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Thermophoresis Effect on MHD Forced Convection on a Fluid over a Continuous Linear Stretching Sheet in Presence of Heat Generation and Power-Law Wall Temperature

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Abstract. An analysis is presented to investigate the effects of thermophoresis and heat generation on MHD forced convection, heat, and mass transfer in a fluid over a linear stretching sheet in the presence of power-law wall temperature. The appropriate similarity transformations are used to transform the considered problem of nonlinear partial differential equations into a system of nonlinear ordinary differential equations which are solved numerically by using Nachtsheim-Swigert shooting iterative technique along with sixth order Runge-Kutta integration scheme. Numerical computations are carried out for the non-dimensional velocity, temperature and thermophoresis profiles for various non-dimensional physical parameters which are shown graphically and discussed from the point of physical view. For physical and engineering interest the effects of skin-friction coefficient, the Nusselt number and the wall deposition heat flux are also studied and presented in tabular form.

Keywords: MHD, Forced convection, Heat generation, Thermophoresis, Nachtsheim-Swigert iteration technique, Power-law wall temperature.

AMS Mathematics Subject Classification (2010): 76A25

1. Introduction

The magnetohydrodynamic (MHD) flow has attracted a great interest to many researchers during the last several decades owing to the effect of magnetic field on the boundary layer flow control and applications in many engineering and physical aspects such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. Flow characteristics and heat transfer over a stretching sheet have been studied extensively by many researchers in the recent past because of many engineering applications of stretching sheet in manufacturing processes such as hot rolling, wire drawing, drawing of plastic films and artificial fibers, metal extrusion, crystal growing, continuous casting, glass fiber production and paper production. Matallurgy lies in the purification of molten metals from nonmetallic inclusions by applying magnetic field is other application of MHD. Tsou et al. [1] reported both analytical and experimental results for the flow and heat transfer in the boundary layer on a continuously moving surface. Crane [2] studied the flow caused by an elastic sheet whose velocity varies linearly with the distance from a fixed point on the sheet. Chen and Char [3] studied the suction and injection on a linearly moving plate with uniform wall temperature and and heat flux. Agarwal et al. [4] studied the flow and heat transfer over a stretching sheet. Chen [5] studied the effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power law stretched sheet with surface heat flux. An analysis is carried out to investigate the radiative heat and mass transfer of an MHD free convection flow along a stretching sheet with chemical reaction, heat generation and viscous dissipation by Ishrat et al. [6].

Industrially the study of heat generation effect on MHD flow, heat and mass transfer in moving fluid is also an important matter. Possible heat generation effect may alter the temperature distribution. Vajravelu and Hadjinicalaou [7] have studied on hydrodynamic convective heat transfer from a stretching surface with heat generation/absorption. Molla et al. [8] studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation are studied by Samad et al. [9]. Reddy [10] studied heat generation and radiation effects on steady MHD free convection flow past a moving surface.

From the technical point of view thermophoresis effect on heat and mass transfer flow of a fluid over a stretching sheet plays an important role for many practical applications. Anderson et al. [11] analyzed the diffusion of a chemically reactive species from a linearly stretching sheet. An effect of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface was studied by Chamkha et al. [12]. Alam et al. [13] have studied transient magnetohydrodynamic free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeable plate in the presence of variable chemical reaction and temperature dependent viscosity. Soret and dufour effects on unsteady MHD flow of a micropolar fluid in the presence of thermophoresis deposition particle were studied by Aurangzaib et al. [14].

Thermophoresis with power-law wall temperature is a case which was not considered in the above mentioned studies. Therefore, the aim of the present paper is to study the effect of thermophoresis in two dimensional MHD convective flow of a fluid over a linearly stretching sheet in presence of heat generation, power-law wall temperature as well as uniform magnetic field which is normal to the sheet. Thus, this present study may be a cause of interest to the concern researchers for further modified investigations in the field of heat and mass transfer flow of a fluid.

