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Multiple Fuzzy Regression Model on Two Wheelers' Mileage with Several independent Factors

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Abstract. Regression is one of the important factors of statistical analysis. But the applications of statistical regression are bounded by the quantitative aspects. It is one of the major drawbacks of normal regression. Because real life applications are mostly based on uncertain interval estimation, than certain point estimation. Fuzzy logic is one of the best ways to deal with such problems. Here in this paper a fuzzy linear multiple regression analysis has been done on the mileage of two wheeler's with several independent factors. In this multiple linear fuzzy regression model input data has been taken in two aspects, one is crisp another is triangular fuzzy data .Statistical level of significance is taken as membership values, ranges from 0 to 1. Defuzzification technique is used to generate the crisp mileage. In this entire work MATLAB, SPSS, MAPLE 6 are used for statistical calculation.

Keywords: Triangular fuzzy number, Yager's ranking index method, t-distribution, Middle of the maxima, Level of Significance.

1. Introduction

Fuzzy linear regression analysis has been used for many years in developing mathematical models for application in marketing, management, and sales forecasting in uncertain environment. A study of the international market sales of cameras, done in 1968 by John Scott Armstrong, utilizing non-fuzzy forecasting techniques. The second sales forecasting technique uses fuzzy linear regression introduced by H. Tanaka, S. Uejima, and K. Asai, in 1980 [1].Multiple linear regression modeling is a very powerful technique and is extensively used in agricultural research (Lalitha et al. 1999, Guo and Sun 2001) [2].Epidemic forecasting, another important applications of fuzzy regression technique, applied by Wan Yeh Hsieh and Rusy Chyn Tsaur [3]. Age model estimation in paleoclimatic research is another realistic approach, done in the year of 1997, using fuzzy regression [4].

Here we apply fuzzy linear multiple regression analysis technique to derive the mileage of two wheelers. Mileage is one of the factors of any vehicle, which always varies for different factors, like road quality, routine service of engine, engine acceleration quality etc. So it is better to quantify the mileage within a range of values, rather a fixed value. The uncertainty of mileage can be explained through fuzzy technique. Because interval estimation can be applied through fuzzy technique to deal with such real life uncertainty. Here we use eight independent variables to estimate the dependent variable mileage. Using fuzzy technique we generate mileage as an interval. After that using defuzzification technique we derive crisp value of mileage.

2. Fuzzy Linear Regression Basic

Fuzzy linear regression is used for creating the relationship of dependent variables with independent variables in some uncertain environment.

2.1. Type of Input data and relationship:

1) Input data are crisp and relationship of dependent and independent variables are fuzzy.
2) Input data and the relationship both are fuzzy.

2.2. Multiple linear regression:

Classical approach of multiple regression takes the form.

$$Y = a + b_1 X_1 + b_2 X_2 + \ldots\ldots\ldots\ldots b_n X_n \quad (1)$$

where b_i (i=1, 2, 3,….., n) are regression coefficients and a is constant value. Y is dependent variable and X_i (i=1, 2,3,……, n) are independent variables.

3. Preliminaries

Here we use fuzzy linear multiple regression analysis with two aspects.

1) Constant and regression coefficients are taken as interval estimator [5],[6].
2) Few inputs are taken as crisp data and few in triangular fuzzy. Using Yager's ranking index method we convert the triangular fuzzy values in crisp data [7].

3.1. Formulation of Fuzzy Multiple linear regression

$$Y_1(X_1, X_2, \ldots\ldots\ldots, X_n)(\alpha) = a_1(\alpha) + X_1 b_1(\alpha) + X_2 c_1(\alpha) + \ldots\ldots\ldots\ldots X_n k_1(\alpha) \quad (2)$$

$$Y_2(X_1, X_2, \ldots\ldots\ldots, X_n)(\alpha) = a_2(\alpha) + X_1 b_2(\alpha) + X_2 c_2(\alpha) + \ldots\ldots\ldots\ldots X_n k_2(\alpha) \quad (3)$$

where

$$\left.\begin{array}{l} a(\alpha) = [\, a_1(\alpha), a_2(\alpha)\,] \\ b(\alpha) = [\, b_1(\alpha), b_2(\alpha)\,] \\ c(\alpha) = [\, c_1(\alpha), c_2(\alpha)\,] \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ k(\alpha) = [\, k_1(\alpha), k_2(\alpha)\,] \end{array}\right\} \textbf{Fuzzy interval estimators}$$

