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Fuzzy Inventory Model for Imperfect Quality Items with Shortages

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Abstract. In this paper, a fuzzy based inventory model for imperfect quality items has been developed with shortages. The parameters fixed cost; holding cost and shortage cost are considered as fuzzy numbers. We considered the triangular fuzzy number to represents fuzzy parameters. The optimum order quantity is obtained in fuzzy sense with the help of signed distance method. The proposed model is illustrated with numerical example.

Keywords: Fuzzy triangular number, signed distance method, defuzzification

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

In the classic economic production quantity (EPQ) models, the underlying assumption is that 100% of items in a produced lot are perfect. However, in most real-life situations, the lot sizes produced may contain some defective products due to imperfect production process, machine malfunction and so on. The imperfect quality has influence on lot sizing policy. Therefore, the research on the inventory problems with imperfect quality products has become a hot issue in enterprises and academia. Rosenblatt and Lee [16] originally proposed an EPQ model that deals with imperfect quality.

They assume that the random deterioration from in-control state is exponentially distributed. Kim and Hong [12] extensive Rosenblatt and Lee's [16] model with the assumption of the elapsed time until process shift is arbitrarily distributed. Chung and Hou [9] further comprehensive Kim and Hong [17] to consider allowable shortages for the imperfect production processes. Chen, Lo and Liao [8] combined abovementioned models by considering the learning effect of the unit production system. Chia-Huei Ho [4] investigates a production/inventory policy in an integrated vendor-buyer inventory system with defective goods in the buyer's arrival order lot. He assumes there be a lead time demand and its distribution is unknown so the minima distribution-free procedure is applied to solve the problem.

Ouyang and Chang [15] have studied an EMQ model with variable lead time and imperfect production process; furthermore they also investigated the impact of setup cost reduction on the EMQ model. Lee and Wu [13] derive an EOQ model for items with Weibull spread deterioration, power demand rate and shortages. All of these models did not consider the time required to rework on defective items to make them good-quality items. Hayek and Salameh [10] assumed that the defective products reworked are all perfect and developed the EPQ model with allowable shortage and random defective rate which follows normal distribution. Chiu [3] extended the Hayek and Salameh's [10] model by considering a proportion of imperfect items and scrap items are produced in regular production process.

Chiu and Chiu [5] investigated EPQ model with random imperfect rate. The basic assumption in his model is that a portion of the imperfect items are reworked to make them good quality items. Chiu [6] combined aforementioned models by considering a portion of imperfect items and scrap items are produced in regular production process and a portion of the imperfect items are reworked to make them good quality items. All of the models derived optimal inventory policy by utilizing conventional approach. Chiu [7] reworked on the paper by Chiu [5] and used a simple algebraic method to derive the optimal solution. Lin, Chiu and Ting [14] used the same approach and solved Chiu's [6] problem. From literature survey, the above models with imperfect items are based on the EPQ inventory systems, where the uncertainty of defective rate is tackled from the traditional probability theory is assessed by a crisp value.

But in practical situations, precise values of the defective rate are seldom achieved as they may be vague and imprecise to certain extent. Thus in inventory system, the decision maker may allow some flexibility in the parameter values in order to tackle the uncertainties which always fit the real situations. As such, these characteristics are better described by the use of fuzzy sets which encompass a specific range of values. In recent years, several researchers have developed various types of inventory problems in fuzzy environments. Vijayan and Kumaran [17] studied fuzzy inventory models with a mixture of backorders and lost sales by introducing fuzziness in the cost parameters.

Vijayan and Kumaran [18] considered Economic order time models in which the time period of sales is a decision variable in fuzzy environments. Bj¨ork [1] contributes to the set of models capturing the economic order quantity with backorders, where both the demand and the lead times were fuzzified as the triangular fuzzy number. Konstantaras, Skouri, and Jaber [11] studied Inventory models for imperfect quality items with shortages and learning in inspection. Specifically, Chang [2] developed an economic order quantity (EOQ) model with fuzzy defective rate and demand, and signed distance method is employed to find the optimal order quantity.

This paper is organized as follows. In Section 2, some basic concepts of fuzzy sets, fuzzy numbers and signed distance method are introduced. Section 3 states the assumptions of the model. In Section 4, mathematical modeling for infinite planning horizon is discussed. Section 5 provides Numerical examples to illustrate the results of the proposed models. Finally the conclusion is given in Section 6.

