Annals of Pure and Applied Mathematics
Vol. 6, No. 1, 2014, 25-35
ISSN: 2279-087X (P), 2279-0888(online)
Published on 5 May 2014
www.researchmathsci.org

Annals of
Pure and Applied
Mathematics

Adjacent Edge Graceful Graphs

***T. Tharmaraj*[1] and *P.B.Sarasija*[2]**

[1]Department of Mathematics, Udaya School of Engineering
Vellamodi, Tamil Nadu, India. E-mail: trajtr@gmail.com

[2]Department of Mathematics, Noorul Islam University
Kumaracoil, Tamil Nadu, India. E-mail: sijavk@gmail.com

Received 6 April 2014; accepted 20 April 2014

Abstract. Let $G(V,E)$ be a graph with p vertices and q edges. A (p,q) graph $G(V,E)$ is said to be an adjacent edge graceful graph if there exists a bijection $f: E(G) \to \{1,2,3,\ldots\ldots, q\}$ such that the induced mapping f^* from $V(G)$ by $f^*(u) = \sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u is an injection. The function f is called an adjacent edge graceful labeling of G.In this paper, we prove path $P_n (n \geq 5)$, cycle $C_n (n \geq 5)$, fan $f_n (n \geq 3)$, Helm $H_n (n \geq 3)$, triangle snake $T_n (n \geq 3)$ and alternate triangle snake $A(T_n)(n = 4,6,8,\ldots)$ are the adjacent edge graceful graphs and we also prove n-bistar $B_{n,n}$, complete bipartite graph $K_{m,n}$, star graph $K_{1,n}$ and graph g_n are not the adjacent edge graceful graphs.

Keywords: adjacent edge graceful graph**,** adjacent edge graceful labeling

AMS Mathematical Subject Classification (2010): 05C78

1. Introduction

All graphs in this paper are finite, simple and undirected graphs. Let (p,q) be a graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges. Graph labeling, where the vertices and edges are assigned real values or subsets of a set are subject to certain conditions. A detailed survey of graph labeling can be found in [1].Terms not defined here are used in the sense of Harary in [3].The concept of edge graceful labeling was first introduced in [2] and the concept of strong edge graceful labeling was introduced in [4]. Some results on strong edge graceful labeling of graphs are discussed in [4]. In this paper, we introduced a new edge graceful labeling. We use the following definitions in the subsequent sections.

Definition 1.1. A (p,q) graph $G(V,E)$ is said to be an adjacent edge graceful graph if there exists a bijection $f : E(G) \to \{1,2,3,\ldots\ldots, q\}$ such that the induced mapping f^*

from $V(G)$ by $f^*(u) = \sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u is an injection. The function f is called an adjacent edge graceful labeling of G.

Definition 1.2.[1] The bistar graph $B_{n,n}$ is the graph obtained from two copies of star $K_{1,n}$ by joining the vertices of maximum degree by an edge.

Definition 1.3.[1] The helm H_n is the graph obtained from a wheel by attaching a pendent edge at each vertex of the n-cycle.

Definition 1.4.[1] The fan $f_n(n \geq 2)$ is obtained by joining all nodes of P_n to a further node called the center and contains $n+1$ nodes and $2n-1$ edges.

Definition 1.5.[1] A graph $g_n(n \geq 2)$ be a graph with $n+2$ nodes and $3n-1$ edges obtained by joining all nodes of P_n to two additional nodes.

Definition 1.6.[1] A triangular snake T_n is obtained from a path $u_1 u_2 ......u_n$ by joining u_i and u_{i+1} to a new vertex v_i for i=1,2,….,n-1.

Definition 1.7.[5] An alternate triangular snake $A(T_n)$ is obtained from a path $u_1 u_2 ......u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i.

2. Main results

Theorem 2.1. The triangle snake $T_n(n \geq 3)$ admits an adjacent edge graceful labeling.