2. Mathematical Formulation

Let us consider a steady, laminar, two-dimensional MHD heat and mass transfer flow of a viscous incompressible electrically conducting fluid of temperature T past a linear stretching sheet with constant heat generation and thermophoretic effect in presence of power-law wall temperature under the influence of a transversely applied magnetic field. Keeping the origin fixed, along the x-axis which is measured along the sheet two equal and opposite forces are introduced to stretch the sheet. The y-axis is taken normal to the flow direction. The continuous sheet is moving with velocity $u = ax$, where arbitrary constant $a(>0)$ called stretching rate varies linearly along x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that induced magnetic field can be neglected in comparison with the applied magnetic field $B = (0, B_0, 0)$ where magnetic field of uniform strength B_0 is imposed in the y-direction. The electrical current flowing in the fluid gives rise to an induced magnetic field if the fluid were an electrical insulator, but here the fluid is considered to be electrically conducting.

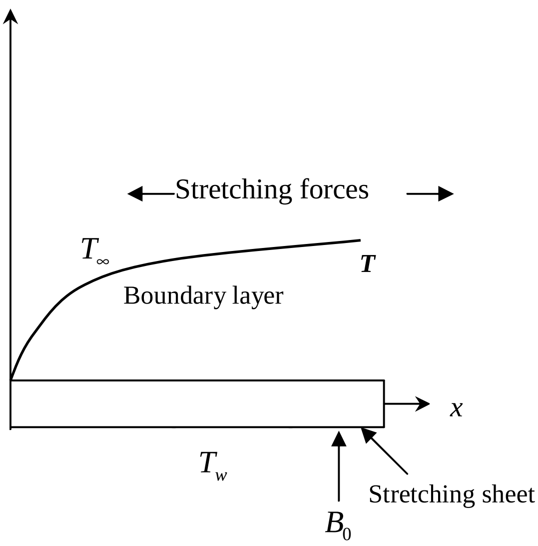

Figure 1: Flow configurations and coordinate system

Under the above assumptions and usual Boussinesq approximation, the governing equations for this problem can be written in the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \text{ (Continuity)} \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2 u}{\rho}, \text{ (Momentum)} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty), \text{(Energy)} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_M\frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}(V_T C), \text{ (Concentration)} \tag{4}$$

The appropriate physical boundary conditions for the above problem are:

$$\left.\begin{aligned} &u = u_w = ax, \quad v = 0, \quad T = T_w(x) = T_\infty + Ax^p, \quad \mathrm{C} = \mathrm{C_w} = 0 \ \text{at} \ \mathrm{y} = 0, \\ &\quad u \to 0, \quad T \to T_\infty, \quad \mathrm{C} \to \mathrm{C}_\infty \quad \text{as y} \to \infty. \end{aligned}\right\} \tag{5}$$

In above equations u, v are the velocity components along x, y co-ordinates respectively, υ is the kinematic viscosity, ρ is the mass density of the fluid, σ is the electric conductivity, B_0 is the uniform magnetic field acting normal to the sheet, T is the temperature of the fluid within the boundary layer, T_∞ is the temperature of the fluid outside the boundary layer, c_p is the specific heat of the fluid at constant pressure, λ_g is the thermal conductivity of the fluid, Q_0 is the heat generation constant, C is the concentration inside the boundary layer, C_w and C_∞ are the concentration of the fluid at the surface and outside the boundary layer respectively. D_M is the chemical molecular diffusivity of the species concentration and V_T is the thermophoretic velocity. The subscripts w and ∞ refer to the surface conditions and conditions far away from the surface respectively. The value $p = 1$ represents constant temperature of the fluid within the boundary layer.

For non-dimensionalisation of the model we introduce the following dimensionless variables:

$$\left.\begin{aligned} &\eta = y\sqrt{\frac{a}{\upsilon}}, \psi = \sqrt{a\upsilon}\, x f(\eta), \\ &\theta = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C}{C_\infty} \end{aligned}\right\} \tag{6}$$

The stream function ψ with $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$ satisfies the continuity equation (1). Thus from equations (6) in terms of the new variable η the velocity components are obtained

$$u = axf'(\eta) \ \text{and} \ v = -\sqrt{a\upsilon}\, f(\eta) \tag{7}$$

Here prime denotes ordinary differentiation with respect to η and f', θ and φ are the dimensionless velocity, temperature and thermophoresis respectively.