2. Preliminaries

2.1. Definition: Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$. In the pair $\{(x, \mu_{\tilde{A}}(x))\}$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function or grade of membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

2.2. α-Cut

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α - cut $A(\alpha) = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$

2.3. Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,

(iii) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,

(iv) $\mu_{\tilde{A}}(x) = 1, a_2 \leq x \leq a_3$,

(v) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

(vi) $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$, where a_1, a_2, a_3 and a_4 are real numbers.

2.4. Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \leq a_2 \leq a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by (Q, r) inventory model with fuzzy lead time

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_1 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{Otherwise} \end{cases}$$

Definition 1. For $0 \leq \alpha \leq 1$, the fuzzy set $\tilde{a}_z$ defined on R = (-∞, +∞) is called an α-level fuzzy point if the membership function of $\tilde{a}_z$ is given by $\mu_{\tilde{a}z}(x) = \begin{cases} \alpha, & if\ x = a \\ 0, & if\ x \neq a \end{cases}$

Remark 1.

(1) When α=1, the membership function of the 1- level fuzzy point $\tilde{a}_1$ becomes the characteristic function that is $\mu_{\tilde{a}1}(x)=\begin{cases}1, \text{ if } x = a\\ 0, \text{ if } x\neq a\end{cases}$ in this case the real number $a\in R$ is the same as the fuzzy point $\tilde{a}_1$ except for their representations

(2) If c=b=a, then the triangular fuzzy number $\tilde{A} = (a, b, c)$ is identical to the 1-level fuzzy point $\tilde{a}_1$

Definition 2. For $0 \leq \alpha \leq 1$, the fuzzy set $[a_\alpha, b_\alpha]\tilde{a}_z$ defined on R is called an α-level fuzzy interval, if the membership function of $[a_\alpha, b_\alpha]$ is given by $\mu_{\alpha B(\alpha)}(x)=\begin{cases}\alpha, a\leq x\leq b\\ 0, \text{otherwise}\end{cases}$

Definition 3. Let be a fuzzy set on R and $0 \leq \alpha \leq 1$. The α- cut B (α) of $\tilde{B}$ consists of points x such that $\mu_{\tilde{B}}(x) \geq \alpha$ that is B (α) = $\{x / \mu_{\tilde{B}}(x) \geq \alpha\}$.

Decomposition principle:
Let be a fuzzy set on R and $0 \leq \alpha \leq 1$. Suppose the α- cut of to be closed interval $[B_L(\alpha), B_U(\alpha)]$, that is $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$.
Then we have $\tilde{B} = \bigcup_{0\leq\alpha\leq1} \alpha B(\alpha)$ or $\mu_{\tilde{B}}(x) = \bigcup_{0\leq\alpha\leq1} \alpha C_{B(\alpha)}(x)$, where α B(α) is a fuzzy set with membership function $\mu_{\alpha B(\alpha)}(x)=\begin{cases}\alpha, a\leq x\leq b\\ 0, \text{otherwise}\end{cases}$ and CB(α)(x) is a characteristic function B(α), that is $C_{B(\alpha)}(x)=\begin{cases}1, \text{ if } x\in B(\alpha)\\ 0, \text{ if } x\notin B(\alpha)\end{cases}$

Remark 2. From the decomposition principle, we obtain
$\tilde{B} = \bigcup_{0\leq\alpha\leq1} \alpha B(\alpha)= \bigcup_{0\leq\alpha\leq1} \left[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha\right]$ or
$\mu_{\tilde{B}}(x)= \bigcup_{0\leq\alpha\leq1} \alpha C_{B(\alpha)}(x)= \bigcup_{0\leq\alpha\leq1} \mu_{[B_L(\alpha), B_U(\alpha)]}(x)$

Interval operations:
For any a, b, c, d, k ∈ R, a< b and c<d, the interval operations are as follows:
(1). $[a, b] \oplus [c, d] = [a + c, b + d]$
(2). $[a, b] \Theta [c, d] = [a- d, b- c]$
(3). $k\otimes [a, b] = \begin{cases}[ka,kb], \text{ if } k>0\\ [kb,ka], \text{ if } k<0\end{cases}$

