Annals of Pure and Applied Mathematics
Vol. 5, No.1, 2013, 37-46
ISSN: 2279-087X (P), 2279-0888(online)
Published on 13 November 2013
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Product of Interval Valued Intuitionistic fuzzy graph

***S. N. Mishra*[1] and *A. Pal*[2]**

Department of Mathematics, National Institute of Technology, Durgapur, India
[1]Email: sachchida.n.mishra@gmail.com
[2]Email: anita.buie@gmail.com

Received 28 September 2013; accepted 5 October2013

Abstract. In this paper we introduce the notion of product of two interval valued intuitionistic fuzzy graphs and investigate some of their properties. We discuss some propositions on Cartesian product and define some properties on it. We also define some other products like *tensor product, lexicographic product of interval valued* intuitionistic fuzzy graphs.

Keywords: tensor product, lexicographic product, interval valued intuitionistic fuzzy set.

AMS Mathematics Subject Classification (2010): 05C72

1.Introduction

In 1965, Zadeh [13] introduced the notion of fuzzy set as a method of presenting uncertainty. Since complete information in science and technology is not always available. Thus we need mathematical models to handle various types of systems containing elements of uncertainty. Atanassov proposed intuitionistic fuzzy set (IFS) [4] which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After three years Atanassov and Gargov[5]introduced interval-valued intuitionistic fuzzy set (IVIFS) which is help full to model the problem precisely. After 10 years of Zadeh's classic 'fuzzy sets' [13] Rosenfeld [14] introduced fuzzy graphs. Yeh and Bang [15] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graphs. It has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, transportation, etc. Mordeson and Nair[6-8], Bhutani and Rosenfeld [6], Sunitha and Vijayakumar [9-12] are among the other main contributors in fuzzy graphs. The operation of union, join, cartesian product and composition on two fuzzy graphs were defined by Mordeson and Peng [6]. Akram and Dudec [3] defined interval-valued fuzzy graphs in 2011. Rashmanlou and Pal define the Product of interval-valued fuzzy graphs and their degree. Methods of interval-valued intuitionistic fuzzy sets and fuzzy logic is useful to handle the problem containing uncertainty like the vehicle travel time or vehicle capacity on a road network may not be known exactly. In this paper we define Cartesian product of two interval valued intuitionistic fuzzy graphs and obtain its degree and on the basis of degree we examine the robustness of the product of these graphs.

2. Preliminaries

Definition 2.1. A fuzzy set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, i.e. $\mu(uv) \le \sigma(u) \wedge \sigma(v)$. The underlying crisp graph of G = (σ, μ) is denoted by $G^* = (V, E)$ where $E \subseteq V \times V$.

Definition 2.2. Let G = (σ, μ) be a fuzzy graph, the degree of a vertex u in G is defined by

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv).$$

Definition 2.3. The order of a fuzzy graph G is defined by $o(G) = \sum_{u \in V} \sigma(u)$.

Let $X=\{x_1, x_2, \ldots\ldots, x_n\}$ be any set. Let D[0, 1] be the set of all closed subintervals of the interval [0, 1] and element of this set are denoted by uppercase letters. If $M \in D[0, 1]$ then it can be represented as $M=[M_L, M_U]$, where M_L and M_U are the lower and upper limits of M. For $M \in D[0, 1]$, $\overline{M} = 1\text{-}m$ represents the interval $[1-M_L, 1-M_U]$.

Definition 2.4. An IVIFS A in X, is given by $A=\{\langle x, M_A(x), N_A(x)\rangle / x \in X\}$ where $M_A: X \to D[0, 1]$, $N_A: X \to D[0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x to the set, where $M_A(x)=[M_{AL}(x), M_{AU}(x)]$ and $N_A(x)=[N_{AL}(x), N_{AU}(x)]$ with the condition $0 \le M_{AU}(x)+N_{AU}(x) \le 1$ for all $x \in X$.

Definition 2.5. Let $G^*_1= (V_1; E_1)$ and $G^*_2= (V_2; E_2)$ be two simple graphs, we can construct several new graphs. The first construction called the Cartesian product of G^*_1 and G^*_2 gives a graph $G^*_1 \quad G^*_2= (V; E)$ with $V = V_1 \times V_2$ and

$E =\{(x; x_2)(x; y_2)|x \in V_1; x_2y_2 \in E_2\} \cup \{(x_1; z)(y_1; z)|x_1y_1 \in E_1; z \in V_2\}$.