Output:

$$Y(X_1, X_2, \ldots\ldots\ldots, X_n)(\alpha) = [\, Y_1(X_1, X_2, \ldots\ldots\ldots, X_n)(\alpha), Y_2(X_1, X_2, \ldots\ldots\ldots, X_n)(\alpha)] \quad (4)$$

α is the membership value, ranges from 0 to 1.

3.2. Yager's Ranking Index Method

Before discussing the Yager's ranking method, we need a brief idea about Triangular fuzzy number.

Triangular fuzzy number: A triangular fuzzy number **a** is defined by a triplet (a1, a2, a3). The membership function is defined as

$$\boldsymbol{\mu_a (x) = \{ (x - a1) / (a2 - a1) \text{ if } a1 \le x \le a2}$$
$$\boldsymbol{(a3 - x) / (a3 - a2) \text{ if } a2 \le x \le a3}$$
$$\boldsymbol{0 \text{ otherwise } \}}$$

The triangular fuzzy number is based on three-value judgment: The minimum possible value a1, the most possible value a2 and the maximum possible value a3

3.2.2. Yager's ranking method mathematical explanation

For the above given triplet (a1, a2, a3) , The α − cut will be

$$[(a2 - a1)\,\alpha + a1 \;,\; -(a3-a2)\,\alpha + a3\,] = [c1(\alpha), c2(\alpha)] \qquad (5)$$

where, $a1^{(\alpha)} = (a2 - a1)\,\alpha + a1$, $a3^{(\alpha)} = -(a3-a2)\,\alpha + a3$

Then,

Yager's ranking index is $\mathbf{Y}_{\mathbf{yager's}} = 0.5\int_0^1 (\; c1(\alpha) + c2(\alpha) \;)\,d\alpha$ (6)

4. Data collection and analysis

Here we generate a data sheet from a survey on two wheeler's details.

two_wheeler.sav [DataSet1] - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

1 : Two_wheeler_name Passion plus(Hero Honda) Visible: 10 of 10 Variables

	Two_wheeler_name	Engine_capacity	milage	performance_engine	load_capacity	lubricant_quality	service_routine	tube_check	Road_quality	Acceleration_quality	var	var
1	Passion plus(Hero Honda)	110.00	45.00	good	good	very good	within every 3 mon...	every four months after	average	medium		
2	Passion plus(Hero Honda)	110.00	45.00	good	good	very good	within every 3 mon...	every four months after	average	medium		
3	Discover(Bajaj)	110.00	45.00	average	good	good	within every 3 mon...	every three months after	average	medium		
4	Discover(Bajaj)	110.00	45.00	average	good	good	within every 3 mon...	every three months after	average	medium		
5	LML(Vespa)	150.00	35.00	good	high	good	within every 6 mon...	every two months after	average	high		
6	Bajaj(Priya)	110.00	35.00	good	good	good	within every 9	every three months after	average	medium		
7	Discover(Bajaj)	110.00	65.00	good	high	very good	within every 3 mon...	every three months after	average	high		
8	Bajaj(Boxer 80)	110.00	65.00	good	good	very good	within every 6 mon...	every months	average	medium		
9	Hero honda Splender	110.00	45.00	good	good	good	within every 6 mon...	every three months after	average	medium		
10	Passion Pro(Hero Honda)	110.00	45.00	good	good	average	within every 6 mon...	every four months after	average	medium		
11	Yamaha(Gladiator)	150.00	35.00	good	high	good	within every 3 mon...	every four months after	good	medium		
12	Passion Pro(Hero Honda)	110.00	65.00	good	high	good	within every 6 mon...	every months	good	medium		
13	Bajaj pulsar	150.00	35.00	good	high	very good	within every 6 mon...	every three months after	good	medium		
14	Hero Glamour F1	150.00	65.00	good	high	very good	within every 6 mon...	every months	average	medium		
15	Glamour(Hero)	125.00	45.00	good	high	average	within every 12 mo...	every months	average	medium		
16	Hero honda Splender plus	110.00	65.00	good	very high	good	within every 3 mon...	every months	average	medium		
17	Hero honda Splender	110.00	65.00	good	good	very good	within every 6 mon...	every months	average	medium		
18	Passion plus(Hero Honda)	120.00	45.00	good	high	average	within every 12 mo...	every months	poor	medium		
19	Spleder pro(Hero Honda)	110.00	65.00	good	high	very good	within every 6 mon...	every months	average	medium		
20	Discover(Bajaj)	110.00	65.00	good	good	good	within every 6 mon...	every months	average	medium		
21	Ignitor(Hero)	125.00	45.00	good	good	good	within every 6 mon...	every months	average	medium		
22	Glamour(Hero)	125.00	45.00	good	high	very good	within every 6 mon...	every months	average	medium		
23	Passion plus(Hero Honda)	110.00	65.00	very good	high	good	within every 3 mon...	every two months after	good	medium		
24	Super splender	125.00	45.00	very good	high	good	within every 3 mon...	every two months after	average	medium		