Further for a > 0 and c > 0

(4). $[a, b] \otimes [c, d] = [ac, bd]$

(5). $[a, b] \oslash [c, d] = \left[\frac{a}{d}, \frac{b}{c}\right]$

Next, we introduce the concept of the signed distance of fuzzy set

Definition 4. For any a and $0 \in R$, define the signed distance from a to 0 as $d_0(a, 0) = a$, if $a > 0$, the distance from a to 0 as $- d_0(a, 0) = -a$, if $a < 0$. Hence $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R with which the α- cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ exists for every $\alpha \in [0, 1]$ and both $B_L(\alpha)$and $B_U(\alpha)$ are continuous functions on $\alpha \in [0, 1]$. Then for any $\tilde{B} \in \Omega$, we have $\tilde{B} = \bigcup_{0 \le \alpha \le 1} \left[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha\right]$

From the above definition, the signed distance of two end points, $B_L(\alpha)$ and $B_U(\alpha)$ of the α- cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ of $\tilde{B}$ to the origin 0 is $d_0(B_L(\alpha), 0) = B_L(\alpha)$ and $d_0(B_U(\alpha), 0) = B_U(\alpha)$ respectively. The average $\frac{B_L(\alpha)+B_U(\alpha)}{2}$

In addition, for every $\alpha \in [0, 1]$, there is a one – one mapping between the α-level fuzzy interval $[B_L(\alpha), B_U(\alpha)]$ and the real interval $[B_L(\alpha), B_U(\alpha)]$, that is $[B_L(\alpha), B_U(\alpha)] \leftrightarrow [B_L(\alpha), B_U(\alpha)]$.

Also the 1-level fuzzy point $\tilde{0}_1$ is mapping to the real number 0. Hence the signed distance of $[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ to $\tilde{0}_1$ can be defined as $= d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1) = d([B_L(\alpha), B_U(\alpha)], 0) = \frac{B_L(\alpha)+B_U(\alpha)}{2}$

Moreover, $\tilde{B} \in \Omega$, Since the above function is continuous on $0 \le \alpha \le 1$, we can use the integration to obtain the mean value of the signed distance as follows;

$$\int_0^1 d\left(\left[B_L(\alpha)_\alpha, B_L(\alpha)_\alpha\right], \tilde{0}_1\right) d\alpha = \frac{1}{2}\int_0^1 \left[B_L(\alpha), B_U(\alpha)\right] d\alpha$$

Definition 5. For, $\tilde{B} \in \Omega$, define the signed distance of $\tilde{B}$ to $\tilde{0}_1$ (that is y axis) as

$$d\left(\tilde{B}, \tilde{0}_1\right) = \int_0^1 d\left(\left[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha\right], \tilde{0}_1\right) d\alpha = \frac{1}{2}\int_0^1 \left[B_L(\alpha), B_U(\alpha)\right] d\alpha$$

Property 1. For the triangular fuzzy number $\tilde{A} = (a, b, c)$, the α-cut of $\tilde{A}$ is $[A_L(\alpha), A_U(\alpha)]$, $\alpha \in [0,1]$, where $A_L(\alpha)=a+(b-a)\alpha$ and $A_U(\alpha)=c-(c-b)\alpha$. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is $d\left(\tilde{A}, \tilde{0}_1\right) = \frac{1}{4}(a+2b+c)$.

Furthermore, for two fuzzy sets $\tilde{B},\tilde{G} \in \Omega$, where $\tilde{B}= \underset{0\le\alpha\le1}{\cup}\left[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha\right]$ and $\tilde{G}= \underset{0\le\alpha\le1}{\cup}\left[G_L(\alpha)_\alpha, G_U(\alpha)_\alpha\right]$ and $k\in R$

Using the interval operations +, - and ., we have

(i) $\tilde{B}(+)\tilde{G}= \underset{0\le\alpha\le1}{\cup}\left[B_L(\alpha)_\alpha + G_L(\alpha)_\alpha, B_U(\alpha)_\alpha + G_U(\alpha)_\alpha\right]$

(ii) $\tilde{B}(-)\tilde{G}= \underset{0\le\alpha\le1}{\cup}\left[B_L(\alpha)_\alpha - G_U(\alpha)_\alpha, B_U(\alpha)_\alpha - G_L(\alpha)_\alpha\right]$

(iii) $k(.)\tilde{B} = \begin{cases} \underset{0\le\alpha\le1}{\cup}\left[(kB_L(\alpha))_\alpha, (kB_U(\alpha))_\alpha\right], k>0 \\ \underset{0\le\alpha\le1}{\cup}\left[(kB_U(\alpha))_\alpha, (kB_L(\alpha))_\alpha\right], k<0 \\ 0, \ k=0 \end{cases}$