Proof: Let $V(T_n) = \{u_i : 1 \leq i \leq n \; ; \; v_i : 1 \leq i \leq n-1\}$

Let $E(T_n) = \begin{cases} e_i = u_i u_{i+1} & if \;\; 1 \leq i \leq n-1 \\ e_{n-1+i} = u_i v_i & if \;\; 1 \leq i \leq n-1 \\ e_{2n-1+i} = u_{i+1} v_i & if \;\; 1 \leq i \leq n-1 \end{cases}$

Define a bijection $f : E(T_n) \to \{1,2,3,....,3n-3\}$ by

$f(e_i) = i \;\; if \;\; 1 \leq i \leq n-1$; $f(e_{n-1+i}) = n + 2(i-1) \;\; if \;\; 1 \leq i \leq n-1$;

$f(e_{2n-1+i}) = 2n - 2 + 2i \;\; if \;\; 1 \leq i \leq n-1$.

Let f^* be the induced vertex labeling of f.

The induced vertex labels are as follows:

If $n = 3$, $f^*(u_1) = 6n+1$; $f^*(u_2) = 10n$; $f^*(u_3) = 7n+2$;

$f^*(v_1) = 5n+1$; $f^*(v_2) = 6n+2$.

If $n = 4$, $f^*(u_1) = 5n+3$; $f^*(u_2) = 11n+3$; $f^*(u_3) = 14n$;

$f*(u_4)=9n+1$; $f*(v_1)=4n+3$; $f*(v_2)=8n+2$; $f*(v_3)=85n$.

If $n \geq 5$, $f*(u_1)=4n+7$; $f*(u_2)=7n+19$;

$f*(u_{n-1})=24n-40$; $f*(u_n)=14n-19$; $f*(v_1)=3n+7$;

$f*(v_{n-1})=12n-16$; $f*(u_{i+2})=8n+18+20i$ *if* $1 \leq i \leq n-4$;

$f*(v_{i+1})=4n+6+12i$ *if* $1 \leq i \leq n-3$.

Example 2.2. An adjacent edge graceful labeling of a triangular snake T_8 is shown in Fig.1.

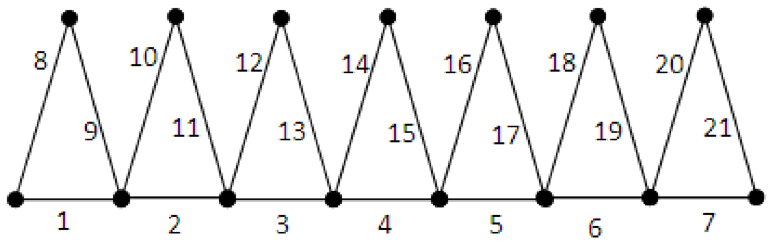

Figure 1:

Theorem 2.3. The alternate triangular snake $A(T_n)(n = 4,6,8,...)$ is an adjacent edge graceful graph.

Proof: Case (i) If the triangle starts from u_1.

Let $V(A(T_n)) = \{u_i : 1 \leq i \leq n \ ; \ v_i : 1 \leq i \leq \frac{n}{2}\}$.

Let $$E(A(T_n)) = \begin{cases} e_i = u_i u_{i+1} & \text{if } 1 \leq i \leq n-1 \\ e_{n-1+i} = u_{2i-1} v_i & \text{if } 1 \leq i \leq \frac{n}{2} \\ e_{\frac{3n-2+2i}{2}} = u_{2i} v_i & \text{if } 1 \leq i \leq \frac{n}{2} \end{cases}$$

Define a bijection $f : E(A(T_n)) \to \{1,2,3,......,2n-1\}$ by

$f(e_i) = 2i$ *if* $1 \leq i \leq n-1$; $f(e_{n-1+i}) = 4i-3$ *if* $1 \leq i \leq \frac{n}{2}$;

$f(e_{\frac{3n-2+2i}{2}}) = 4i-1$ *if* $1 \leq i \leq \frac{n}{2}$.