Data View Variable View

SPSS Processor is ready

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Table 1: Two wheelers survey report

Using Yager's Ranking Index method of equations (5) and (6) in Table 1, we generate Table 2 (crisp format). The detail process for generating Table 2 is given below through equations (7)- (10)

4.1. Derived Table (Crisp format)

Two wheeler name	Engine capacity	Mileage	Performance engine	Load capacity	lubricant quality	service routine	tube check	Road quality	Accelera Tion quality
Passion plus(H.H	110.0	45.0	6.0	90.0	9.0	9.0	2.0	6.0	5.0
Passion plus(HH)	110.0	45.0	6.0	90.0	9.0	9.0	2.0	6.0	5.0
Discover(Bajaj)	110.0	45.0	3.0	90.0	6.0	9.0	5.0	6.0	5.0
Discover(Bajaj)	110.0	45.0	3.0	90.0	6.0	9.0	5.0	6.0	5.0
LML(Vespa)	150.0	35.0	6.0	127.5	6.0	6.0	6.0	6.0	8.25
Bajaj(Priya)	110.0	35.0	6.0	90.0	6.0	4.0	5.0	6.0	5.0
Discover(Bajaj)	110.0	65.0	6.0	127.5	9.0	9.0	5.0	6.0	8.25
Bajaj(Boxer 80)	110.0	65.0	6.0	90.0	9.0	6.0	9.0	6.0	5.0
HH Splender	110.0	45.0	6.0	90.0	6.0	6.0	5.0	6.0	5.0
Passion Pro(HH)	110.0	45.0	6.0	90.0	4.0	6.0	2.0	6.0	5.0
Yamaha(Gladiat)	150.0	35.0	6.0	127.5	6.0	9.0	2.0	9.0	5.0
Passion Pro(HH)	110.0	65.0	6.0	127.5	6.0	6.0	9.0	9.0	5.0
Bajaj pulsar	150.0	35.0	6.0	127.5	9.0	6.0	5.0	9.0	5.0
Hero Glamour F1	150.0	65.0	6.0	127.5	9.0	6.0	9.0	6.0	5.0
Glamour(Hero)	125.0	45.0	6.0	127.5	4.0	1.0	9.0	6.0	5.0
HH Splenderpl	110.0	65.0	6.0	177.5	6.0	9.0	9.0	6.0	5.0
HH Splender	110.0	65.0	6.0	90.0	9.0	6.0	9.0	6.0	5.0
Passion plus(HH)	120.0	45.0	6.0	127.5	4.0	1.0	9.0	3.0	5.0
Spleder pro(HH)	110.0	65.0	6.0	127.5	9.0	6.0	9.0	6.0	5.0
Discover(Bajaj)	110.0	65.0	6.0	90.0	6.0	6.0	9.0	6.0	5.0
Ignitor(Hero)	125.0	45.0	6.0	90.0	6.0	6.0	9.0	6.0	5.0
Glamour(Hero)	125.0	45.0	6.0	127.5	9.0	6.0	9.0	6.0	5.0
Passion plus(HH)	110.0	65.0	9.0	127.5	6.0	9.0	6.0	9.0	5.0
Super splender	125.0	45.0	9.0	127.5	6.0	9.0	6.0	6.0	5.0
Spleder pro(HH)	110.0	65.0	6.0	90.0	6.0	9.0	2.0	6.0	5.0
Platina(Bajaj)	110.0	65.0	3.0	90.0	6.0	1.0	6.0	3.0	5.0
Hero Honda	120.0	45.0	6.0	90.0	6.0	9.0	2.0	6.0	5.0
HH Splender super	120.0	45.0	6.0	90.0	4.0	6.0	6.0	6.0	5.0
Pulsar(Bajaj)	150.0	35.0	6.0	127.5	9.0	9.0	9.0	9.0	8.25
Super splender(HH)	125.0	45.0	6.0	127.5	6.0	9.0	9.0	9.0	5.0
Hero Honda	110.0	45.0	6.0	90.0	4.0	9.0	2.0	6.0	5.0
HHPassion	110.0	35.0	6.0	127.5	6.0	6.0	2.0	6.0	5.0
Hero	150.0	35.0	6.0	127.5	6.0	4.0	2.0	6.0	8.25
Hero	125.0	45.0	9.0	127.5	6.0	9.0	6.0	9.0	8.25

