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τ_α*-gα Closed Sets in Topological Spaces

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Abstract. In this paper, we introduce a new class of sets called τ_α^*-gα closed sets and τ_α^*-gα open sets in topological spaces and study some of their properties.

Keywords: αcl^*- operator,τ_α^*- topology,τ_α^*-gα closed sets and τ_α^*-gα open sets

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1. Introduction

In 1970, Levine [6] introduced the concepts of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham [5] introduced the concept of the closure operator cl* and a new topology τ* and studied some of their properties. P. Bhattacharya and B.K. Lahiri[3] introduced the concept of semi generalized closed sets in topological spaces. J. Dontchev [4] introduced generalized semi-open sets, H. Maki, R. Devi and K. Balachandran [9] introduced generalized α-closed sets in topological spaces.

In this paper, we obtain a new generalization of α-closed sets in the topological space (X,τ*). Throughout this paper X and Y are topological spaces in which no separation axioms areassumed unless otherwise explicitly stated. For a subset A of a topological space X, int(A), cl(A), cl*(A) and A^c denote the interior, closure, closure* and complement of A respectively.

2. Preliminaries

Definition 2.1. A subset A of a topological space (X,τ) is called

(i) Generalized closed(briefly g-closed)[6] if cl(A)⊆G whenever A ⊆ G and G is open in X.

(ii) Semi-generalized closed(briefly sg-closed)[3] if scl(A)⊆ G whenever A⊆ G and G issemi-open in X.

(iii) Generalized semi-closed(briefly gs-closed)[2] if scl(A)$\subseteq$ G whenever A$\subseteq$G and G is openin X.

(iv) α –closed[8] if cl(int(cl(A)))$\subseteq$ A.

(v) α - generalized closed(briefly g-closed)[9] if αcl(A)$\subseteq$G whenever A$\subseteq$ G and G inX.

(vi) Generalized α – closed(briefly gα –closed)[12]if αcl(A)$\subseteq$ G whenever A$\subseteq$ G and G isα-open in X.

(vii) Generalized semi-pre closed(briefly gsp-closed)[2] if spcl(A)$\subseteq$ G whenever A$\subseteq$ G and Gis open in X.

(viii) Pre closed[11] if cl(int(A))$\subseteq$ A.

(ix) Semi-closed[7] if int(cl(A))$\subseteq$ A.

(x) Semi-pre closed(briefly sp-closed)[1] if int(cl(int(A))) $\subseteq$A.

The complements of the above mentioned sets are called their respective open sets.

Definition 2.2. For the subset A of a topological space X, the generalized α-closure operator αcl*(A) is defined by the intersection of allgα-closed sets containing A.

Definition 2.3. For a topological space X, the topology τ_α* is defined by
τ_α* = {G :αcl*(G^c) = G^c }.

Example 2.1. Let X={a,b,c}and τ={X,Φ,{a}}.Thenthe collection of subsets {X,Φ, {a},{b},{c},{a,b},{b,c},{a,c}} is a τ_α* - topology on X.

Definition 2.4. For the subset A of a topological X, the α-closure of A (cl_α(A)) is defined as the intersection of all α -closed sets containing A.

3. τ_α*- gα closed sets in topological spaces

In this section, we introduce the concept of τ_α*-gα closed sets in topological spaces.

Definition 3.1. A subset A of a topological space X is called τ_α*- Generalized α-closed set(briefly τ_α*-gαclosed) if cl_α*(A) $\subseteq$G (or) αcl*(A) $\subseteq$G whenever A$\subseteq$G and G is τ_α*-open.

The complement of τ_α*-generalized α-closed set is called the τ_α*- generalized α -open set(briefly τ_α*-gα open).

Theorem 3.2. Every closed set in X is τ_α*-gα closed.

Proof: Let A be a closed set.

Let A $\subseteq$ G, since A is closed,

Cl(A) = A $\subseteq$ G, where G is τ_α*- open.

But αcl*(A)$\subseteq$cl(A) $\subseteq$ G

αcl*(A)$\subseteq$G , whenever A$\subseteq$ G and G is τ_α*- open.

Hence A is τ_α*- gα closed.

Theorem 3.3. Every τ_α* - closed set in X is τ_α* -gα closed.
Proof : Let A be a τ_α * - closed set.
Let $A \subseteq G$, where G is τ_α* - open.
Since A is τ_α* - closed, αcl*(A) = A $\subseteq$ G
αcl*(A) $\subseteq$ G
Hence A is τ_α* - gα closed.

Theorem 3.4. Every α - closed set in X is a τ_α*- gα closed but not conversely.
Proof : Let A be a α - closed set.
Assume that A$\subseteq$G, G is τ_α* - open in X,
Since A is α-closed $\Longrightarrow$ A is gα-closed.
αcl*(A) = A
Since A$\subseteq$G, then αcl*(A)$\subseteq$ G.
Hence A is τ_α* - αg closed.
The converse of the above theorem need not be true as seen from the following example.