Property 2. For two fuzzy sets $\tilde{B},\tilde{G} \in \Omega$ and $k\in R$, we have

$$d\left(\tilde{B},\tilde{0}_1\right) = \int_0^1 d\left(\left[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha\right],\tilde{0}_1\right)d\alpha = \frac{1}{2}\int_0^1 \left(B_L(\alpha)+B_U(\alpha)\right)d\alpha$$

$$d\left(\tilde{G},\tilde{0}_1\right) = \int_0^1 d\left(\left[G_L(\alpha)_\alpha, G_U(\alpha)_\alpha\right],\tilde{0}_1\right)d\alpha = \frac{1}{2}\int_0^1 \left(G_L(\alpha)+G_U(\alpha)\right)d\alpha$$

(i) $d\left(\left[\tilde{B}(+)\tilde{G}\right],\tilde{0}_1\right) = d\left(\tilde{B},\tilde{0}_1\right)+d\left(\tilde{G},\tilde{0}_1\right)$

(*ii*) $d\left(\left[\tilde{B}(-)\tilde{G}\right],\tilde{0}_1\right) = d\left(\tilde{B},\tilde{0}_1\right) - d\left(\tilde{G},\tilde{0}_1\right)$

(*iii*) $d\left(\left[k.\tilde{B}\right],\tilde{0}_1\right) = kd\left(\tilde{B},\tilde{0}_1\right)$

An inventory model with shortages is developed with fuzzy parameters fixed cost, holding cost and shortage cost are represented by fuzzy triangular membership function and the fuzzy total cost is obtained by applying signed distance method.

3. Assumption and Notations

Assumptions:

1. The demand rate is constant.
2. Shortages are allowed.
3. Lead time is zero.
4. 100% inspection of items is performed for each shipment.
5. The screening rate is faster than the demand rate.
6. The defective items are sold at a discounted price.
7. The planning horizon is infinite.

Notations Used:

q - Order quantity per shipment

B	-	Maximum back ordering quantity in units per cycle
c	-	Unit purchasing cost
K	-	Fixed cost per order
h	-	Holding cost per unit of time
b	-	Shortage cost per unit per unit of time
p	-	Percentage of defectives per shipments
w	-	Unit selling price per good quality unit
v	-	Unit discounted price per defective unit, v< c
y	-	Screening rate in units per unit of time y>D
d	-	Unit screening cost
*	-	The superscript representing optimal value

4. Mathematical Model

Crisp Mathematical Model

The profit per unit time is given by

$$TP(q,B) = wD + vD\frac{p}{1-p} - \frac{(c+d)D}{1-p} - \frac{KD}{q(1-p)} + hB - \frac{(h+b)B^2}{2q(1-p)} - \frac{hDpq}{(1-p)y} - \frac{h(1-p)q}{2} \quad (1)$$

The objective is to maximize the profit per unit time. By taking partial derivatives of TP (q, B) w.r.to q and B and by setting the results to zero, we have

$$\frac{\partial TP}{\partial q} = 0 \Rightarrow \frac{KD}{q^2(1-p)} + \frac{(h+b)B^2}{2q^2(1-p)} - \frac{hDp}{(1-p)y} - \frac{h(1-p)}{2} \quad (2)$$

$$\text{and } \frac{\partial TP}{\partial B} = 0 \Rightarrow B = \frac{h(1-p)q}{h+b}$$

Substitute B in equation (2), we obtain

$$\frac{\partial TP}{\partial q} = 0 \Rightarrow \frac{KD}{q^2(1-p)} + \frac{(1-p)h^2}{2(h+b)} - \frac{hDp}{(1-p)y} - \frac{h(1-p)}{2} = 0$$

Hence $$q^* = \sqrt{\frac{2KDy(h+b)}{h\left[(1-p)^2by + 2DP(h+b)\right]}} \quad (3)$$

Fuzzy Mathematical Model

Equation (1) can be written in terms of fuzzy number, we get

$$\tilde{TP}(q,B) = w\tilde{D} + v\tilde{D}\frac{p}{1-p} - \frac{(c+d)\tilde{D}}{1-p} - \frac{\tilde{K}\tilde{D}}{q(1-p)} + \tilde{h}B - \frac{(\tilde{h}+\tilde{b})B^2}{2q(1-p)} - \frac{\tilde{h}\tilde{D}pq}{(1-p)y} - \frac{\tilde{h}(1-p)q}{2}$$

Suppose the demand rate lies in the interval $[D-\Delta_1, D+\Delta_2]$, we can find a fuzzy triangular number to represent the vagueness in demand rate as $\tilde{D} = (D-\Delta_1, D, D+\Delta_2)$, $0 < \Delta_1 < D$ and $\Delta_1\Delta_2 > 0$.