Table 2: Two wheelers survey report-in crisp format

4.2. Data Analysis

a) Dependent variable: Engine Mileage.

b) Independent Variables:

Engine Capacity, Performance of Engine, Load Capacity, Lubricant Quality, Routine Service, Tube Checking, Road Quality, Acceleration Quality are the independent variables and all are taken as fuzzy except last one.

We apply Yager's ranking index method for converting the fuzzy data to crisp value. The above table-2 is generated after converting fuzzy data into crisp values.

Independent Variable	**Triangular Fuzzy Data**
Performance of Engine	Poor(0,1,2), average(2,3,4), good(5,6,7), very good(8,9,10)
Load Capacity	Very low(40,50,60), low(60,70,80), good(80,90,100), high(110,130,140), very high(150,180,200)
Lubricant Quality	Very poor(1,2,3),Poor(2,3,4), average (3,4,5), good(5,6,7), very good(8,9,10)
Routine Service	Within 12 months(0,1,2), Within 9 months(3,4,5), Within 6 months(5,6,7), Within 3 months(8,9,10).
Tube Checking	Within 4 months(1,2,3), Within 3 months(4,5,6), Within 2 months(5,6,7), Within 1 month(8,9,10)
Road Quality	Very poor(0,1,2),Poor(2,3,4), average (5,6,7), good(8,9,10)
Acceleration Quality	Low(1,2,3),mid(4,5,6),high(7,8,10)

Table 3: Triangular fuzzy values in linguistic terms

We calculate the triangular fuzzy value good (5, 6, 7) in "performance of engine" with Yager's rule **[**7] using the following process.

$$\mu_{poor}(x) = \left\{ \begin{array}{ll} (x - 5) / (6- 5) & \text{if } 5 \le x \le 6 \\ (7 - x) / (7 - 6) & \text{if } 6 \le x \le 7 \end{array} \right\}$$

Membership value from Triangular Fuzzy Numbers.

