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Use of Fuzzy Relational Maps and Intuitionistic Fuzzy Sets to Analyze Health Problem of Agricultural Labourers

***Dhrubajyoti Ghosh*[1] and *Anita Pal*[2]**

[1]Department of Mathematics, National Institute of Technology Durgapur,
Durgapur-712209, West Bengal, India
Email: krizz27@gmail.com
[2]Department of Mathematics, National Institute of Technology Durgapur,
Durgapur-712209, West Bengal, India
Email: anita.buie@gmail.com

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Abstract. In this paper we use similarity measure for intuitionistic fuzzy sets and apply in real life problem to analyze health problem of agricultural labourers those who are suffering from health hazards due to chemical pollution.

Keywords: Fuzzy Relational Maps, Intuitionistic Fuzzy Sets

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1. Introduction

The fuzzy model is a finite set of fuzzy relations that form an algorithm for determining the outputs of a process from some finite number of past inputs and outputs. Fuzzy model can be used in applied mathematics, to study social and psychological problem and also used by doctors, engineer, scientists, industrialists and statisticians. There are various types of fuzzy models. In this paper we use two fuzzy models and give their application to a real world problem. The paper is organized in four sections. First section defines Fuzzy Relational Maps (FRMs). In section two draws a relationship between diseases and symptom of the agricultural labourers who are suffering from health hazards. Section three gives the idea of intuitionistic fuzzy set (IFS) are generalized fuzzy sets whose elements are characterized by a membership, as well as non-membership value. The membership value indicates the degree of belongingness, whereas the non membership value indicates the degree of non – belongingness of an element to the set. The utility of IFSs theory in computer vision is increasingly becoming apparent, especially as a means to copy the noise.

1. Fuzzy Relational Maps (FRMs)

FRMs are a directed graph or a map from domain space to range space with concepts and causalities as edges [1].

Let, Domain space = n
Range space = m [$m \neq n$]
$R_1, R_2,\ldots,R_m$ be the nodes of range space.
$R = \{ (x_1, x_2 ,\ldots,x_m) \mid x_j = 0 \text{ or } 1\}$ for j = 1,2…. ,m.
$x_j = 1$ i.e. R_j is on state and $x_j = 0$ i.e. R_j is off state
Similarly,
$D_1 , D_2 , \ldots, D_n$ be the nodes of domain space.
$D = \{ (x_1, x_2 ,\ldots,x_n) \mid x_i = 0 \text{ or } 1\}$ for i = 1,2…. ,n.
$x_i = 1$ i.e. D_i is on state and $x_i= 0$ i.e. D_i is off state

1.1. Formation of FRMs

Let D_i and R_j denotes the two nodes of FRM. Let e_{ij} be the weight of the edge D_iR_j (or R_jD_i), then $e_{ij} \in \{0 ,1, -1\}$. The relational matrix E be defined as $E = (e_{ij})$.

a) **Instantaneous state vector**

Let $A = (a_1 \ldots,a_n)$, $a_i \in \{ 0, 1\}$ where i = 1,2,..,n. A is called the instantaneous state vector of the domain space and it denotes the on-off position of the nodes at any instant i.e. $a_i = 0$ if a_i is off and $a_i = 1$ if a_i is on for i = 1, 2,.., n for domain space. Similarly, $B = (b_1,\ldots,b_m)$,$b_j \in \{0,1\}$ where j = 1, 2,..,m. B is the instantaneous state vector of the range space. $b_j = 0$ if b_j is off and $b_j = 1$ if b_j is on for j = 1, 2,..,m for range space.

b) **Directed Cycle**

Let D_iR_j (or $R_j D_i$) be the edges of an FRM where j = 1, 2,…,m and i = 1, 2,..,n form a directed cycle, FRM is said to be cycle if it possesses a directed cycle. Otherwise, it is acyclic.

c) **Dynamical system of FRM**

An FRM with cycle is said to be an FRM with feedback. When there is feedback in the FRM. The FRM is called a dynamical system.