Throughout this paper, we denote G^* a crisp graph, and G an intuitionistic fuzzy graph.

Definition 2.6. An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair G =(A; B) where

(i) the functions $M_A : V \longrightarrow D[0; 1]$ and $N_A : V \longrightarrow D[0; 1]$ denote the degree of membership and nonmembership of the element $x \in V$, respectively such that $0 < M_A(x) + N_A(x) \le 1$ for all $x \in V$,

(ii) the functions $M_B : E \subseteq V \times V \to D[0; 1]$ and $N_B : E \subseteq V \times V \to D[0; 1]$ are defined by

$M_{BL}(\{x; y\}) \le \min(M_{AL}(x); M_{AL}(y))$ and $N_{BL}(\{x; y\}) \ge \max(N_{AL}(x); N_{AL}(y))$
and $M_{BU}(\{x; y\}) \le \min(M_{AU}(x); M_{AU}(y))$ and $N_{BU}(\{x; y\}) \ge \max(N_{AU}(x); N_{AU}(y))$
such that $0 < M_{BU}(\{x; y\}) + N_{BU}(\{x; y\}) \le 1$ for all $\{x; y\} \in E$.

Example 2.1. *Consider a graph G*= (*V*, *E*) *such that* V = {x, y, z}, E = {xy, yz, zx}. Let A be an interval value intiutionistic fuzzy set of V and let B be an interval-valued *fuzzy set of E⊆ V × V defined by*
A ={⟨x, [0.4, 0.6],[0.2, 0.4]⟩, ⟨y,[0.5, 0.7], [0.1, 0.3]⟩, ⟨z, [0.6, 0.7], [0.1, 0.3]⟩},

B ={⟨xy, [0.3, 0.5], [0.2, 0.4]⟩, ⟨yz, [0.4, 0.6], [0.1, 0.4]⟩, ⟨zx, [0.3, 0.6], [0.2, 0.4]⟩}

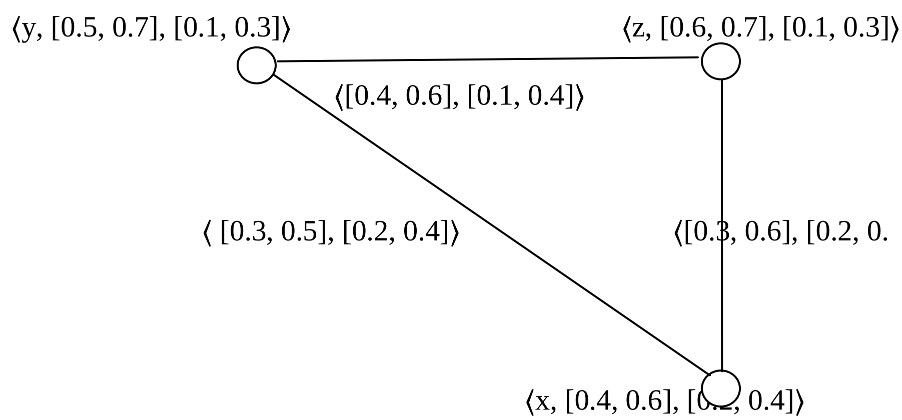


Figure 1. Interval-valued intuitionistic fuzzy graph

Definition 2.8. The cartesian product G = $G_1 \times G_2$ of two interval-valued intuitionistic fuzzy graphs $G_1 = (A_1, B_1)$ and $G_2 = (A_2,B_2)$ of the graphs $G^*_1 = (V_1, E_1)$ and $G^*_2 = (V_2,E_2)$ is defined as pair $(A_1 \times A_2, B_1 \times B_2)$ such that:

(*i*)
$(M_{A1L}\times M_{A2L})(u_1, u_2) = \min (M_{A1L}(u_1), M_{A2L}(u_2))$
$(M_{A1U}\times M_{A2U})(u_1, u_2) = \min (M_{A1U}(u_1), M_{A2U}(u_2))$
$(N_{A1L}\times N_{A2L})(u_1, u_2) = \max (N_{A1L}(u_1), N_{A2L}(u_2))$
$(N_{A1U}\times N_{A2U})(u_1, u_2) = \max (N_{A1U}(u_1), N_{A2U}(u_2))$ *for all* $(u_1, u_2) \in V$

