONN_{G-e}$ >x then e is δ- accidental zone.
4. Repeat steps 1–4 for all arcs in G.

We can apply this method to classify the accidental zone of a traffic flows in a city. We apply the method to our problem (Fig 5.3). In this fuzzy graph α-strong accidental zone are FG,AF,BC,CH and β-strong accidental zone are DF,GE,FH,AC,AG,BD,BH and δ- accidental zone are DE,DG,AH,CE. This classification will help control the traffic flow of a city.

7. Coloring the fuzzy graph

We color this fuzzy graph using the concept of Eslahchi and Onagh [9]. They defined fuzzy chromatic number as the least value of *k* for which the fuzzy graph *G* has *k*-fuzzy coloring and *k*-fuzzy coloring is defined as follows.

A family $\Gamma=\{\gamma_1, \gamma_2 \ldots. \gamma_k\}$ of fuzzy sets on V is called a k-coloring of fuzzy graph G= (V,E,σ,μ)

(a) $\vee \Gamma = \sigma$,

(b) $\gamma_1 \square \gamma_2 = 0$

(c) for every strong edge xy of G ,min{($\gamma_1(x)$, $\gamma_2(y)$}=0 (1≤i≤k).

Let $\Gamma=\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be a family of fuzzy sets defined on *V* by

$$\gamma_1(v_i)=\begin{cases} M & \text{if } i=A \\ M & \text{if } i=E \\ 0 & \text{otherwise} \end{cases} \qquad \gamma_2(v_i)=\begin{cases} H & \text{if } i=B \\ H & \text{if } i=F \\ 0 & \text{otherwise} \end{cases}$$

$$\Gamma_3(v_i)=\begin{cases} L & \text{if } i=D \\ L & \text{if } i=D \\ 0 & \text{otherwise} \end{cases} \qquad \Gamma_4(v_i)=\begin{cases} M & \text{if } i=C \\ M & \text{if } i=G \\ 0 & \text{otherwise} \end{cases}$$

Here we can see that the family $\Gamma=\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ satisfies the conditions of definition of vertex coloring[1]. Thus the fuzzy graph in our example have 4-fuzzy vertex coloring and this is the minimal vertex fuzzy coloring since any family with less than 4 members does not satisfy the conditions of the definition. Thus the fuzzy vertex chromatic number $\chi(G) = 4$.

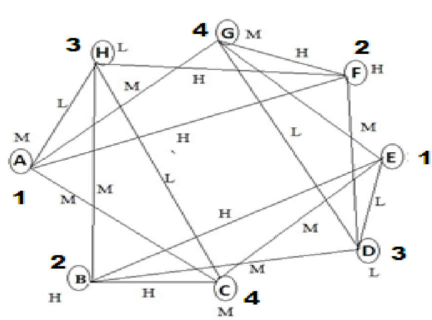


Fig. 7.1:

Fig 7.1 shows a coloring of graph with exactly four colors, which depicts an efficient way of designing the traffic signal pattern. It consists of four phases:

Traffic light pattern			
Phase 1	Phase 2	Phase 3	Phase 4
Only B and F proceed	Only D and H proceed	Only A and E proceed	Only C and G proceed

Table 3: Traffic light control pattern

In our traditional road traffic lights have the same cycle time T. So the duration of the all the phase will be equal and that is T. In this problem, we assume that number of vehicles in B and F direction is maximum and D and H direction is minimum. That means number of vehicles in all paths are not equal. If D and H need T time to pass all the vehicles then B and F will need more time than T. So total waiting time of vehicles on the roads will be increase and there may be a possibility of traffic jam or accident. Using fuzzy graph, we can solve this problem. We can give the duration time of the traffic light depends upon on the vehicle number (it is represented by vertex membership value). If the node membership value is high then it needs more time to flow the all vehicles. In this problem duration of the Phase 2 will be maximum and Phase 3 will be minimum. Using this concept, total waiting time of the vehicles will be minimizing**.**

8. Coloring function of fuzzy graph (crisp mode)

Definition 8.1. Given a fuzzy graph G= (V,σ,μ), its chromatic number is a fuzzy set $\chi(G)=\{(x_\alpha ,\alpha)\}$ where x_α is the chromatic number of G_α and values α are the different membership values of vertices and edges of graph G.