d) **Hidden pattern**

Let D_iR_j (or $R_j D_i$) where $1 \leq j \leq m$ and $1 \leq i \leq n$. When D_j (or R_i) is switched on and if causality flows and if again causes D_j (or R_i). This equilibrium state is called hidden pattern.

e) **Fixed Point**

The equilibrium state of this system is a unique state vector then it is called fixed point. Example: -Let us assume dynamical system by switching on R_1 (or D_1). FRM settles down with R_1 and R_m (D_1 and D_n) on i.e. state vector remains as (1,0,…0,1) as in R (1,0,..,0,1). This state vector is called the fixed point.

f) **Limit cycle**

If the FRM settles down with a state vector in the forms: $D_1 \rightarrow D_2 \rightarrow \ldots \rightarrow D_i \rightarrow D_1$ ($R_1 \rightarrow R_2 \rightarrow \ldots \rightarrow R_i \rightarrow R_1$). This form is called limit cycle.

g) **Combined FRMs**

Let us $E_1, E_2\ldots E_p$ be the relational matrices of the FRMs. Combined FRMs denotes be the relational matrix by $E = E_1 +\ldots+E_p$.

1.2. Methods of determining the hidden pattern

Let $R_1, R_2,\ldots,R_m$ and $D_1 , D_2, \ldots, D_n$ be the nodes of FRM. Let us assume D_1 is switched on i.e. when an input is given as vector $A_1 = (1, 0,\ldots, 0)$ in $D_1$and the relational matrix is E. Now $A_1 E = (r_1, r_2,\ldots, r_m)$, after thresholding and updating the resultant vector $A_1E \in$

R. Now let $B = A_1E$, passing B into E^T and obtain BE^T. After threshold and update the vector $BE^T \in D$. The procedure repeated till we get a fixed point or limit cycle.

1.3. Application of FRMs

FRMs are used in the following areas**: -**

a) Relation between Doctor and Patient.
b) Relation between quality condition and academic condition of students
c) Relational between teacher and poor rural students in City Colleges
d) Study of employee-employer relationship
e) finding out the diseases among various ages of agricultural labourers due to chemical pollution

2. Model illustrating FRMs

Real data is collected from the agricultural labourers on small village of state West Bengal suffering from health hazards due to chemical pollution is studied using the FRMs. Using the model we estimate the maximum age group in which the agricultural labourers suffer health hazards due to chemical pollution. The term chemical pollution we mean the pollution we mean the pollution due to spray of pesticides and insecticides, also the pollution of the grain due to the use of fertilizer, which are mainly chemicals.
According from their information victims fainted in the field and before they could be taken to the hospital they died of suffocation due to the spray. Further, the older people were very angry at this event. In fact they expressed in those days the agriculture methods used by them were really eco-friendly. Now days due to modernization the pesticides and insecticides are sprayed on the field using helicopters, which has largely affected the health conditions of the agricultural labourers. For the reason, total atmosphere is polluted badly by the methods. To best of our knowledge no one has ever cared to study the health hazards suffered by these people due to chemical pollution. Thus our study can be adopted to any agricultural field in India. We approach the problem of pollution by determining the peak age group in which they are maximum affected by pollution. By knowing this age group the rehabilitate them and give medicines which will antinodes the chemicals due to which they are suffering these health problems. We analyze this problem using IFSs. At first, we have taken 4 nodes in domain space which denote the problem and 16 nodes to represent the symptom of the problem.

P_1 Cardiovascular problem
- S_1 = Pain (chest, back, shoulder, and leg)
- S_2 = Blood pressure
- S_3 = Burning chest
- S_4 = Swollen limb

P_2 Digestive problem
- S_5 = Vomiting
- S_6 = Diarrhea
- S_7 = Constipation
- S_8 = Indigestion

P_3 Nervous problem
- S_9 = Faint
- S_{10} = Headache, getting angry, irritated
- S_{11} = Loss of energy
- S_{12} = Fits

P_4 Respiratory problem
- S_{13} = Allergy
- S_{14} = Coughing
- S_{15} = Asthma
- S_{16} = Breathing difficulty