(*ii*) $(M_{B1L}\times M_{B2L})((u, u_2)(u, v_2)) = \min (M_{A1L}(u), M_{B2L}(u_2v_2))$
$(M_{B1U}\times M_{B2U})(u, u_2)(u, v_2)) = \min(M_{A1U}(u), M_{B2U}(u_2v_2))$
$(N_{B1L}\times N_{B2L})((u, u_2)(u, v_2)) = \max (N_{A1L}(u), N_{B2L}(u_2v_2))$
$(N_{B1U}\times N_{B2U} ((u, u_2)(u, v_2)) = \max (N_{A1U}(u), N_{B2U}(u_2v_2))$ *f or all* $u\in V_1$ *and* $u_2v_2 \in E_2$,

(*iii*) $(M_{B1L}\times M_{B2L})((u_1, z)(v_1, z)) = \min (M_{B1L}(u_1v_1), M_{A2L}(z))$
$(M_{B1U}\times M_{B2U})((u_1, z)(v_1, z)) = \min(M_{B1U}(u_1v_1), M_{A2U}(z))$
$(N_{B1L}\times N_{B2L})((u_1, z)(v_1, z)) = \max (N_{B1L}(u_1v_1), N_{A2L}(z))$
$(N_{B1U}\times N_{B2U})((u_1, z)(v_1, z)) = \max (N_{B1U}(u_1v_1), N_{A2U}(z))$ for all $z \in V_2$ and $u_1v_1\in E_1$,

Definition 2.9. The tensor product $G_1 \otimes G_2$ of two interval-valued intuitionistic fuzzy graphs $G_1 = (A_1,B_1)$ and $G_2 =(A_2,B_2)$ of $G*_1 = (V_1,E_1)$ and $G*_2 = (V_2,E_2)$ respectively is defined as a pair (A,B), where $A = (M_A, N_A)$ and $B = (M_B, N_B)$ are interval-valued intuitionistic fuzzy sets on $V = V_1 \times V_2$ and $E = \{(u_1, u_2), (v_1, v_2) \mid (u_1, v_1) \in E_1, (u_2v_2) \in E_2\}$, respectively which satisfies the followings:
(*i*) $(M_{A1L}\otimes M_{A2L})(u_1, u_2) = \min(M_{A1L}(u_1), M_{A2L}(u_2))$

$(M_{A1U} \otimes M_{A2U})(u_1, u_2) = \min(M_{A1U}(u_1), M_{A2U}(u_2))$
$(N_{A1L} \otimes N_{A2L})(u_1, u_2) = \max(N_{A1L}(u_1), N_{A2L}(u_2))$
$(N_{A1U} \otimes N_{A2U})(u_1, u_2) = \max(N_{A1U}(u_1), N_{A2U}(u_2))$ *for all* $(u_1, u_2) \in V_1 \times V_2$,
(*ii*) $(M_{B1L} \otimes M_{B2L})((u_1, u_2)(v_1, v_2)) = \min(M_{B1L}(u_1v_1), M_{B2L}(u_2v_2))$
$(M_{B1U} \otimes M_{B2U})((u_1, u_2)(v_1, v_2)) = \min(M_{B1U}(u_1v_1), M_{B2U}(u_2v_2))$
$(N_{B1L} \otimes N_{B2L})((u_1, u_2)(v_1, v_2)) = \max(N_{B1L}(u_1v_1), N_{B2L}(u_2v_2))$
$(N_{B1U} \otimes N_{B2U})((u_1, u_2)(v_1, v_2)) = \max(N_{B1U}(u_1v_1), N_{B2U}(u_2v_2))$ *for all* $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$.