Let $f*$ be the induced vertex labeling of f .In this case the induced vertex label are as follows:

If $n = 4$, $f*(u_1)=3n+1$; $f*(u_2)=5n+2$; $f*(u_3)=8n+2$;

$f*(u_4)= 6n+3$; $f*(v_1)=3n$; $f*(v_2)=7n$.

If $n \geq 6$, $f*(u_1)=13$; $f*(u_2)=22$; $f*(u_{2i+1})=32i+10$ *if* $1 \leq i \leq \left(\frac{n-4}{2}\right)$;

$f*(u_{n-1})=14n-22$; $f*(u_n)=10n-13$; $f*(u_{2i+2})=32i+22$ *if* $1 \leq i \leq \left(\frac{n-4}{2}\right)$;

$f*(v_1)=12$; $f*(v_{i+1})=24i+12$ *if* $1 \leq i \leq \left(\frac{n-4}{2}\right)$; $f*(v_{\frac{n}{2}})=10n-12$.

Case (ii) If the triangle starts from u_2

Let $V(A(T_n)) = \{u_i : 1 \leq i \leq n \ ; \ v_i : 1 \leq i \leq \frac{n}{2}\}$.

Let $$E(A(T_n)) = \begin{cases} e_i = u_i u_{i+1} & \text{if } 1 \leq i \leq n-1 \\ e_{n-1+i} = u_{2i} v_i & \text{if } 1 \leq i \leq \left(\frac{n-2}{2}\right) \\ e_{\frac{3n-2+2i}{2}} = u_{2i+1} v_i & \text{if } 1 \leq i \leq \left(\frac{n-2}{2}\right) \end{cases}$$

Define a bijection $f : E(A(T_n)) \rightarrow \{1,2,3,......,2n-3\}$ by

$f(e_i) = 2i-1$ *if* $1 \leq i \leq n-1$; $f(e_{n-1+i}) = 4i-2$ *if* $1 \leq i \leq \left(\frac{n-2}{2}\right)$;

$f(e_{\frac{3n-4+2i}{2}}) = 4i$ *if* $1 \leq i \leq \left(\frac{n-2}{2}\right)$.

In this case the induced vertex label are as follows:

If $n = 4$, $f*(u_1)=n+2$; $f*(u_2)=4n+3$; $f*(u_3)=4n+1$; $f*(u_4)=3n$;

$f*(v_1)=4n+2$.

If $n \geq 5$, $f*(u_1)=6$; $f*(u_2)=19$; $f*(v_i)=24i-6$ *if* $1 \leq i \leq \left(\frac{n-2}{2}\right)$;

$f*(u_n)=6n-12$; $f*(u_{2i+1})=32i-2$ *if* $1 \leq i \leq \left(\frac{n-4}{2}\right)$;

$f*(u_{2i+2})=32i+18$ *if* $1 \leq i \leq \left(\frac{n-4}{2}\right)$; $f*(u_{n-1})=12n-31$.

Example 2.4. An adjacent edge graceful labeling of an alternate triangular snake $A(T_8)$ are shown in Fig. 2 and Fig. 3 respectively.

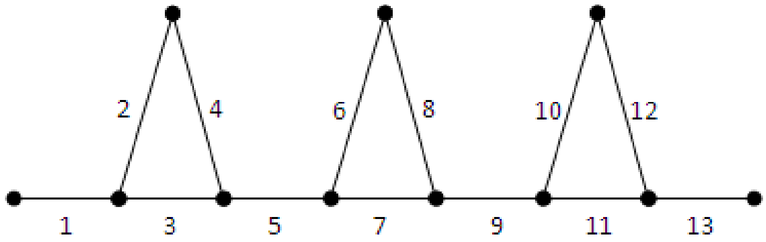

Figure 2:

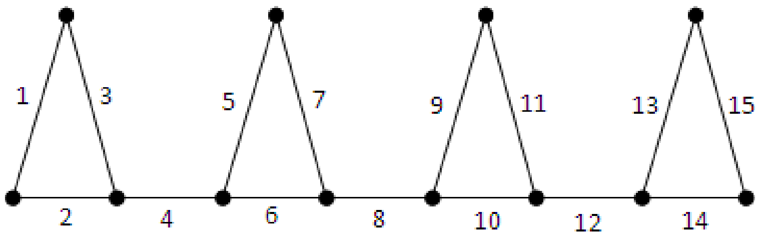

Figure 3:

Theorem 2.5. The Path $P_n (n \geq 5)$ is an adjacent edge graceful graph.

Proof: Let $V(P_n) = \{v_i : 1 \leq i \leq n\}$.Let $E(P_n) = \{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$.

Define $f : E(P_n) \to \{1,2,3,......,n-1\}$ by

Case (i) n is odd , $f(e_i) = i \quad if \quad 1 \leq i \leq n-1$.

Case (ii) n is even, $f(e_i) = 2i-1 \quad if \quad 1 \leq i \leq \frac{n}{2}$;

$f(e_{\frac{n+2i}{2}}) = n-2i \quad if \quad 1 \leq i \leq \frac{n-2}{2}$.

Let f^* be the induced vertex labeling of f .The induced vertex labels are as follows:

If n is odd , $f^*(v_1) = 3$; $f^*(v_2) = 6$; $f^*(v_{n-1}) = 3(n-2)$;

$f^*(v_n) = 2n-3$; $f^*(v_{i+2}) = 4i+6 \quad if \quad 1 \leq i \leq n-4$.

If $n = 6$, $f^*(v_1) = n-2$; $f^*(v_2) = n+3$; $f^*(v_3) = 2n+1$;

$f^*(v_4) = 2n$; $f^*(v_5) = 2n-5$; $f^*(v_6) = 6$.

If $n = 8$, $f^*(v_1) = n-4$; $f^*(v_2) = n+1$; $f^*(v_3) = 2n$; $f^*(v_4) = 2n+5$;

$f^*(v_5) = 3n$; $f^*(v_6) = 2n+3$; $f^*(v_7) = 2n+2$; $f^*(v_8) = n-2$.

If $n \geq 10$, $f^*(v_1) = 4$; $f^*(v_2) = 9$; $f^*(v_n) = 6$; $f^*(v_{n-1}) = 12$;

$f^*(v_{i+2}) = 8(i+1)$ *if* $1 \le i \le \frac{n-6}{2}$; $f^*(v_{\frac{n}{2}}) = 4n-11$; $f^*(v_{\frac{n+2}{2}}) = 4n-10$;

$f^*(v_{\frac{n+4}{2}}) = 4n-13$; $f^*(v_{\frac{n+4+2i}{2}}) = 4n-12-8i$ *if* $1 \le i \le \frac{n-8}{2}$.

Example 2.6. The adjacent edge graceful labelings of P_9 and P_{10} are shown in the Fig. 4 and Fig. 5 respectively.

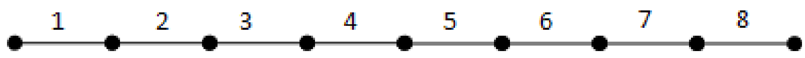

Figure 4:

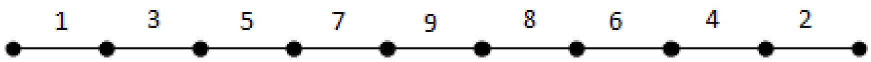

Figure 5:

Theorem 2.7. The cycle is an adjacent edge graceful graph.

Proof: Let $V(C_n) = \{v_i : 1 \le i \le n\}$. Let

$E(C_n) = \{e_i = v_i v_{i+1} : 1 \le i \le n-1 \; ; e_n = v_1 v_n\}$.