The α – cut (formulation) will be

$$[\,(6 - 5)\,\alpha + 5\ ,\ -(6-7)\,\alpha + 7\,] = [\ c1(\alpha)\ ,\ c2(\alpha)\ ] = [\,\alpha + 5\ ,\ 7 - \alpha\,] \tag{7}$$

Then,

$$Y_{yager's(poor)} = 0.5\int_0^1 (c1(\alpha) + c2(\alpha))\,d\alpha = 0.5\int_0^1 [\,\alpha + 5 + 7 - \alpha\,]\,d\alpha = 6 \tag{8}$$

The α − cut for very high (150,180,200) in "Load Capacity" will be

$$[(180 - 150)\,\alpha + 150\ ,\ -(200-180)\,\alpha + 200\,] = [c1(\alpha)\ , c2(\alpha)] = [30\alpha + 150\ ,\ 200 - 20\alpha\,] \tag{9}$$

Then,

$$Y_{yager's(very\ high)} = 0.5\int_0^1 (\ c1(c1(\alpha) + c2(\alpha))\,d\alpha = 0.5\int_0^1 [30\alpha + 150 + 200 - 20\alpha\,]\,d\alpha = 177.5 \tag{10}$$

Same way we calculate all the triangular fuzzy values of different independent variables and generate Table-2.

After this we apply fuzzy regression technique on Table-2, which is explained below.

5. Methodology of Fuzzy Multiple linear Regression

Step 1:

From the table-2 we generate the matrix X where all the independent variables are taken into consideration in the following way.

$$X=\begin{pmatrix} 1 & X_{11} & X_{21} & \dots\dots & X_{k1} \\ 1 & X_{12} & X_{22} & \dots\dots & X_{k2} \\ \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \\ 1 & X_{1n} & X_{2n} & \dots\dots & X_{kn} \end{pmatrix} \tag{11}$$

Here k=8 and n=34

X_{ij} (i=1,2,....k, j=1,2,..,n) are independent variables.

Y= [$y_{1,}$ $y_{2,\dots\dots}$$y_n$] is dependent variable.

Mileage is dependent variable.

All the values are taken from Table 2.

Step 2:

Now we calculate the constant and regression coefficients as point estimator.

So we can define it as a vector θ^t=[a , b , c , d , e, f , g , h , i], where

$$\theta=(X^t X)^{-1} X^t Y^t \tag{12}$$

Putting the values from Table-2 we get (Using Maple 6)

a=74.6718 , b=0.4021, c= 0.5575 , d=0.0533 , e= 1.5494 , f= 0.4678 , g= 1.5963

h= -1.0910 , i= -0.3353

Step 3:

Now we calculate $(X^t X)^{-1}$ and get the matrix

$$\begin{pmatrix} 2.9220 & -0.0167 & -0.0829 & -0.0002 & -0.0272 & -0.0533 & -0.0270 & 0.0564 & -0.0038 \\ -0.0167 & 0.0002 & 0.0003 & -0.0000 & -0.0003 & 0.0006 & 0.0002 & -0.0010 & -0.0010 \\ -0.0829 & 0.0003 & 0.0251 & -0.0004 & 0.0016 & -0.0005 & 0.0002 & -0.0076 & -0.0029 \\ -0.0002 & -0.0000 & -0.0004 & 0.0001 & 0.0001 & -0.0001 & -0.0003 & -0.0001 & -0.0003 \\ -0.0272 & -0.0003 & 0.0016 & 0.0001 & 0.0126 & -0.0037 & -0.0027 & 0.0009 & -0.0019 \\ -0.0533 & 0.0006 & -0.0005 & -0.0001 & -0.0037 & 0.0103 & 0.0033 & -0.0095 & -0.0014 \\ -0.0270 & 0.0002 & 0.0002 & -0.0003 & -0.0027 & 0.0033 & 0.0056 & -0.0023 & 0.0014 \\ 0.0564 & -0.0010 & -0.0076 & -0.0001 & 0.0009 & -0.0095 & -0.0023 & 0.0278 & 0.0021 \\ -0.0038 & -0.0010 & -0.0029 & -0.0003 & -0.0019 & -0.0014 & 0.0014 & 0.0021 & 0.0304 \end{pmatrix} \tag{13}$$