Definition 2.10. The lexicographic product $G_1 \star G_2$ of two interval-valued intuitionistic fuzzy graphs $G_1 = (A_1,B_1)$ and $G_2 = (A_2,B_2)$ of $G^*_1 = (V_1,E_1)$ and $G^*_2 = (V_2,E_2)$ respectively is defined as a pair (A,B), where $A = (M_A, N_A)$ and $B = (M_B, N_B)$ are interval-valued intuitionistic fuzzy sets on $V = V_1 \times V_2$ and $E = \{(u, u_2)(u, v_2) \mid u \in V_1, (u_2v_2) \in E_2\} \cup \{((u_1, u_2)(v_1, v_2)) \mid (u_1v_1) \in E_1, u_2v_2 \in E_2\}$ respectively which satisfies the followings:

(*i*) $(M_{A1L} \star M_{A2L})(u_1, u_2) = \min(M_{A1L}(u_1), M_{A2L}(u_2))$
$(M_{A1U} \star M_{A2U})(u_1, u_2) = \min(M_{A1U}(u_1), M_{A2U}(u_2))$
$(N_{A1L} \star N_{A2L})(u_1, u_2) = \max(N_{A1L}(u_1), N_{A2L}(u_2))$
$(N_{A1U} \star N_{A2U})(u_1, u_2) = \max (N_{A1U}(u_1), N_{A2U}(u_2))$ *for all* $(u_1, u_2) \in V_1 \times V_2$,

(*ii*) $(M_{B1L} \star M_{B2L})((u, u_2)(u, v_2)) = \min(M_{A1L}(u), M_{B2L}(u_2v_2))$
$(M_{B1U} \star M_{B2U})((u, u_2)(u, v_2)) = \min(M_{A1U}(u), M_{B2U}(u_2v_2))$
$(N_{B1L} \star N_{B2L})((u, u_2)(u, v_2)) = \max (N_{A1L}(u), N_{B2L}(u_2v_2))$
$(N_{B1U} \star N_{B2U})((u, u_2)(u, v_2)) = \max (N_{A1U}(u), N_{B2U}(u_2v_2))$ for all $u \in V_1$ and $u_2v_2 \in E_2$,

(*iii*) $(M_{B1L} \star M_{B2L})((u_1, u_2)(v_1, v_2)) = \min(M_{B1L}(u_1v_1), M_{B2L}(u_2v_2))$
$(M_{B1U} \star M_{B2U})((u_1, u_2)(v_1, v_2)) = \min(M_{B1U}(u_1v_1), M_{B2U}(u_2v_2))$
$(N_{B1L} \star N_{B2L})((u_1, u_2)(v_1, v_2)) = \max (N_{B1L}(u_1v_1), N_{B2L}(u_2v_2))$
$(N_{B1U} \star N_{B2U})((u_1, u_2)(v_1, v_2)) = \max (N_{B1U}(u_1v_1), N_{B2U}(u_2v_2))$ *for all* $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$.

3. Degree of a vertex in cartesian product

By the definition, for any vertex $(u_1, u_2) \in V_1 \times V_2$,

$d^-{}_{G_1 \times G_2}(u_1, u_2) =$

$$\sum_{(u_1;u_2)(v_1;v_2)\in E} \left(M_{B_1L} \times M_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right) - \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(N_{B_1L} \times N_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right)$$

$= \sum_{u_1=v_1;\, u_2v_2\in E_2} M_{A_{1L}}(u_1) \wedge M_{B_{2L}}(u_2v_2) + \sum_{u_2=v_2;\, u_1v_1\in E_1} M_{A_{2L}}(u_2) \wedge M_{B_{1L}}(u_1v_1) - \{\sum_{u_1=v_1;\, u_2v_2\in E_2} N_{A_{1L}}(u_1) \vee N_{B_{2L}}(u_2v_2) + \sum_{u_2=v_2;\, u_1v_1\in E_1} N_{A_{2L}}(u_2) \vee$ $NB1Lu1v1$

Similarly,

$$d^{+}{}_{G_1\times G_2}(u_1,u_2)$$

$$= \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(M_{B_1U} \times M_{B_2U}\right)\left((u_1;u_2)(v_1;v_2)\right) - \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(N_{B_1U} \times N_{B_2U}\right)\left((u_1;u_2)(v_1;v_2)\right)$$

$$= \sum_{u_1=v_1;\, u_2v_2\in E_2} M_{A_{1U}}\,(u_1) \wedge M_{B_{2U}}\,(u_2v_2) + \sum_{u_2=v_2;\, u_1v_1\in E_1} M_{A_{2U}}\,(u_2) \wedge M_{B_{1U}}\,(u_1v_1) - \left\{ \sum_{u_1=v_1;\, u_2v_2\in E_2} N_{A_{1U}}\,(u_1) \vee N_{B_{2U}}(u_2v_2) + \sum_{u_2=v_2;\, u_1v_1\in E_1} N_{A_{2U}}\,(u_2) \vee N_{B_{1U}}(u_1v_1) \right\}$$

Example 3.1. Consider the interval-valued intuitionistic fuzzy graphs G_1, G_2 and $G_1 \times G_2$ in Figure 2.