Define $f : E(C_n) \to \{1,2,3,......,n\}$ by

Case (i) n is odd , $f(e_i) = i$ *if* $1 \le i \le n$.

Case (ii) n is even , $f(e_i) = 2i-1$ *if* $1 \le i \le \frac{n}{2}$;

$f(e_{\frac{n+2i}{2}}) = n-2(i-1)$ *if* $1 \le i \le \frac{n}{2}$.

Let f^* be the induced vertex labeling of f .The induced vertex labels are as follows:

If n is odd , $f^*(v_1) = 2n+2$; $f^*(v_2) = n+6$;

$f^*(v_n) = 3n-2$; $f^*(v_{i+2}) = 4i+6$ *if* $1 \le i \le n-3$;

If $n = 6$, $f^*(v_1) = n+4$; $f^*(v_2) = n+5$; $f^*(v_3) = 2n+3$;

$f^*(v_4) = 3n$; $f^*(v_5) = 2n+5$; $f^*(v_6) = 2n+1$.

If $n \ge 8$, $f^*(v_1) = 10$; $f^*(v_2) = 11$; $f^*(v_{i+2}) = 8(i+1)$ *if* $1 \le i \le \frac{n-6}{2}$;

$f^*(v_{\frac{n}{2}}) = 4n-9$; $f^*(v_{\frac{n+2}{2}}) = 4n-6$; $f^*(v_{\frac{n+4}{2}}) = 4n-7$;

$f^*(v_{n-i}) = 4(2i+3)$ *if* $1 \le i \le \frac{n-6}{2}$; $f^*(v_n) = 13$.

Example 2.8. The adjacent edge graceful labelings of C_{14} and C_{11} are shown in the Fig. 5 and Fig. 6 respectively.

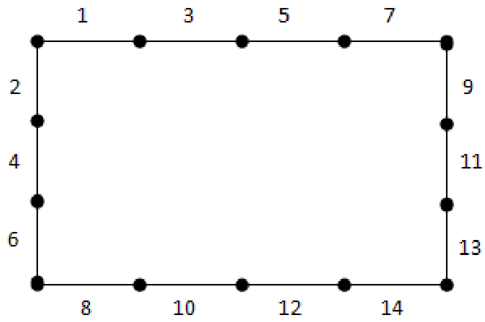

Figure 5:

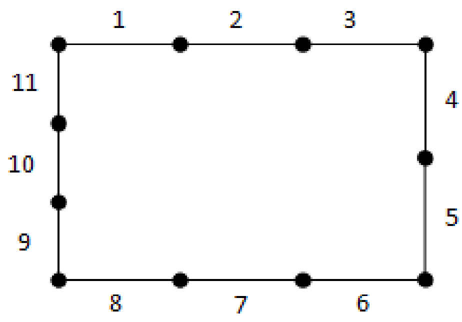

Figure 6:

Theorem 2.9. The graph g_n is not an adjacent edge graceful graph.

Proof: Let $V(g_n)=\{v_i : 1\le i\le n\ ;\ v_{n+1}\ ;\ v_{n+2}\}$.

$$\text{Let } E(g_n)=\begin{cases} e_i = v_i v_{i+1} \ \ if \ \ 1\le i\le n-1 \\ e_{n-1+i} = v_i v_{n+1} \ \ if \ \ 1\le i\le n \\ e_{2n-1+i} = v_i v_{n+2} \ \ if \ \ 1\le i\le n \end{cases}$$

Let f be a bijection from $E(g_n)$ to $\{1,2,3,.......,3n-1\}$.

Let $l_1, l_2, ..., l_n, l_{n+1}, ..., l_{3n-1}$ be the label of edges $e_1, e_2, ..., e_n, e_{n+1}, ..., e_{3n-1}$ by f respectively. And f induces that $f^*: V(g_n)\rightarrow\{1,2,3,......\}$ by $f^*(u)=\sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u.