Example 3.5. Consider the topological space X = {a,b,c} with topology τ ={Y,Φ,{a},{a,b},{a,b,c},{a,b,d}}.Thenthe collection of sets {{a},{b},{c},{d},{a,b},{b,c},{c,d},{a,c},{a,d}{b,d},{a,b,c},{a,b,d}{b,c,d},{a,c,d}}are τ_α* -gα closed but not α- closed.

Example 3.6. Let X = {a,b,c} and Y = {a,b,c,d} be the two non-empty sets.
(i) Consider the topology τ = {X,Φ,{a}}. Then the set {a} is not both gs- closed and τ_α* -gα closed.
(ii) Consider the topology τ = {X,Φ,{a}}. Then the set {a}is not both g-closedand τ_α* -gα closed.
(iii) Consider the topology τ = {X,Φ,{a}}. Then the set {a} is not both gsp-closed andτ_α* -gα closed.
(iv) Consider the topology τ = {X,Φ,{b},{a,b}}. Then the sets {{c},{b,c},{a,c}}are not both pre- closed andτ_α* - gα closed.
(v) Consider the topology τ = {X,Φ,{a}}. Then the set {a} is not both gα - closed andτ_α* -gα closed.
(vi) Consider the topology τ = {X,Φ,{a},{b},{a,b}}. Then the set {a,b} is bothgsp-closed andτ_α* -gα closed.
(vii) Consider the topology τ = {X,Φ,{a},{b},{a,b}}.Then the set {a} is sg-closed but notτ_α* - gα closed.
(viii) Consider the topology τ = {Y,Φ,{a},{a,b},{a,b,c},{a,b,d}}. Then the sets {{c},{d},{b,c},{a,b,c},{a,b,d}} are τ_α* -gα closed but not αg - closed.
(ix) Consider the topology τ = {X,Φ}. Then the set {{a},{b},{c},{a,b},{b,c},{a,c}} are τ_α* -gα closed but not α-closed.

(x) Consider the topology $\tau = \{X,\Phi\}$. Then the sets{{a},{b},{c},{a,b},{b,c},{a,c}}areτ_α*-gα closed but not closed.

(xi) Consider the topology $\tau = \{X,\Phi,\{a\}\}$. Then the sets {{a},{b}} are not both sp-closed andτ_α* -gα closed.

Remark 3.7. From the above discussion, we obtain the following implications.

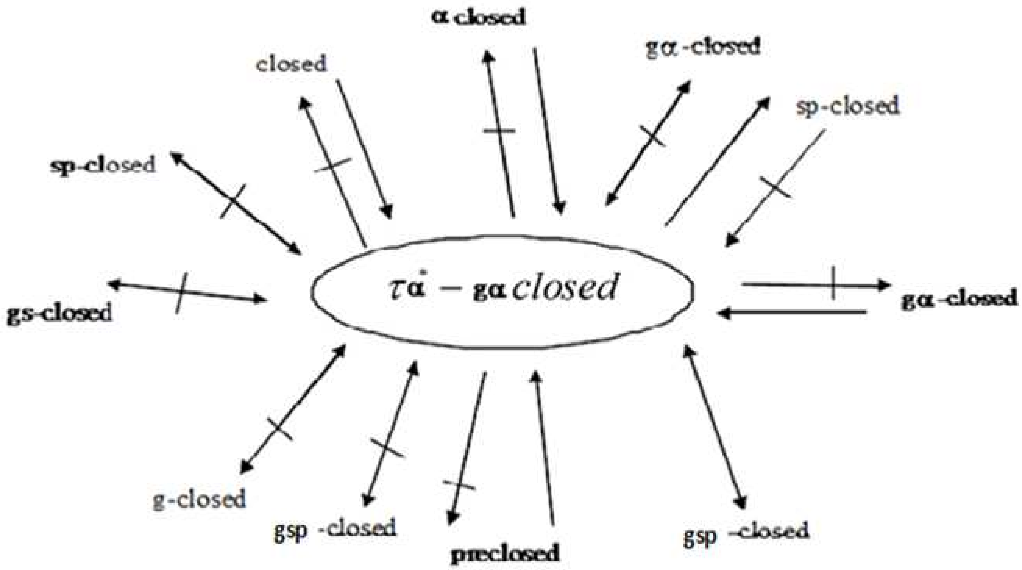

Figure 1:

Theorem 3.8. For any two sets A and B,αcl*(A∪B) = αcl*(A)∪ αcl*(B).

Proof : Since A⊆A∪B and B ⊆ A∪B thenαcl*(A) ⊆ αcl*(A∪ B) and αcl*(B) ⊆ αcl*(A∪B)

Hence αcl*(A) ∪ αcl*(B) ⊆ αcl*(A∪B) (1)

αcl*(A) and αcl*(B) are gα- closed sets.

⟹αcl*(A) ∪αcl*(B) is alsogα-closed set.

Again, A ⊆αcl*(A) and B⊆αcl*(B)

⟹A ∪ B ⊆αcl*(A) ∪αcl*(B).