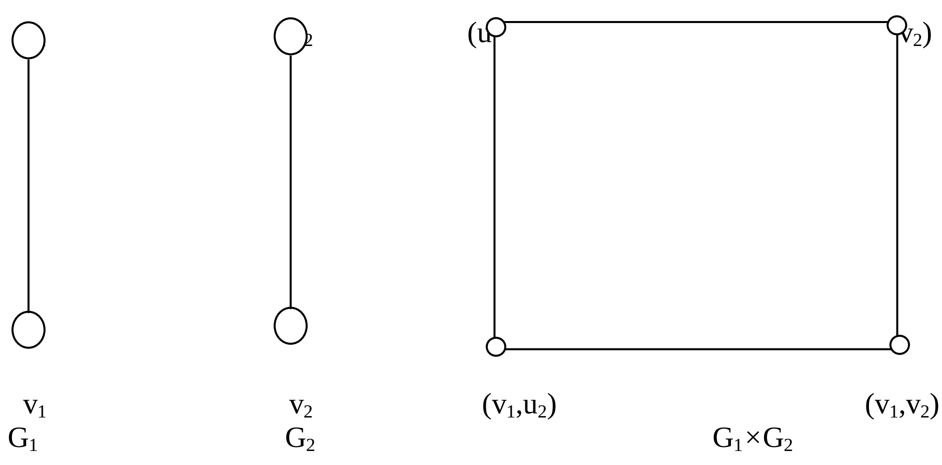


Figure 2: Cartesian product of G_1 and G_2 ($G_1 \times G_2$).

The membership and non membership of the vertices are given as:
u_1=⟨[0.2, 0.4], [0.4, 0.6]⟩; v_1=⟨[0.3, 0.5], [0.3, 0.5]⟩; u_2=⟨[0.1, 0.4], [0.3, 0.6]⟩; v_2=⟨[0.2, 0.6], [0.1, 0.4]⟩ and edges u_1 v_1=⟨[0.2, 0.4], [0.4, 0.6]⟩; and u_2 v_2=⟨[0.1, 0.3], [0.3, 0.7]⟩.
Then the membership value of the vertices in cartesian product is calculated as follows:
(u_1, u_1) =⟨[0.1, 0.4], [0.4, 0.6]⟩; (u_1, v_2) =⟨[0.2, 0.4], [0.4, 0.6]⟩; (v_1, u_2) =⟨[0.1, 0.4], [0.3, 0.6]⟩; (v_1, v_2) =⟨[0.2, 0.5], [0.3, 0.5]⟩.

and edges of $G_1 \times G_2$ is calculated as:
(u_1u_2, v_1u_2)= ⟨[0.1, 0.4], [0.4, 0.6]⟩; (u_1u_2, u_1v_2)= ⟨[0.1, 0.3], [0.3, 0.7]⟩;

(v_1u_2, v_1v_2)= ⟨[0.1, 0.3], [0.3, 0.7]⟩; (u_1v_2, v_1v_2)= ⟨[0.2, 0.4], [0.4, 0.6]⟩;

Here $d^-{}_{G_1 \times G_2}(u_1, u_2)$ = (0.2∧ 0.1) + (0.3∧ 0.1)+ (0.2∧ 0.2)+ (0.2∧ 0.2) –{(0.4 ∨ 0.3)+(0.3 ∨ 0.3)
+ (0.3 ∨ 0.4)+ (0.1 ∨ 0.4)}
= 0.6 – 1.5= -0.9
Similarly, $d^+{}_{G_1 \times G_2}(u_1, u_2)$ = {(0.4∧ 0.3) + (0.3∧ 0.5)+ (0.4∧ 0.4)+ (0.6∧ 0.4)}-{(0.6 ∨ 0.7)+(0.5 ∨ 0.7) +(0.6 ∨ 0.6)+(0.4 ∨ 0.6)}
= 1.4 – 2.6 = -1.2