Diagonals of the matrix are $a_{11}=2.9220$ $a_{22}=0.0002$ $a_{33}=0.0251$ $a_{44}=0.0001$ $a_{55}=0.0126$ $a_{66}=0.0103$ $a_{77}=0.0056$ $a_{88}=0.0278$ $a_{99}=0.0304$

Step 4:

Now calculate $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}(y_i - y\_cap_i)^2$ (14)

And $б^2 = \sum_{i=1}^{n} e_i^2 /(n-9)$, here total number of estimators are 9. (15)

where $y\_cap_i=(a+bx_{1+}cx_{2+} dx_{3+}ex_{4+} fx_{5+}gx_{6+} hx_{7+}ix_8)$ (16)

Using Maple 6, we get the value of 1) $\sum_{i=1}^{n} e_i^2 = 2229.309735$

2) $б^2=89.17238940$ Here n=34

Step 5:

In this step we are deriving the interval estimators from point estimators (a,b,c,d,e,f,g,h,i) **[5].**

We may find the interval estimators through statistical confidence interval, where statistical level of significance can be used to generate the membership value, ranges from 0 to 1. Here we take t distribution technique to generate the value of certain significance level.

6. Calculation of Interval estimator:

A (1-β) 100% confidence interval for a is

$$[\quad a - t_{β/2}\, б\sqrt{a_{11}} \;,\; a + t_{β/2}\, б\sqrt{a_{11}} \quad]$$

(1-β) 100% confidence interval for b is

$$[\quad b - t_{β/2}\, б\sqrt{a_{22}} \;,\; b + t_{β/2}\, б\sqrt{a_{22}} \quad]$$

(1-β) 100% confidence interval for c is

$$[\quad c - t_{β/2}\, б\sqrt{a_{33}} \;,\; c + t_{β/2}\, б\sqrt{a_{33}} \quad]$$

(1-β) 100% confidence interval for d is

$$[\quad d - t_{β/2}\, б\sqrt{a_{44}} \;,\; d + t_{β/2}\, б\sqrt{a_{44}} \quad]$$

A (1-β) 100% confidence interval for e is

$$[\quad e - t_{β/2}\, б\sqrt{a_{55}} \;,\; e + t_{β/2}\, б\sqrt{a_{55}} \quad] \qquad (17)$$

(1-β) 100% confidence interval for f is

$$\left[\ f - t_{\beta/2}\, \sigma\sqrt{a_{66}}\ ,\ f + t_{\beta/2}\, \sigma\sqrt{a_{66}}\ \right]$$

(1-β) 100% confidence interval for g is

$$\left[\ g - t_{\beta/2}\, \sigma\sqrt{a_{77}}\ ,\ g + t_{\beta/2}\, \sigma\sqrt{a_{77}}\ \right]$$

(1-β) 100% confidence interval for h is

$$\left[\ h - t_{\beta/2}\, \sigma\sqrt{a_{88}}\ ,\ h + t_{\beta/2}\, \sigma\sqrt{a_{88}}\ \right]$$

(1-β) 100% confidence interval for i is

$$\left[\ i - t_{\beta/2}\, \sigma\sqrt{a_{99}}\ ,\ i + t_{\beta/2}\, \sigma\sqrt{a_{99}}\ \right]$$

Using all previous equations at (17) we can generate the following table (Table-4) where β is considered as membership value [0, 1] and corresponding value at β-level of significance for certain degree of freedom (here d.o.f is 25) from t-table is, $t_{\beta/2}$(For Two Tailed Test) . Here we denote $t_{\beta/2}$ as α.