When small particles are exhibited to a temperature gradient gain an average velocity by which usually the effect of thermophoresis is recommended. In boundary layer flow, the temperature gradient in x-direction is very much low than in the y-direction i.e., $\dfrac{\partial T}{\partial x} << \dfrac{\partial T}{\partial y}$. The component of thermophoretic velocity is thus insignificant along the surface of the sheet compared to that of its normal to the surface of the sheet. As a consequence, the thermophoretic velocity V_T, which appears in equation (4) can be expressed in the following form:

$$V_T = -\frac{k_t \upsilon}{T_r}\frac{\partial T}{\partial y} \quad (8)$$

where T_r is some reference temperature and k_t is the thermophoretic coefficient whose values range from 0.2 to 1.2 as indicated by Batchelor and Shen [15] and is defined from the theory of Talbot et al. [16] by

$$k_t = \frac{2C_s\left(\frac{\lambda_g}{k_p} + C_t Kn\right)\left[1 + Kn(C_1 + C_2 e^{\frac{-C_3}{Kn}})\right]}{(1+3C_m Kn)(1+\frac{2\lambda_g}{k_p} + 2C_t Kn)} \quad (9)$$

where $C_1 = 1.2$, $C_2 = 0.41$, $C_3 = 0.88$, $C_m = 1.146$, $C_S = 1.147$, $C_t = 2.20$ are constants, λ_g and k_p are the thermal conductivities of the fluid and diffused particles respectively and Kn is the Knudsen number of the particle.
A thermophoretic parameter τ can be given by the relation (see Tsai [17])):

$$\tau = -\frac{k_t (T_w - T_\infty)}{T_r} \quad (10)$$

Typical values of τ are 0.01, 0.1 and 1.0 corresponding to approximate values of $-k_t(T_w - T_\infty)$ equal to 3, 30 and 300 K for a reference temperature of $T_r = 300$ K. It is to be noticed that positive τ indicate that the sheet surface is cold whereas negative τ indicate that sheet surface is hot. In the present problem we have considered the sheet surface is cold.

Now substituting equations (6), (7), (8) and (10) the nonlinear equations (2)-(4) reduced to

$$f''' + ff'' - f'^2 - Mf' = 0, \quad (11)$$
$$\theta'' + \Pr\left(f\theta' - pf'\theta\right) + \Pr Q\theta = 0 \quad , \quad (12)$$
$$\varphi'' + Sc(f - \tau\theta')\varphi' - Sc\tau\theta''\varphi = 0, \quad (13)$$

where $M = \frac{\sigma B_0{}^2}{\rho a}$, $\Pr = \frac{\mu c_P}{\lambda_g}$, $Q = \frac{Q_0}{\rho c_P a}$, $Sc = \frac{\upsilon}{D_M}$ and $\tau = -\frac{k_t\left(T_w - T_\infty\right)x^p}{T_r}$

represent respectively the magnetic field parameter, Prandtl number, heat generation parameter, Schmidt number and the local thermophoretic parameter.
The corresponding boundary conditions transformed into:

$$f = 0,\ f' = 1, \theta = 1, \phi = 0 \quad \text{at} \quad \eta = 0 \quad (14a)$$
$$f' \to 0, \theta \to 0, \phi \to 1 \quad \text{as } \eta \to \infty \quad (14b)$$

3. Important Physical Parameters

The chief physical quantities of engineering interest for the present problem are the local skin-friction coefficient Cf_x in x-direction, local Nusselt number Nu_x and local

Stanton number St_x are defined respectively as follows.