4. Some properties of Cartesian product
We say Cartesian product of graph is Super strong, if $d^-{}_{G_1 \times G_2} > 0$ and $d^+{}_{G_1 \times G_2} > 0$, product of graph is Strong, if $d^-{}_{G_1 \times G_2} \geq 0$ and $d^+{}_{G_1 \times G_2} > 0$. Similarly, we define Cartesian product of graph is very week if $d^-{}_{G_1 \times G_2} < 0$ and $d^+{}_{G_1 \times G_2} < 0$ and week if $d^-{}_{G_1 \times G_2} < 0$ and $d^+{}_{G_1 \times G_2} \leq 0$.
So, from example2 we say that it is a very week Cartesian product.

Theorem 4.1. Let $G_1 \times G_2$=(A, B) be the Cartesian product of two interval valued intuitionistic fuzzy graphs then $G_1 \times G_2$ is super strong if $min\{M_{AiL}(u_i)\} \geq \max\{N_{AiL}(u_i)\}$ and $min\{M_{AiU}(u_i)\} \geq \max\{N_{AiU}(u_i)\}$.

Proof: We know that $G_1 \times G_2$ is super strong iff $d^-{}_{G_1 \times G_2} > 0$ and $d^+{}_{G_1 \times G_2} > 0$.
For $d^-{}_{G_1 \times G_2} > 0$

$$\sum_{(u_1;u_2)(v_1;v_2)\in E} (M_{B_1L} \times M_{B_2L})\big((u_1;u_2)(v_1;v_2)\big) - \sum_{(u_1;u_2)(v_1;v_2)\in E} (N_{B_1L} \times N_{B_2L})\big((u_1;u_2)(v_1;v_2)\big) > 0$$

$$\sum_{(u_1;u_2)(v_1;v_2)\in E} (M_{B_1L} \times M_{B_2L})\big((u_1;u_2)(v_1;v_2)\big) > \sum_{(u_1;u_2)(v_1;v_2)\in E} (N_{B_1L} \times N_{B_2L})\big((u_1;u_2)(v_1;v_2)\big) \quad \text{….(1)}$$

Now
$\sum_{(u_1;u_2)(v_1;v_2)\in E}(M_{B_1L} \times M_{B_2L})\big((u_1;u_2)(v_1;v_2)\big)=$
$\sum_{u_1=v_1;\, u_2v_2\in E_2} M_{A_{1L}}(u_1) \wedge M_{B_{2L}}(u_2v_2) + \sum_{u_2=v_2;\, u_1v_1\in E_1} M_{A_{2L}}(u_2) \wedge M_{B_{1L}}(u_1v_1)$

$= \sum_{u_1=v_1;\, u_2v_2\in E_2} M_{A_{1L}}(u_1) \wedge M_{A_{2L}}(u_2) \wedge M_{A_{2L}}(v_2) + \sum_{u_2=v_2;\, u_1v_1\in E_1} M_{A_{2L}}(u_2) \wedge M_{A_{1L}}(u_1) \wedge M_{A_{1L}}(v_1)$

$= min\{M_{A_{1L}}(u_1),\ M_{A_{2L}}(u_2),\ M_{A_{2L}}(v_2)\}$+ $\min\{M_{A_{2L}}(u_2),\ M_{A_{1L}}(u_1),\ M_{A_{1L}}(v_1)\}$

= n($min\{M_{A_{iL}}(u_i)\}$)

where n= number of edges and i=1,2,3,…,n.

Similarly, we get

$$\sum_{(u_1;u_2)(v_1;v_2)\in E} \left(N_{B_1L} \times N_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right) = n(\max\{N_{AiL}(u_i)\})$$

Hence from eq.(1) we get $min\{M_{A_{iL}}(u_i)\} \geq \max\{N_{AiL}(u_i)\}$

Similarly, we prove $min\{M_{AiU}(u_i)\} \geq \max\{N_{AiU}(u_i)\}$. Hence the theorem.