Thus αcl*(A) ∪αcl*(B) is a gα - closed set containing A∪B.

Since αcl*(A ∪ B) is the smallest gα - closed set containing A ∪ B

We have αcl*(A ∪ B) ⊆αcl*(A) ∪αcl*(B)⟶ (2)

From (1) and (2), αcl*(A ∪ B) =αcl*(A) ∪αcl*(B).

Theorem 3.9. Union of two τ_α*- gα closed sets in X is a τ_α*-gα closed set in X.

Proof : Let A and B be two τ_α*-gα closed sets.

Let $A \cup B \subseteq G$, where G is τ_α *- open.

Since A and B are τ_α*-gα closed sets,then $\alpha cl^*(A) \subseteq G$ and $\alpha cl^*(B) \subseteq G$.

$\alpha cl^*(A) \cup \alpha cl^*(B) \subseteq G$.

But by the above theorem,$\alpha cl^*(A) \cup \alpha cl^*(B) = \alpha cl^*(A \cup B)$

$\alpha cl^*(A \cup B) \subseteq G$, where G is τ_α *- open.

Hence $A \cup B$ is a τ_α*-gα closed set.

Theorem 3.10. A subset A of X is τ_α*-gαclosed if and only if $\alpha cl^*(A)$-A contains no non-empty τ_α* - closed set in X.

Proof : Let A be a τ_α* - gα closed set.

Suppose that F is a non-empty τ_α* - closed subset of $\alpha cl^*(A) – A$.

Now, $F \subseteq \alpha cl^*(A) – A$.

Then $F \subseteq \alpha cl^*(A) \cap A^c$

Since$\alpha cl^*(A) – A = \alpha cl^*(A) \cap A^c$

$F \subseteq \alpha cl^*(A)$ and $F \subseteq A^c$

$A \subseteq F^c$

Since F^c is a τ_α* - open set and A is τ_α*-gα closed.

$\alpha cl^*(A) \subseteq F^c$, i.e. $F \subseteq [\alpha cl^*(A)]^c$.

Hence, $F \subseteq \alpha cl^*(A) \cap [\alpha cl^*(A)]^c = \Phi$, i.e. $F = \Phi$,

which is a contradiction.

$\alpha cl^*(A)$-A contains no non-empty τ_α* - closed set in X.

Conversely, assume that $\alpha cl^*(A) – A$ contains no non-empty τ_α* - closed set.

Let $A \subseteq G$, G is τ_α* - open.

Suppose that $\alpha cl^*(A)$ is not contained in G then $\alpha cl^*(A) \cap G^c$ is a non-empty τ_α* - closed set of $\alpha cl^*(A) – A$, which is a contradiction.

$\alpha cl^*(A) \subseteq G$, G is τ_α* - open.

Hence A is τ_α* - αg closed.

Corollary 3.11. A subset A of X is τ_α* - gα closed if and only if $\alpha cl^*(A) – A$ contain no non-empty closed set in X.

Proof : The proof follows from the theorem 3.10 and the fact that every closed set is τ_α* - gα closed set in X.

Theorem 3.12. If a subset A of X is τ_α* - gα closed and $A \subseteq B \subseteq \alpha cl^*(A)$ then B is τ_α* - gα closed set in X.

Proof : Let A be a τ_α* - gα closed set such that $A \subseteq B \subseteq \alpha cl^*(A)$. Let U be a τ_α* - open set of X such that $B \subseteq U$.

Since A is τ_α* - gα closed,

We have $\alpha cl^*(A) \subseteq U$.

Now, $\alpha cl^*(A) \subseteq \alpha cl^*(B) \subseteq \alpha cl^*[\alpha cl^*(A)] = \alpha cl^*(A) \subseteq U$.

$\alpha cl^*(B) \subseteq U$, U is τ_α* - open set.

B is τ_α* - gα closed set in X.

Theorem 3.13. Let A be a τ_α* - gα closed set in (X,τ*). Then A is gα - closed if and only if $\alpha cl^*(A)$ isτ_α* - open.

Proof : Suppose A is gα – closed in X.

Then $\alpha cl^*(A) = A$ and so $\alpha cl^*(A) - A = \Phi$ which is τ_α* - open in X.

Conversely, suppose $\alpha cl^*(A) - A$ is τ_α* - open in X.

Since A is τ_α*-gα closed, by theorem: 3.10, $\alpha cl^*(A) - A$ contains no non-empty τ_α* - closed set in X.

Then $\alpha cl^*(A) - A = \Phi$.

Hence A is gα - closed.

Theorem 3.14. For x$\in$ X, the set X-{x} is τ_α * - gα closed or τ_α* - open.

Proof : Suppose X – {x} is not τ_α* - open. Then X is the only τ_α* - open set containing X – {x}. This implies $\alpha cl^*(X - \{x\}) \subseteq X$.

Hence X – {x} is a τ_α* - gα closed in X.

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