The vertices v_{n+1} and v_{n+2} have the same adjacent vertices $\{v_1, v_2, ..., v_n\}$.

Then $f^*(v_{n+1}) = 2(l_1 + l_2 + ... + l_{n-1}) + (l_n + l_{n+1} + ... + l_{3n-1})$

$$= \sum_{i=1}^{n-1} l_i + \sum_{i=1}^{3n-1} l_i = \sum_{i=1}^{n-1} l_i + \sum_{i=1}^{3n-1} i = m + \frac{3n(3n-1)}{2} \text{ , where } m = \sum_{i=1}^{n-1} l_i$$

Since v_{n+1} and v_{n+2} have the same adjacent vertices,

$f^*(v_{n+2}) = m + \dfrac{3n(3n-1)}{2}$. That is f^* is not injective. Hence g_n is not an adjacent edge graceful graph.

Theorem 2.10. The graph n-bistar $B_{n,n}$ is not an adjacent edge graceful graph.

Proof: Let $V(B_{n,n})=\{v_i : 1\le i\le n+1\ ;\ u_i : 1\le i\le n+1\}$.

Let $E(B_{n,n}) = \begin{cases} e_i = u_i u_{n+1} & if \ \ 1 \le i \le n \\ e_{n+i} = v_i v_{n+1} & if \ \ 1 \le i \le n \\ e_{2n+1} = u_{n+1} v_{n+1} \end{cases}$

Let f be a bijection from $E(B_{n,n})$ to $\{1,2,3,.......,2n+1\}$.

And f induces that $f^*: V(B_{n,n}) \rightarrow \{1,2,3,......\}$ by $f^*(u) = \sum_i f(e_i)$

over all edges e_i incident to adjacent vertices of u.

Step 1:

The adjacent vertices of u_{n+1} are $u_1, u_2, ..., u_n, v_{n+1}$.

$$f^*(u_{n+1}) = \sum_{i=1}^{n} f(e_i) + \sum_{i=n+1}^{2n+1} f(e_i) = \sum_{i=1}^{2n+1} f(e_i)$$

$$= 1 + 2 + 3 + ... + (2n+1) = (n+1)(2n+1).$$

Step 2:

The adjacent vertices of v_{n+1} are $v_1, v_2, ..., v_n, u_{n+1}$.

$$f^*(v_{n+1}) = \sum_{i=1}^{n} f(e_i) + \sum_{i=n+1}^{2n} f(e_i) + f(e_{2n+1}) = \sum_{i=1}^{2n+1} f(e_i)$$

$$= 1 + 2 + 3 + ... + (2n+1) = (n+1)(2n+1).$$

From step(1) and step(2), the vertex labels of u_{n+1} and v_{n+1} are same.

That is f^* is not injective.Hence $B_{n,n}$ is not an adjacent edge graceful graph.

Theorem 2.11. The Helm $H_n (n \ge 3)$ is an adjacent edge graceful graph.

Proof: Let $V(H_n) = \{u_i : 1 \le i \le n+1 ; \ v_i : 1 \le i \le n\}$

Let $E(H_n) = \begin{cases} e_i = u_i u_{i+1} & if \ \ 1 \le i \le n-1 \\ e_n = u_1 u_n \\ e_{n+i} = v_i v_i & if \ \ 1 \le i \le n \\ e_{2n+i} = u_i u_{n+1} & if \ \ 1 \le i \le n \end{cases}$

Define a bijection $f : E(H_n) \rightarrow \{1,2,3,......,3n\}$ by $f(e_i) = 2i \ \ if \ \ 1 \le i \le n$;

$f(e_{n+i}) = 2i - 1 \ \ if \ \ 1 \le i \le n$; $f(e_{2n+i}) = 2n + i \ \ if \ \ 1 \le i \le n$.