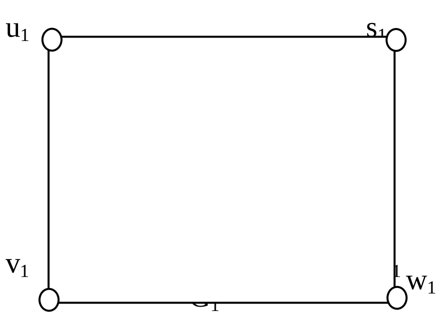
Theorem 4.2. Cartesian product of two super strong Cartesian product graph is always super strong.

Proof: Suppose G_1 and G_2 be two super strong Cartesian product graph then $d_{G1}^- > 0, d_{G1}^+ > 0$ and $d_{G2}^- > 0, d_{G2}^+ > 0$.

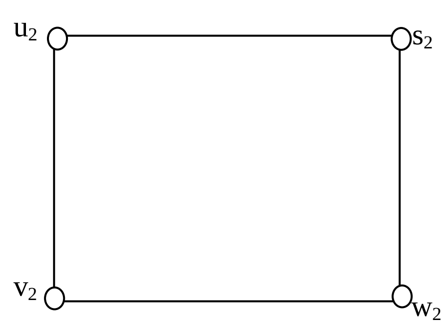
We know that, $d^-{}_{G_1\times G_2} = d_{G1}^- + d_{G2}^-$ and $d^+{}_{G_1\times G_2} = d_{G1}^+ + d_{G2}^+$ for the Cartesian product of interval valued fuzzy graph. Thus by using theorem1 we can prove that $d^-{}_{G_1\times G_2} > 0$ and $d^+{}_{G_1\times G_2} > 0$. Hence the theorem.

Corollary 4.3. Cartesian product of two very week Cartesian product graph is always very week.

Example 4.4. Consider the two super strong Cartesian product graph G_1, G_2 and $G_1 \times G_2$ in Figure 3.



G_1



G_2

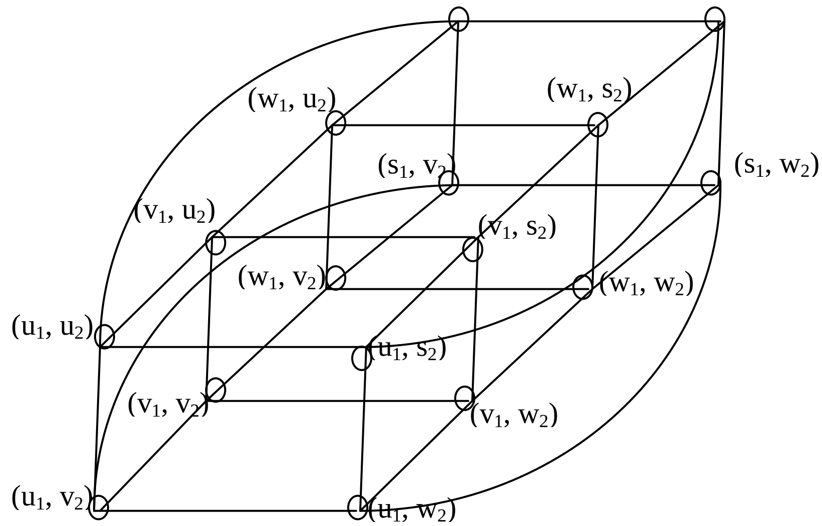


Figure 3: Cartesian product of two strong Cartesian product graph

Here suppose u_1=u_2= ⟨[0.3, 0.7], [0.1, 0.3]⟩, v_1= v_2=⟨[0.3, 0.6], [02, 0.4]⟩, w_1=w_2=⟨[0.4, 0.6], [0.2, 0.4]⟩ and s_1=s_2=⟨[0.5, 0.7], [0.1, 0.3]⟩ where u_1, v_1, w_1, $s_1 \epsilon$ G_1 and u_2, v_2, w_2, s_2 ϵ G_2.

Hence the vertex set of $G_1 \times G_2$ is given as

(u_1, u_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩, (u_1, v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (u_1, w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (u_1, s_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩, (v_1, u_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (v_1, v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (v_1, w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (v_1, s_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (w_1, u_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (w_1, v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (w_1, w_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩, (w_1, s_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩, (s_1, u_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩, (s_1, v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩, (s_1, w_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩, (s_1, u_2) =⟨[0.5, 0.7], [0.1, 0.3]⟩.