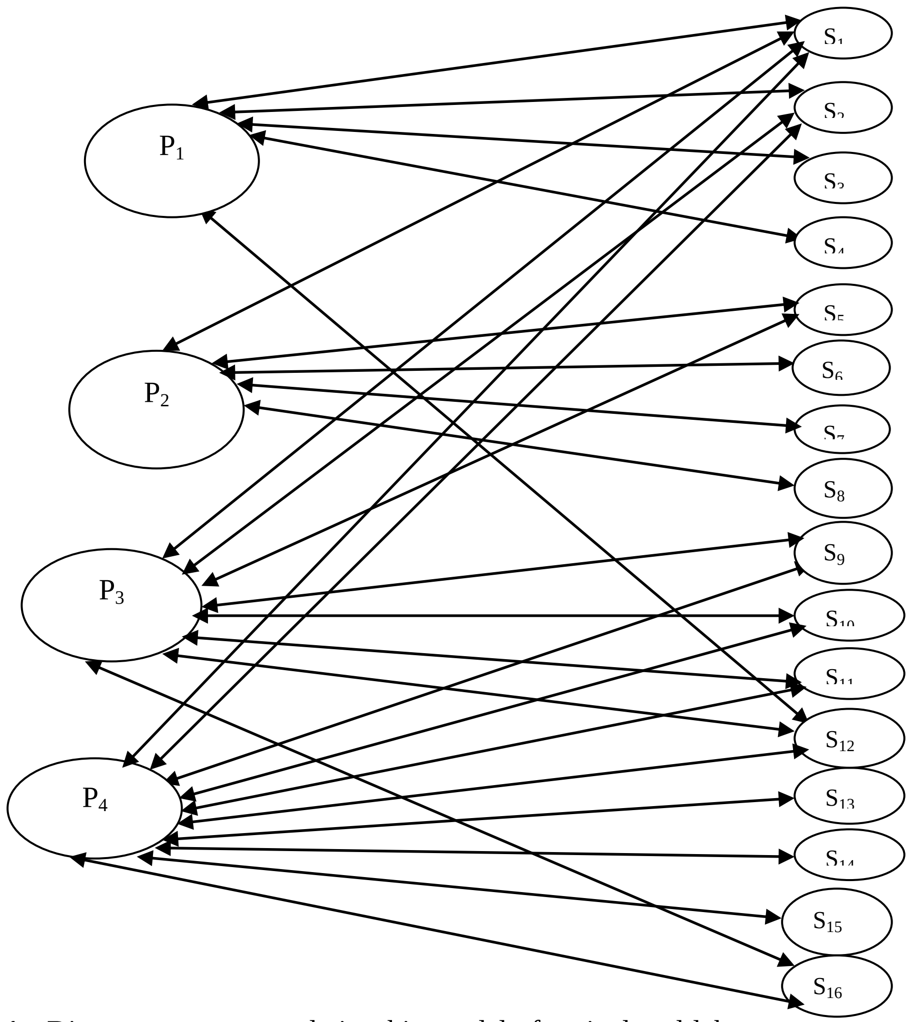

Figure 1: Diseases -symptom relationship model of agricultural labourers representation by a directed graph

	R_1	R_2	R_3	R_4
S_1	1	1	0	1
S_2	1	0	0	1
S_3	1	0	0	0
S_4	1	0	0	0
S_5	0	1	0	0

$R_1 =$

S_6	0	1	0	0
S_7	0	1	0	0
S_8	0	1	0	0
S_9	0	0	1	0
S_{10}	0	0	1	1
S_{11}	0	0	1	1
S_{12}	1	0	1	1
S_{13}	0	0	0	1
S_{14}	0	0	0	1
S_{15}	0	0	0	1
S_{16}	1	0	0	1

Table 1: Associated relational matrix of FRMs

Here we have taken one case on this model taking some nodes are on or off and try to find out some hidden pattern.