Tabulated Results:

	Lower Limit values	**Upper Limit values**
a(α)	74.6718-16.1419 α $a_1(\alpha)$	74.6718+16.1419 α $a_2(\alpha)$
b(α)	-0.4021-0.13354α $b_1(\alpha)$	-0.4021+0.13354α $b_2(\alpha)$
c(α)	0.5575-1.4960 α $c_1(\alpha)$	0.5575+1.4960 α $c_2(\alpha)$
d(α)	0.0533-0.09443 α $d_1(\alpha)$	0.0533+0.09443 α $d_2(\alpha)$
e(α)	1.5494-1.0599 α $e_1(\alpha)$	1.5494+1.0599 α $e_2(\alpha)$
f(α)	0.4678-0.9583 α $f_1(\alpha)$	0.4678+0.9583 α $f_2(\alpha)$
g(α)	1.5963-0.70665 α $g_1(\alpha)$	1.5963+0.70665 α $g_2(\alpha)$
h(α)	-1.0910-1.5744 α $h_1(\alpha)$	-1.0910+1.5744 α $h_2(\alpha)$
i(α)	-0.3353-1.6464 α $i_1(\alpha)$	-0.3353+1.6464 α $i_2(\alpha)$

Table 4: Fuzzy interval estimators

If we put these tabulated values (Table-4) in equation (2) and (3) and take values from Table-2, we can get all 34 mileages with fuzzy interval. We have shown here few of them.

$$\left.\begin{array}{l} Y_1 1(\alpha)= [\, 51.70770 - 85.56493076\,\alpha\,]\ Y_1 2(\alpha)=[\, 51.70770 + 85.56493076\alpha] \\ Y_2 1(\alpha)= [51.70770 - 85.56493076\,\alpha\,]\ Y_2 2(\alpha)=[\, 51.70770 + 85.56493076\alpha] \\ Y_3 1(\alpha)= [50.17590 - 80.01673237\alpha\,]\ Y_3 2(\alpha)= [\, 50.17590 + 80.01673237\alpha] \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y_{34} 1(\alpha)= [46.721725 - 105.3186176\alpha] Y_{34} 2(\alpha)=[\, 46.721725 + 105.3186176\alpha] \end{array}\right\} \quad (18)$$

7. Results and Discussion

At 5% level of significance and 25 degree of freedom (34-number of estimators) we get the value of $t_{\beta/2}$=2.060 (At 5% level of significance for Two Tailed Test)
As we take $\alpha = t_{\beta/2} =$ 2.060, from series of equations (18) we get the values

$$\left.\begin{array}{ll} Y_1 1(\alpha)= -124.5560574 & Y_1 2(\alpha)= 227.9714574 \\ Y_2 1(\alpha)= -124.5560574 & Y_2 2(\alpha)= 227.9714574 \\ Y_3 1(\alpha)= -114.6585687 & Y_3 2(\alpha)= 215.0103687 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots \\ Y_{34} 1(\alpha)= -170.2346272 & Y_{34} 2(\alpha)= 263.6780772 \end{array}\right\} \quad - \quad (19)$$

The following results can be represented as

$$\left.\begin{array}{l} Y_1(\alpha) = [\, 45.44 , 57.97] \\ Y_2(\alpha) = [\, 45.44 , 57.97] \\ Y_3(\alpha) = [\, 45.34 , 55.01] \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ Y_{34}(\alpha)=[\, 39.76 , 53.67] \end{array}\right\}$$

All the negative values in the intervals at (19) are converted to positive value, keeping the midpoint unchanged. Now using middle of the maxima technique we can defuzzify all the interval values into crisp data and get the best possible mileage.

Example: From $Y_1(\alpha)$ we get Y_1=(45.44 + 57.97)/2=51.07, others can be calculated in same way.

We can calculate these values at any statistical significance level.

8. Conclusion and Future Work

Through this article we tried to show that any real life application always gives a result, based on approximation and uncertainty. Fuzzy interval estimation is one of the best ways to solve this problem. Here we use fuzzy multiple linear regression technique to derive the mileages of two wheelers in the form of interval, within which mileage varies for other independent factors.

As we go with our future work, two important aspects are factor analysis and fuzzy MANOVA applications. With factor analysis we will be able to generate individual factors of similar types of independent variables which are highly correlated. Then those factors can be used in fuzzy regression to give more perfect analysis. Multivariate Analysis of Variance (MANOVA) consists of calculations that provide information about levels of variability within a regression model and form a basis for tests of significance. Using Fuzzy MANOVA further we can analysis our fuzzy multiple linear regression model to deal with residuals and other factors.

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