Similarly edges of $G_1 \times G_2$ are:

(u_1u_2, u_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (u_1u_2, u_1s_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩; (u_1v_2, u_1w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (u_1w_2, u_1s_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1u_2, v_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1v_2, v_1w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1u_2, v_1s_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1w_2, v_1s_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (w_1u_2, w_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (w_1v_2, w_1w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (w_1w_2, w_1s_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩; (w_1s_2, w_1u_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (s_1u_2, s_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (s_1v_2, s_1w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (s_1w_2, s_1s_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩; (s_1s_2, s_1u_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩; (u_1u_2, v_1u_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (u_1v_2, v_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (u_1w_2, v_1w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (u_1s_2, v_1s_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1u_2, w_1s_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1v_2, w_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (v_1w_2, w_1w_2) =⟨[0.3, 0.6],[0.2, 0.4]⟩; (v_1s_2, w_1s_2) =⟨[0.3, 0.6],[0.2, 0.4]⟩; (w_1u_2, s_1u_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (w_1v_2, s_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (w_1w_2, s_1w_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩; (w_1s_2, s_1s_2) =⟨[0.4, 0.6], [0.2, 0.4]⟩; (s_1u_2, u_1u_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩; (s_1v_2, u_1v_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩; (s_1s_2, u_1s_2) =⟨[0.3, 0.7], [0.1, 0.3]⟩; (s_1w_2, u_1w_2) =⟨[0.3, 0.6], [0.2, 0.4]⟩.

Now the degree of $G_1 \times G_2$ is calculated as
$d^-{}_{G_1\times G_2}(u_1,u_2)$= 32[min{0.3, 0.4, 0.5}- max{0.1, 0.2}]= 32
and $d^+{}_{G_1\times G_2} = 32[\min\{0.6,\ 0.7\} - \max\{0.3, 0.4\}]$ =64.
Hence, the degree of the product of two strong Cartesian product graph is always strong.

5.Degree of a vertex in tensor product
By the definition, for any vertex (u1, u2) ∈ $V_1 \otimes V_2$,

$$d^-{}_{G_1\otimes G_2}(u_1,u_2) = \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(M_{B_1L} \otimes\ M_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right) - \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(N_{B_1L} \otimes\ N_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right)$$

$$= \sum_{u_1v_{1\in E_1};\, u_2v_2\in E_2} M_{B_{1L}}\,(u_1v_1) \wedge\ M_{B_{2L}}\,(u_2v_2) - \left\{\sum_{u_1v_1\ \in E_1;\, u_2v_2\in E_2} N_{B_{1L}}(u_1v_1)\ \vee N_{B_{2L}}(u_2v_2)\right\}$$

6. Degree of a vertex in lexicographic product
By the definition, for any vertex (u1, u2) ∈ V1 ⋆ V2,

$$d^-{}_{G_1\star\, G_2}(u_1,u_2) = \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(M_{B_1L} \times\ M_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right) - \sum_{(u_1;u_2)(v_1;v_2)\in E} \left(N_{B_1L} \times\ N_{B_2L}\right)\left((u_1;u_2)(v_1;v_2)\right)$$

$$= \max\left\{\sum_{u_1=v_1;\, u_2v_2\in E_2} M_{A_{1L}}\,(u_1) \wedge\ M_{B_{2L}}\,(u_2v_2), \sum_{u_2=v_2;\, u_1v_1\in E_1} M_{A_{2L}}\,(u_2) \wedge\ M_{B_{1L}}\,(u_1v_1)\right\} - max\left\{\sum_{u_1=v_1;\, u_2v_2\in E_2} N_{A_{1L}}\,(u_1)\ \vee N_{B_{2L}}(u_2v_2), \sum_{u_2=v_2;\, u_1v_1\in E_1} N_{A_{2L}}\,(u_2) \vee N_{B_{1L}}(u_1v_1)\right\}$$

7. Conclusion

In this paper we find the degree and classify it for Cartesian product of two interval valued intuitionistic fuzzy graphs. We can also extend it to other product like Strong product, tensor product, lexicographic product, etc. We may implement this concept to find the strength of the product of two algorithms which is also useful to solve the problem containing combinatorics. It is useful to the areas including geometry, algebra, number theory, topology, operations research, and computer science.

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