Case 1:

Consider the node A_1 i.e. blood pressure of cardiovascular problem is on the state and rest of the nodes in the off state. Now A_1 = (0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0), passing A_1 into the matrix R_1 we get:

A_1 = (0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0) `

A_1R_1 = (1 0 0 1) = H_1

$H_1R_1^T$ = (1 1 1 1 0 0 0 0 0 1 1 1 1 1 1 1) = A_2

A_2R_1 = (1 1 1 1) = H_2

$H_2R_1^T$ = (1 1 1 1 0 0 0 <u>0 0 1</u> 1 1 1 1 1 1) = A_3

A_3R_1 = (1 1 1 1) = H_3 $\rightarrow$ H_2

Here A_1 is a unique state vector of the equilibrium state of this system. Thus A_1 is the fixed point. We have seen that if pain, burning chest, swollen limb, headache, getting angry, irritating, loss of energy, fits, allergy, coughing, asthma, breathing difficulty will be increased then blood pressure must be checked.

Similarly we have to consider the other cases to find the hidden pattern of the system.

3. Intuitionistic Fuzzy Sets (IFSs)

After the introduction of fuzzy sets by Zadeh [2], a number of generalizations of this fundamental concept come up. The notion of intuitionistic fuzzy sets introduced by Attanassov [3] ([4],[5]) is one among of them. The main advantage of IFS is their property to cope with the hesitancy that mat exist due to information impression. This is achieved by incorporating a second function, along with the membership function of the conventional FSs, called non-membership function. In this way, apart from the degree of the belongingness, the IFSs also combine the notation of the non-belongingness in order to better describe the real status of the information. Since the first introduction of the IFSs and the consequent study on the fundamentals of the IFSs, a lot of attention has been paid on developing distance or similarity measures between the IFSs, as a way to apply them on several problems of the real life. IFSs as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. For example, in decision making problems, particularly in the case of medial diagnosis, sales analysis, new product marketing, financial services, etc. there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. To be more precise - IFSs let us express e.g., the fact that the temperature of a patient changes, and other symptoms are not quite clear.In this article we will present IFs sets as a tool for reasoning in the presence of imperfect facts and imprecise knowledge. An example of agricultural labourers suffering from health hazards due to chemical pollution will be presented assuming there is a database, i.e. description of a set of symptoms A, and a set of diagnoses P. We will describe a state of group of ages of labourers knowing results of his/her medical tests. Description of the problem uses the notion of an intuitionistic fuzzy set. The proposed method of diagnosis involves intuitionistic fuzzy distances as introduced in (Szmidt and Kacprzyk [6, 7]). Advantages of such an approach are pointed out in comparison with the method presented in (De, Biswas and Roy [8]) in which the max-min-max composition rule was applied.

According to a fuzzy set (Zadeh [9]) X= x, given by

$$A' = \{< x, \mu_{A'} (x) > \mid x \in X\} \tag{1}$$

where $\mu_{A'} : X \rightarrow [0, 1]$ is the membership function of the fuzzy set $A' : \mu_{A'} (x) \in [0, 1]$; is the membership of $x \in X$ in A', an IFSs (Attanosov [10]) $A \in X$ is given by

$$A = \{x, \mu_A (x), v_A (x) > \mid x \in X\} \tag{2}$$

where $\mu_A : X \rightarrow [0, 1]$ and $v_A : X \rightarrow [0, 1]$ such that

$$0 \leq \mu_A (x) + v_A (x) \leq 1 \tag{3}$$

And $\mu_A (x), v_A (x) \in [0, 1]$ denote the degree of membership and non- membership $x \in A$, respectively.

Obviously each fuzzy set corresponds to the following IFS

$$A' = \{< x, \mu_{A'} (x), 1- \mu_{A'} (x) > \mid x \in X\} \tag{4}$$

For each IFSs in X, we will call

$$\pi_A (x) = 1 - \mu_A (x) - v_A (x) \tag{5}$$

a hesitation margin (or intuitionistic fuzzy index) of $x \in A$, and it is a hesitation degree of whether x belongs to A or not [cf. Atanassov [2]]. It is obvious that $0 \leq \pi_A (x) \leq 1$ for each $x \in X$. On the other hand, for each fuzzy set A' in X, we evidently have

$$\pi_{A'}\text{ (x) = 1- }\mu_{A'}\text{ (x) – [1- }\mu_{A'}\text{ (x)] = 0, for each x} \in \text{X} \qquad (6)$$

Therefore, we can state that if we want to fully describe an IFSs, we must use any two functions from the triplet:

1) Membership function,
2) Non- membership function, and
3) Hesitation margin. In other words, the application of IFSs instead of fuzzy sets means the introduction of another degree of freedom into a set description (i.e. in addition to μ_A we also have v_A or π_A).