Then the signed distance of $\tilde{D}$ is given by $d\left(\tilde{D},\tilde{0}\right) = D + \frac{1}{4}\left(\Delta_2 - \Delta_1\right)$ (4)

In practical problems, it is not easy to decide the fixed cost for long period due to some uncountable factors. Therefore it becomes reasonable to locate a fixed cost in an interval $[K-\Delta_3, K+\Delta_4]$, where $0 < \Delta_3 < K$ and $\Delta_3\Delta_4 > 0$ such as Δ_3 and Δ_4 are chosen appropriately.

Once interval is chosen then there is need to find an appropriate value in the interval [K-Δ_3, K+Δ_4]: If"K" is chosen then there is need to find an appropriate value in the interval. If K is chosen then it is coincident with the ordering cost. In crisp case error is considered as 0. If the value is within interval, then the error is larger when the value deviates from 'K' farther. Obviously error will attain its maximum at K-Δ_3 and K+Δ_4. From fuzzy point of view, error can be changed to confidence level such as: if error is zero then confidence level to be 1. If error is maximum which attains at K-Δ_3 and K+Δ_4, then confidence level is zero. If value is chosen from an interval [K-Δ_3, K+Δ_4], then confidence level is any real value between 0 and 1.

Hence, above condition can be suitably represented by fuzzy triangular number, $\tilde{K}$ = (K-Δ_3, K, K+Δ_4) and $0 < \Delta_3 < K$ and $\Delta_3\Delta_4 > 0$, Where the membership grade of K for $\tilde{K}$ is 1. For the points in the interval [K-Δ_3, K+Δ_4], as value is far from 'K' membership grade is less, and at the points $0 < \Delta_3 < K$ and $\Delta_3\Delta_4 > 0$ membership grade is zero. Therefore, it is natural and reasonable to the interval [K-Δ_3, K+Δ_4] to the fuzzy number $\tilde{K}$ in equation (4), when we respond membership grade to confidence level. The signed distance of $\tilde{K}$ is given by,

$$d\left(\tilde{K},\tilde{0}\right)=K+\frac{1}{4}\left(\Delta_4-\Delta_3\right) \tag{5}$$

$d\left(\tilde{K},\tilde{0}\right) > 0$ and $d\left(\tilde{K},\tilde{0}\right) \in$ [K-Δ_3, K+Δ_4]: $d\left(\tilde{K},\tilde{0}\right)$ can be taken as the estimate of total fixed cost in the fuzzy sense based on the singed distance.

As a fact, we know that in a perfect competitive market, the cost of storing a unit per may fluctuate a little from its actual value. Suppose it lies in the interval [h-Δ_5, h+Δ_6]. Similarly, as discussed above in the case of fixed cost, we can find a fuzzy triangular number to represent the vagueness in holding cost as: $\tilde{h}$ = (h-Δ_5, h, h+Δ_6), where $0 < \Delta_5 <$ h and $\Delta_5\Delta_6 > 0$, where the membership grade of h for $\tilde{h}$ is 1. For the points in the interval [h-Δ_5, h+Δ_6] membership grade is less as values within interval farther from 'h'. For the point's h-Δ_5 and h+Δ_6: membership grade is 0. Then the signed distance of $\tilde{h}$ is given by

$$d\left(\tilde{h},\tilde{0}\right)= h+\frac{1}{4}\left(\Delta_6-\Delta_5\right) \tag{6}$$

$d\left(\tilde{h},\tilde{0}\right) > 0$ and $d\left(\tilde{h},\tilde{0}\right) \in$ [h-Δ_5, h+Δ_6]: $d\left(\tilde{h},\tilde{0}\right)$ can be taken as the estimate of total holding cost in the fuzzy sense based on the singed distance.

Suppose the shortage cost lies in the interval [b-Δ_7, b+Δ_8], we can find a fuzzy triangular number to represent the vagueness in shortage cost as $\tilde{b}$ = (b-Δ_7, b, b+Δ_8), $0 <$ $\Delta_7 <$ b and $\Delta_7\Delta_8 > 0$.