Let f^* be the induced vertex labeling of f.

The induced vertex label are as follows:

If $n = 3$, $f^*(u_1) = 22n$; $f^*(u_2) = 22n + 1$; $f^*(u_3) = 20n + 2$;

$f^*(u_4) = 19n$; $f^*(v_1) = 5n + 1$; $f^*(v_2) = 5n + 2$; $f^*(v_3) = 8n$.

If $n \ge 4$, $f^*(u_1) = \frac{5n^2 + 23n + 18}{2}$; $f^*(u_2) = \frac{5n^2 + 13n + 50}{2}$;

$f^*(u_n) = \frac{5n^2+31n-14}{2}$; $f^*(u_{n+1}) = \frac{11n^2+5n}{2}$

; $f^*(u_{i+2}) = \frac{5n^2+9n+50+32i}{2}$ $\quad if \quad 1 \le i \le n-3$;

$f^*(v_1) = 4n+4$; $f^*(v_{i+1}) = 2n+4+7i \quad if \quad 1 \le i \le n-1$.

Theorem 2.12. Complete bipartite graph $K_{m,n}$ is not an adjacent edge graceful graph.

Proof: Let $V(K_{m,n}) = \{u_i : 1 \le i \le m \;\; ; v_i : 1 \le i \le n\}$.

Let $E(K_{m,n}) = \{e_{i_j} = u_i v_j \;\; : 1 \le i \le m, 1 \le i \le n\}$.

Define a bijection f from $E(K_{m,n})$ to $\{1,2,3,......., n\}$.

And f induces that $f^* : V(K_{m,n}) \to \{1,2,3,......\}$ by $f^*(u) = \sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u ..

Case (i) $m < n$

$$f^*(v_i) = \sum_{i=1}^{m}\sum_{j=1}^{n} f(e_{i_j}) = 1+2+3+.........+mn = \frac{mn(mn+1)}{2} \text{ for } 1 \le i \le m .$$

The label of each vertex v_i is $\frac{mn(mn+1)}{2}$ for $1 \le i \le m$.

Case (ii) $m = n$

$$f^*(u_i) = f^*(v_i) = \sum_{i=1}^{n}\sum_{j=1}^{n} f(e_{i_j}) = 1+2+.........+n^2 = \frac{n^2(n^2+1)}{2} \text{ for } 1 \le i \le n .$$

The label of each vertex u_i and v_i are $\frac{n^2(n^2+1)}{2}$ for $1 \le i \le n$.

Case (iii) $m > n$

$$f^*(u_i) = \sum_{i=1}^{m}\sum_{j=1}^{n} f(e_{i_j}) = 1+2+3+.........+mn = \frac{mn(mn+1)}{2} \text{ for } 1 \le i \le n .$$

The label of each vertex u_i is $\frac{mn(mn+1)}{2}$ for $1 \le i \le n$.

In the above three cases , f^* is not injective.

Hence the complete bipartite graph $K_{m,n}$ is not an adjacent edge graceful graph.

Corollary 2.13. The Star graph $K_{1,n}$ is not an adjacent edge graceful graph.

Proof: By putting $m = 1$ in the result of the above theorem 2.12 , we can get the result of this corollary.

Theorem 2.14. Every fan $f_n (n \ge 4)$ is an adjacent edge graceful graph.

Proof: Let $V(f_n) = \{v_i : 1 \le i \le n+1\}$.

Let $E(f_n) = \{e_i = v_i v_{i+1} : 1 \le i \le n-1 ; e_{n-1+i} = v_i v_{n+1} : 1 \le i \le n\}$.

Define a bijection $f : E(f_n) \to \{1,2,3,......,2n-1\}$ by

Case (i) n is odd , $f(e_i) = i \quad if \quad 1 \le i \le n-1$ and

$f(e_{n-1+i}) = n-1+i \quad if \quad 1 \le i \le n$.