3.1. Application of IFSs

1) decision making problems particularly in the case of medial diagnosis,
2) sales analysis and financial services,
3) new product marketing,
4) finding out the diseases among various ages of agricultural labourers due to chemical pollution

3.2. Model using IFSs

To solve the same problem of agricultural labourers due to chemical pollution using IFSs we have taken P_1, P_2, P_3, P_4 variables represent cardiovascular problem, digestive problem, nervous problem, and respiratory problem and S_1, S_2,…,S_{16} variables represent Pain, blood pressure, burning chest, swollen limb, vomiting, diarrhea, constipation, indigestion, faint, headache or getting angry, loss of energy, fits, allergy, coughing, asthma, breathing difficulty. Using these variables we have drawn a matrix, and show their relationship using IFSs.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
P_1	(0.8, 0.1, 0.1)	(0.7, 0.0, 0.3)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.3, 0.6, 0.1)	(0.1, 0.8, 0.1)	(0.0, 0.9, 0.1)	(0.4, 0.5, 0.1)	(0.1, 0.8, 0.1)	(0.4, 0.2, 0.4)	(0.3, 0.1, 0.6)	(0.7, 0.2, 0.1)	(0.2, 0.7, 0.1)	(0.3, 0.5, 0.2)	(0.1, 0.8, 0.1)	(0.8, 0.0, 0.2)
P_2	(0.5, 0.4, 0.1)	(0.0, 0.8, 0.2)	(0.0, 0.9, 0.1)	(0.0, 0.9, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.9, 0.0, 0.1)	(0.3, 0.5, 0.2)	(0.3, 0.6, 0.1)	(0.2, 0.7, 0.1)	(0.4, 0.2, 0.4)	(0.2, 0.1, 0.7)	(0.1, 0.0, 0.9)	(0.0, 0.8, 0.2)	(0.1, 0.8, 0.1)
P_3	(0.4, 0.2, 0.4)	(0.2, 0.7, 0.1)	(0.0, 0.6, 0.4)	(0.0, 0.7, 0.3)	(0.2, 0.5, 0.3)	(0.0, 0.9, 0.1)	(0.0, 0.8, 0.2)	(0.0, 0.7, 0.3)	(0.6, 0.0, 0.4)	(0.7, 0.1, 0.2)	(0.6, 0.1, 0.3)	(0.6, 0.0, 0.4)	(0.0, 0.1, 0.9)	(0.1, 0.8, 0.1)	(0.4, 0.1, 0.5)	(0.5, 0.1, 0.4)
P_4	(0.6, 0.0, 0.4)	(0.5, 0.3, 0.2)	(0.0, 0.1, 0.9)	(0.0, 0.1, 0.9)	(0.3, 0.2, 0.5)	(0.1, 0.0, 0.9)	(0.0, 0.1, 0.9)	(0.1, 0.0, 0.9)	(0.4, 0.1, 0.5)	(0.5, 0.2, 0.3)	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.1, 0.2)	(0.8, 0.0, 0.2)	(0.9, 0.0, 0.1)

Table 2:

The data are given in Table 2 {disease problem is described by three numbers: membership μ, non- membership ϑ, hesition margin π. For example, for Breathing difficulty the Cardiovascular problem is high (μ = 0.8, ϑ = 0.0, π = 0.2), whereas for the Cardiovascular problem Constipation is low (μ = 0.0, ϑ = 0.9, π = 0.1).