The wall shear stress

$$\tau_w = (\mu)(\frac{\partial u}{\partial y})_{y=0} \tag{15}$$

The local skin-friction coefficient is defined as

$$Cf_x = \frac{2\tau_w}{\rho(ax)^2} = (2\,\mathrm{Re}_x)^{\frac{-1}{2}} f''(0) \tag{16}$$

Thus from equation (16) we see that local values of the skin-friction coefficient Cf_x is proportional to $f''(0)$.

The local heat flux may be written by Fourier's law as

$$q_w(x) = -\lambda_g \left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{17}$$

The local Nusselt number is defined as

$$Nu_x = \frac{x q_w(x)}{\lambda_g (T_w - T_\infty)} = -(\mathrm{Re}_x)^{\frac{-1}{2}} \theta'(0) \tag{18}$$

Thus from equation (18) we see that the local Nusselt number Nu_x is proportional to $-\theta'(0)$.

The rate of transfer of species concentration is given by

$$J_S = -D_M \left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{19}$$

The local Stanton number is defined by $St_x = -\frac{J_S}{(ax)C_\infty} = \frac{1}{Sc}(\mathrm{Re}_x)^{\frac{-1}{2}} \varphi'(0)$ (20)

where $\mathrm{Re}_x = \frac{ax^2}{\upsilon}$ is the local Reynolds number. The effects of the various parameters on Cf_x, Nu_x and St_x are calculated from equations (16), (18) and (20) and presented in **Table 1-3.** These effects found in agreement with those on velocity, temperature and thermophoresis profiles and seem to be redundant for any further discussion.

4. Results and Discussion

In order to discuss the results of the present problem, the numerical solutions those obtained from equations (11)-(13) along with corresponding boundary conditions (14a)-(14b) using Nachtsheim-Swigert [18] shooting iteration technique with sixth order Runge-Kutta-Butcher initial value solver for various values of the parameters such as local thermophoretic parameter τ, Schmidt number Sc, heat generation parameter Q, power-law temperature parameter p, Prandtl number $\Pr$ and magnetic field parameter M are presented in the form of non-dimensional velocity, temperature and thermophoresis profiles. The physical explanations of the appropriate change of the parameters are given in details below. Since experimental data of the physical parameters are not available therefore in the numerical simulations the choice of the values of the parameters was dictated by the values chosen by the previous investigators. The values of the above-mentioned parameters are chosen arbitrarily as $\Pr = 0.71$, $M = 1.0$, $Q = 0.5$, $p = 0.50$, $Sc = 1.0$ and $\tau = 0.8$ unless otherwise specified. To be mentioned that $\Pr = 0.71$ corresponds physically to air at $20^0 C$, $\Pr = 1.0$ corresponds to electrolyte solution such as salt water and $\Pr = 7.0$ corresponds to water and $Sc = 0.22$, 0.6 and 1.0 corresponds to hydrogen, water vapor and methanol respectively at approximate $25^0 C$ and 1 atmosphere.

Figures 2 depict the velocity profiles for distinct values of different parameters respectively, local thermophoretic parameter τ, Schmidt number Sc, heat generation parameter Q, power-law temperature parameter p, Prandtl number $\Pr$ and magnetic field parameter M. From Figures 2 it is seen that there is a significant decreasing effect in the velocity fields with the increase values of M while the other parameters τ, Sc, Q, p and $\Pr$ show no effect with their increasing values on velocity profiles. The fluid velocity decreases with the increase of the magnetic field parameter indicating that magnetic field tends to retard the motion of the fluid.