In this paper, we use α values are all different membership value of vertex and edge of fuzzy graph G. We find the all graph G_α which is a crisp graph for all α. Then we find minimum number of color needed to color the graph G_α. In such way, we find the fuzzy chromatic number which is a fuzzy number is calculated by its α cut.

For $\alpha=L\rightarrow G_L=(V_L,E_L)$where $V_L=\{A,B,C,D,E,F,G,H|\sigma(v)\geq L\}$and

$E_L=\{AC,AF,AG,AH,BC,BD,BE,BH,CE,CH,DE,DF,DG,EG,FG,FH\ |\mu \geq L\}$

Thus, for fuzzy graph of Fig 8.1 , $\chi_L = \chi(G_L)=4$.

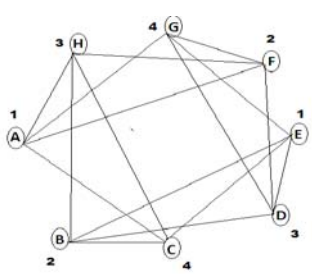


Fig 8.1:

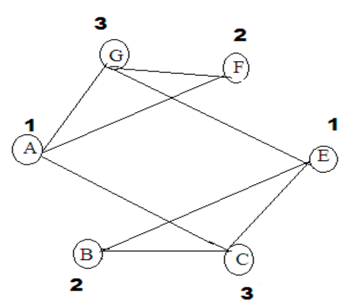


Fig 8.2:

For α=M , G_M= (V_M ,E_M) where V_M={ A ,B ,C, E,F, G |σ(v)≥M} and

E_M={AC,AF,AG,BC,BE, CE,EG,FG | $\mu \geq$ M }

Thus, for fuzzy graph of Fig 8.2 , $\chi_M = \chi(G_M)$=3

For α=H ,G_H= (V_H ,E_H) where V_L={ B, F |σ(v)≥H}and E_L = { □ | $\mu \geq$H }

Thus, for fuzzy graph of Fig 8.3 , $\chi_H = \chi(G_H)$=1.

Fig 8.3:

The chromatic number of a fuzzy graph is a normalized fuzzy number whose modal value is associated with the empty edge-set graph. Its meaning depends on the sense of index α, and it can be interpreted in the following way: for lower values of α there are many number of node and high number of intersection links between nodes and, consequently, more colors are needed in order to consider these intersection; on the other hand, for higher values of α there are fewer number of nodes and low value of intersection links between nodes and less colors are needed. The chromatic number sums up all this information in order to manage the fuzzy problem.

The fuzzy coloring problem consists of determining the chromatic number of a fuzzy graph and an associated coloring function. In this approach, for any level α, the minimum number of colors needed to color the crisp graph $G\alpha$ will be computed. In this way, the fuzzy chromatic number can be defined as a fuzzy number through its α-cuts.

9. Conclusion

In the concept of crisp incompatibly among the nodes of graph we cannot describe the vagueness or partial information about a problem. In this paper, we represent the traffic flows using a fuzzy graph whose vertices and edges both are fuzzy vertices and fuzzy arcs. So we can describe vagueness in vertices and also in edges. Using this membership value of edges and vertices, we have introduced a method to classify the accidental zone of a traffic flows. We can give a speed limit of vehicle according to accidental zone.

The chromatic number of G is $\chi\ (G)$ = {(4, L), (3,M),(1,H)}. The interpretation of $\chi\ (G)$ is the following: lower values of α are associated to lower driver aptitude levels and, consequently, the traffic lights must be controlled conservatively and the chromatic number is high; on the other hand, for higher values of α, the driver aptitude levels increase and the chromatic number is lower, allowing a less conservative control of the traffic lights and a more fluid traffic flow.

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