Case (ii) n is even , $f(e_i) = 2i-1 \quad if \quad 1 \le i \le n-1$;

$f(e_{n-1+i}) = 2i \quad if \quad 1 \le i \le n-1$; $f(e_{2n-1}) = 2n-1$.

Let f^* be the induced vertex labeling of f.

The induced vertex labels are as follows:

If n is odd , $f^*(v_1) = \frac{3n^2+n+8}{2}$; $f^*(v_{n-1}) = \frac{3n^2+13n-20}{2}$;

$f^*(v_n) = \frac{3n^2+7n-10}{2}$; $f^*(v_{n+1}) = \frac{n(5n-3)}{2}$;

$f^*(v_{i+1}) = \frac{3n^2+3n+16}{2} + 6(i-1) \quad if \quad 1 \le i \le n-3$.

If $n = 4$, $f^*(v_1) = 6n+3$; $f^*(v_2) = 9n$; $f^*(v_3) = 9n+3$;

$f^*(v_4) = 8n+1$; $f^*(v_5) = 9n+1$.

If $n \ge 6$, $f^*(v_1) = n^2+n+7$; $f^*(v_2) = n^2+n+16$;

$f^*(v_{n-1}) = n^2+11n-21$; $f^*(v_n) = n^2+7n-11$;

$f^*(v_{n+1}) = 3n^2-3n+1$; $f^*(v_{i+2}) = n^2+n+15+12i \quad if \quad 1 \le i \le n-4$.

REFERENCES

1. J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 5(2002) # DS6, 1-144.
2. B.Gayathri and M.Subbiah, Strong edge graceful labelings of some graphs, *Bull. Pure Appl. Sci*., 27E (2008) 89-98.
3. H.Harary, *Graph Theory*, Addison-Wesley, Reading, Massachusetts, 1972.
4. S.Lo, On edge-graceful labelings of graphs, *Congr. Numer*., 50 (1985) 231-241.
5. S.S.Sandhya, S.Somasundaram and R.Ponraj, Harmonic mean labeling of some cycle related graphs, *Int. Journal of Math. Analysis*, 6 (40) (2012) 1997-2005.
6. Ali Ahmad, E.T.Baskoro and M.Imran, Total vertex irregularity strength of disjoint union of Helm graphs, *Discussiones Mathematicae Graph Theory*, 32 (2012) 427–434.
7. S. K. Vaidya and N. H. Shah , Some star and bistar related divisor cordial graphs, A*nnals of Pure and Applied Mathematics*, 3(1) (2013) 67-77.
8. L.W.Beineke and S.M.Hegde, Strongly multiplicative graphs, *Discuss. Math. Graph Theory*, 21 (2001) 63-75.

9. R. L. Graham and N. J. A. Sloane, On additive bases and harmonious graphs, *SIAM J. Alg. Disc. Meth.*, 1 (4) (1980) 382-404.
10. Z. Liang and Z. Bai, On the odd harmonious graphs with applications, *J. Appl. Math. Comput.*, 29 (2009) 105-116.
11. V. Yegnanaryanan and P. Vaidhyanathan, Some interesting applications of graph labellings, *J. Math. Comput. Sci.*, 2 (5) (2012) 1522-1531.
12. I. Cahit, Cordial Graphs, A weaker version of graceful and harmonious Graphs, *Ars Combinatoria*, 23 (1987) 201-207.
13. M. Andar, S. Boxwala and N. B. Limaye, Cordial labelings of some wheel related graphs, *J. Combin. Math. Combin. Comput*., 41 (2002) 203-208.
14. M. Andar, S. Boxwala and N. B. Limaye, A note on cordial labeling of multiple shells, *Trends Math*., (2002) 77-80.
15. S. K. Vaidya and N. A. Dani, Some new star related graphs and their cordial as well as 3-equitable labeling, *Journal of Sciences*, 1 (1) (2010) 111-114.