Figures 3 show the temperature profiles for different values of different parameters. With an increase of local thermophoretic parameter τ temperature profiles do not alter i.e. there is no effect of thermophoretic parameter on temperature profiles. The effect of Schmidt number Sc on temperature profiles is magnifying with the extended values of Sc. Heat generation parameter Q has a notable impact on temperature profiles. For increasing values of $Q > 0$ temperature profiles decrease first within short interval of η and then increases for next short interval of η. This pattern of temperature profiles occur cyclically through the whole range of η for increasing values of $Q > 0$. It reveals from the figure that for increasing values of power-law temperature parameter p, temperature profiles decrease within $0 \le \eta < 2$ and then the profiles increase within the remaining interval of $2 \le \eta < 10$. The effect of the Prandtl number $\Pr$ on the thermal boundary layer is similar to those of the viscous boundary layer. Therefore, Prandtl number decreases thermal boundary layers thickness within $0 < \eta < 2$. But within $2 < \eta < 10$ temperature profiles increase with increase of $\Pr$. Figure concerns to the effect of M on the temperature profiles reveals that temperature profiles decrease significantly with the increase of M i.e., thermal boundary layer thickness decreases with the increase of magnetic field parameter M. This is due to the fact that magnetic field tends to retard the velocity field which induces the temperature field resulting in decrease of temperature profiles. Thus we can use the magnetic field to control the flow and heat transfer characteristics.

Figures 2: Velocity variations for distinct values of different parameters τ , Sc , Q , p , $\Pr$ and M .

Figure 3: Temperature variations for distinct values of different parameters τ, Sc, Q, p, $\Pr$ and M.

In the following Figures 4, respectively we have presented the effects of τ, Sc, Q, p, $\Pr$ and M on thermophoresis profiles. It is noticed from the figures that due to increase of the values of τ and M the thermophoresis profiles decrease while the thermophoresis profiles increase with the increased values of Sc, p and $\Pr$. A phenomenon of successive increasing-decreasing profiles of thermophoresis entire the whole interval of η is observed for various increasing values of Q>0.

Finally, we have presented the values of the physical parameters $f''(0)$, $-\theta'(0)$ and $\varphi'(0)$ which are respectively proportional to the local skin-friction coefficient Cf_x,the rate of heat transfer Nu_x and local Stanton number St_x in the following **Tables 1-3** for Sc, Q and p respectively.

Figures 4: Thermophoresis variations for distinct values of different parameters τ , Sc , Q , p , $\Pr$ and M .

Sc	Cf_x	Nu_x	St_x
0.10	-1.414213	16.033323	0.06843
0.25	-1.414213	0.688048	0.15994
0.50	-1.414213	0.473184	0.294034
0.75	-1.414213	0.476461	0.414771
0.10	-1.414213	0.466880	0.525750

Table 1:

Q	Cf_x	Nu_x	St_x
0.0	-1.414213	0.548179	0.519810
0.2	-1.414213	0.548072	0.523748
0.4	-1.414213	0.227817	0.514797
0.6	-1.414213	0.563808	0.528978
0.8	-1.414213	0.529264	0.530794

Table 2:

p	Cf_x	Nu_x	St_x
0.00	-1.414213	0.296314	0.525047
0.10	-1.414213	0.335998	0.525325
0.30	-1.414213	0.402818	0.525553
0.50	-1.414213	0.466880	0.525750
0.70	-1.414213	0.536033	0.526164

Table 3:

5. Conclusions

From the present numerical investigation of the forced convection flow of heat and mass transfer the following particular conclusions are drawn:

- Local skin-friction coefficient unaltered with the increase of Schmidt number, heat generation parameter and power-law temperature parameter.
- Local Nusselt number increases with the increase of power-law temperature parameter.
- The effect of magnetic field parameter leads significantly to a decrease in the fluid velocity, temperature profiles as well as thermophoresis profiles. Thus magnetic field can be used significantly to control the flow and heat transfer characteristics.
- For increased heat generation parameter, decrease-increase pattern in thermal state of the fluid while a reverse pattern in thermophoresis profiles is observed cyclically though heat generation yields no effect on velocity profiles. So temperature profiles can be controlled by using heat source parameter.
- Thermophoresis profiles increase, temperature profiles first decrease and then increase and velocity profiles do not have any change for increased values of Prandtl number and power-law temperature parameter.

- For increasing values of thermophoretic parameter thermophoresis profiles decrease though thermophophoretic parameter has no effect on velocity and temperature profiles.
- Increase in temperature profiles while decrease in thermophoresis profiles and no effect in velocity profiles can be seen when values of Schmidt numbers are increased. Thus Schmidt number can be used to control mass transfer in the flow field.

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