	P_1 (Cardiovascular problem)	P_2 (Digestive problem)	P_3 (Nervous problem)	P_4 (Respiratory problem)
20-24	(0.0,0.8,0.2)	(0.6,0.3,0.1)	(0.2,0.6,0.2)	(0.3,0.6,0.1)
25-30	(0.3,0.4,0.3)	(0.5,0.2,0.3)	(0.6,0.3,0.1)	(0.4,0.2,0.4)
31-36	(0.5,0.2,0.3)	(0.7,0.1,0.2)	(0.6,0.2,0.3)	(0.6,0.3,0.1)
37-43	(0.6,0.2,0.2)	(0.4,0.4,0.2)	(0.6,0.0,0.4))	(0.5,0.0,0.5)
44-65	(0.8,0.1,0.1)	(0.4,0.2,0.4)	(0.7,0.2,0.1)	(0.8,0.0,0.2)

Table 3:

Our task is to find the actual problems of disease for each age of labourers pi, i = 1; . . . ; 5. To fulfill the task we calculate for each age of the labourers, a distances of their health symptoms (Table 1) from a set of symptoms s_j = 1; . . . ; 5 characteristic for each symptoms dk, k = 1; . . . ; 16 (Table 2). The lowest obtained distance points out a proper diagnosis.

We showed that the only proper way of calculating the most widely used distances for intuitionistic fuzzy sets is to take into account all three parameters: the membership function, the non-membership function, and the hesitation margin. To be more precise, the normalised Hamming distance for all the symptoms of the i-th age of the labourers from the k-th disease is equal to

$$l(s(p_i), d_k) = \frac{1}{32}\sum_{j=1}^{16}(|\mu_j(p_i) - (\mu_j(d_k)| + |(v_j(p_i) - (v_j(d_k)| + |(\pi_j(p_i) - (\pi_j(d_k)| \qquad (7)$$

The distances (7) for each patient from the considered set of possible diagnoses are given in Table 2. The lowest distance point out the proper problem of the labourers:

	P_1 (Cardiovascular problem)	P_2 (Digestive problem)	P_3 (Nervous problem)	P_4 (Respiratory problem)
20-24	0.425	0.4	0.381	0.569
25-30	0.341	0.419	0.444	0.352
31-36	0.353	0.481	0.388	0.403
37-43	0.334	0.344	0.447	0.331
44-65	0.391	0.431	0.488	0.381

Table 4:

At the age of 20-24, the labourers suffer from nervous problems, at the age 25-30, they suffer from cardiovascular problems, at the age 31-36 the suffer from cardiovascular problems, at the age 37-43 they suffer from respiratory problems and at the 44-65, the labourers suffer from respiratory problems .

We obtained the same results, i.e. the same quality diagnosis for each patient when looking for the solution while applying the normalized Euclidean distance [cf. Szmidt and Kacprzyk]:

$$l(s(p_i),d_k)=\frac{1}{32}\sum_{j=1}^{16}((\mu_j(p_i) - (\mu_j(d_k))^2 + ((v_j(p_i) - (v_j(d_k))^2 + ((\pi_j(p_i) - (d_k))^2)^{\frac{1}{2}} \qquad (8)$$

	P_1 (Cardiovascular problem)	P_2 (Digestive problem)	P_3 (Nervous problem)	P_4 (Respiratory problem)
20-24	0.224	0.188	0.147	0.295
25-30	0.101	0.177	0.191	0.110
31-36	0.122	0.226	0.179	0.182
37-43	0.136	0.121	0.242	0.114
44-65	0.210	0.166	0.244	0.162

Table 5:

The results are given in Table 5 {the lowest distance for each labourers pi from possible symptoms D points out a solution}. At the age of 20-24, the labourers suffer from nervous problems, at the age 25-30, they suffer from cardiovascular problems, at the age 31-36 the suffer from cardiovascular problems, at the age 37-43 they suffer from respiratory problems and at the 44-65, the labourers suffer from respiratory problems .

4. Conclusions

In this paper by using this FRM model we have represented a diagram to show the relationship between the health problem and their symptoms of agricultural labourers and in the next part of this paper we have collected data from the experts and representing in matrix from of those health problems and symptoms. With the help of this database and using similarity measure for intuitionistic fuzzy sets we have explained their actual health problem according to their proper ages.

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