Then the signed distance of $\tilde{b}$ is given by $d\left(\tilde{b},\tilde{0}\right)= b+\frac{1}{4}\left(\Delta_8-\Delta_7\right)$ (7)

Now, we defuzzify TP (q, B) using signed distance method. The signed distance of $\tilde{TP}\ (q,\ B)$ to $\tilde{0}$ is given by

$$d\left(\tilde{T}P(q,B),\tilde{0}\right) = wd\left(\tilde{D},\tilde{0}\right) + vd\left(\tilde{D},\tilde{0}\right)\frac{p}{1-p} - \frac{(c+d)d\left(\tilde{D},\tilde{0}\right)}{1-p} - \frac{d\left(\tilde{K},\tilde{0}\right)d\left(\tilde{D},\tilde{0}\right)}{q(1-p)} + d\left(\tilde{h},\tilde{0}\right)B$$

$$- \frac{\left[d\left(\tilde{h},\tilde{0}\right)+d\left(\tilde{b},\tilde{0}\right)\right]B^2}{2q(1-p)} - \frac{d\left(\tilde{h},\tilde{0}\right)d\left(\tilde{D},\tilde{0}\right)pq}{(1-p)y} - \frac{d\left(\tilde{h},\tilde{0}\right)(1-p)q}{2}$$

Substitute the equations (4), (5), (6) and (7) in the above one, we get

$$TP^*(q,B) = d\left(\tilde{T}P(q,B),\tilde{0}\right) = w\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right] + v\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right]\frac{p}{1-p}$$

$$- \frac{(c+d)\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right]}{1-p} - \frac{\left[K+\frac{1}{4}(\Delta_4-\Delta_3)\right]\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right]}{q(1-p)}$$

$$+ \left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right]B - \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)+b+\frac{1}{4}(\Delta_8-\Delta_7)\right]B^2}{2q(1-p)}$$

$$- \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right]\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right]pq}{(1-p)y} - \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right](1-p)q}{2}$$

Differentiate partially with respect to q and B and equating them to zero, we have

$$\frac{\partial TP^*(q,B)}{\partial q} = 0 \Rightarrow \frac{\left[K+\frac{1}{4}(\Delta_4-\Delta_3)\right]\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right]}{q^2(1-p)} + \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)+b+\frac{1}{4}(\Delta_8-\Delta_7)\right]B^2}{2q^2(1-p)}$$

$$- \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right]\left[D+\frac{1}{4}(\Delta_2-\Delta_1)\right]p}{(1-p)y} - \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right](1-p)}{2} = 0 \qquad (8)$$

$$\frac{\partial TP^*(q,B)}{\partial B} = 0 \Rightarrow \left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right] - \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)+b+\frac{1}{4}(\Delta_8-\Delta_7)\right]B}{q(1-p)} = 0$$

We obtain $$B = \frac{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)\right]q(1-p)}{\left[h+\frac{1}{4}(\Delta_6-\Delta_5)+b+\frac{1}{4}(\Delta_8-\Delta_7)\right]}$$

Substitute B in (8), we get

$$q^* = \sqrt{\frac{2\left(D+\frac{\Delta_2-\Delta_1}{4}\right)y\left(K+\frac{\Delta_4-\Delta_3}{4}\right)\left(h+\frac{\Delta_6-\Delta_5}{4}+b+\frac{\Delta_8-\Delta_7}{4}\right)}{\left(h+\frac{\Delta_6-\Delta_5}{4}\right)\left[2\left(D+\frac{\Delta_2-\Delta_1}{4}\right)p\left(h+\frac{\Delta_6-\Delta_5}{4}+b+\frac{\Delta_8-\Delta_7}{4}\right)+(1-p)^2y\left(b+\frac{\Delta_8-\Delta_7}{4}\right)\right]}} \qquad (9)$$

5. Numerical Example

To illustrate the result of the proposed model, we consider an inventory system with the following data : D=500 units per year; y=17, 520 units per year; K= \$300; h=\$1 per year;

b=$5 per unit per year; P=0.1;Δ_1=0.0005, Δ_2= 0.01, Δ_3 = 0.0004, Δ_4 = 0.02, Δ_5 = 0.0006, Δ_6 = 0.03, Δ_7 = 0.007, Δ_8 = 0.04 Substitute these values in (9), we obtain q* = 664 units.

6. Conclusion

This paper deals a fuzzy based inventory model for imperfect quality items with shortages. For this fuzzy model, a method of defuzzification, namely the signed distance method is employed to find the estimate of total profit per unit time in the fuzzy sense. Then the corresponding optimal order is derived to maximize the total profit. Numerical example is carried out to investigate the behavior of our proposed model and the result is compared with those obtained from the crisp model.

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