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r*g*-Closed Sets in Topological Spaces

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Abstract. The aim of this paper is to introduce a new class of sets called r*g*− closed sets and investigate some of the basic properties of this class of sets which is obtained by generalizing regular closed sets via g-open sets.

Keywords: rcl, rint, r*g*-closed sets, r*g*-open sets,r*g*int, r*g*cl.

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1. Introduction

Levine introduced generalized closed sets in topology. b- open sets have been introduced and investigated by Andrijevic. Veerakumar introduced g* closed sets and studied its properties. The aim of this paper is to introduce r*g* closed sets and investigate some fundamental properties and the relations with the predefined sets like g* closed, regular closed , α g-closed, , g* Ψ -closed, gsp-closed ,gs-closed etc.

2. Preliminaries

Definition 2.1. A subset A of a space (X, τ) is called

(1) a generalized closed (briefly g-closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$U and U is open in (X,τ); the complement of a g-closed set is called a g-open set.

(2) a generalized semi-closed (briefly gs- closed) set if scl(A)$\subseteq$U whenever A $\subseteq$ U and U is open in (X, τ). ; the complement of a gs-closed set is called a gs-open set

(3) a semi-generalized closed (briefly sg- closed) set if scl(A)$\subseteq$U and A $\subseteq$ U and U is semi-open in (X, τ); **t**he complement of a sg-closed set A is called a sg-open set.

(4) a Ψ -closed set if scl(A)$\subseteq$U whenever A$\subseteq$U and U is sg-open in (X, τ). ; the complement of a Ψ -closed set is called a Ψ -open set

(5) a α-generalized closed set (briefly α g- closed) if α cl(A)$\subseteq$U whenever A $\subseteq$ U and U is open in (X, τ); the complement of an α g-closed set is called a α g-open set.

(6) a generalized α − closed set (briefly gα - closed) if α cl(A)$\subseteq$U whenever A $\subseteq$ U and U is α - open in (X, τ). ; the complement of a gα -closed set is called a gα - open set.

(7) a generalized pre-closed set (briefly gp- closed) if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). ; the complement of a gp-closed set is called a gp-open set.

(8) a generalized semi- pre closed set (briefly gsp- closed) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). ; the complement of a gsp closed set is called a gsp-open set.

(9) a g*-closed set if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g open in (X, τ); the complement of a g*-closed set is called a g*-open set.

(10) a gp*-closed set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is gp-open in (X, τ); the complement of a gp*-closed set is called a gp*-open set.

Definition 2.2. rcl (A) is defined as the intersection of all regular closed sets containing A.

3. r* g*-closed sets and their properties

Definition 3.1. A subset A of a space (X,τ) is called a r* g*-closed set if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open.

Example 3.2. Let X = {a, b, c} and τ = {φ, X, {a}, {b}, {a,b}}.
Regular open sets = { φ, X {a}, {b}, }, Regular closed sets are {φ, X, {a,c}, {b,c}}
g-closed sets = {φ, X, {c},{b,c}, {a,c}}, g-open sets = {φ, X, {a,b}, {a}, {b}}. Then r*g*-closed sets are {φ, X, {c}, {b,c}, {a,c}}.

Theorem 3.3. If A is regular closed then A is r*g*-closed.
Proof: Let A $\subseteq$ U where U is g-open. TPT: A is r*g*-closed.
Since A is Regular closed, rcl (A) = A $\subseteq$ U.
⇨ A is r*g*closed.

Note: Every open set is g-open. (1)

Theorem 3.4. If A is r*g*-closed then A is g-closed.
Proof: Let A $\subseteq$ U where U is open.
Now cl (A) $\subseteq$ rcl (A) $\subseteq$ U Where U is g-open.
∴ cl (A) $\subseteq$ U which implies that, A is g-closed.

Theorem 3.5. If A is r*g* -closed then A is g*-closed. The converse is true whenever A is regular closed.
Proof: Let A $\subseteq$ U where U is g-open.
To prove that, A is g*-closed.
Now cl(A) $\subseteq$ rcl(A) $\subseteq$ U.
Hence cl(A) $\subseteq$ U which implies that A is g*-closed.
Conversely, suppose A is regular closed. By theorem 3.3, A is r*g*-closed.

Theorem 3.6. If A is r*g* -closed then A is g* Ψ -closed.

Proof: Let A $\subseteq$ U where U is g-open. Then rcl (A) $\subseteq$ U where U is g-open

Ψ Cl (A) $\subseteq$ rcl (A) $\subseteq$ U

$\therefore$ Ψ Cl (A) $\subseteq$ U

$\Rightarrow$ A is g* Ψ -closed.

Theorem 3.7. If A is r*g* -closed then A is gp-closed.
Proof: Let A $\subseteq$ U where U is open.
Now pcl (A) $\subseteq$ cl (A) $\subseteq$ rcl (A) $\subseteq$ U

$\therefore$ pcl (A) $\subseteq$ U where U is open.

$\therefore$ A is gp-closed.

Theorem 3.8. If A is r*g*-closed then A is gsp-closed.
Proof: Let A $\subseteq$ U where U is open. TPT: A is gsp – closed.
Now spcl (A) $\subseteq$ rcl (A) $\subseteq$ U where U is g-open.

$\Rightarrow$ spcl (A) $\subseteq$ U where U is open By(1)

$\therefore$ A is gsp-closed.

Theorem 3.9. If A is r*g*-closed then A is gs-closed.
Proof: Let A $\subseteq$ U where U is open.
Now scl (A) $\subseteq$ cl (A) $\subseteq$ rcl(A) $\subseteq$ U

$\therefore$ scl(A) $\subseteq$ U where U is open.

$\therefore$ A is gs-closed.

Note: Regular open $\Rightarrow$ g-open.

Theorem 3.10. If A is r*g*-closed then A is rg-closed.
Proof: Let A be r*g*-closed. Then rcl (A) $\subseteq$ U where U is g-open.
Let A $\subseteq$ U where U is regular open.
But cl (A) $\subseteq$ rcl (A) $\subseteq$ U where U is regular open
$\Rightarrow$ A is rg-closed.

Theorem 3.11. If A is r*g* closed then A is α g-closed.
Proof: Let A $\subseteq$ U where U is open.
Now α cl(A) $\subseteq$ cl(A) $\subseteq$ rcl(A) $\subseteq$ U

$\therefore$ α cl(A) $\subseteq$ U where U is g-open by(1)

$\therefore$ A is α g-closed.

Theorem 3.12. If A is r*g*-closed then A is gp*-closed.
Proof: Follows from the fact that pcl(A) $\subseteq$ cl(A).

The converse of the above theorems need not be true as seen from the following examples.

Example 3.13. X = {a, b, c} and τ = {φ, X, {a}, {a,c}}. Here {b} is g-closed but not r*g*-closed.

Example 3.14. Let X = {a, b, c} and τ = {φ,X,{a},{b},{a,b}}. Here {a} is g* Ψ -closed but not r*g*-closed.

Example 3.15. Let X = {a, b, c, d} and τ = {φ, X, {a}, {b}, {a,b}, {a,b,c}}. Here {c} is gp-closed but not r*g*- closed.

Example 3.16. Let X = {a, b, c} and τ = {φ, X, {a}, {a,b}}. Here A = {b} is gsp -closed but not r*g*-closed.

Example 3.17. Let X = {a, b, c} and τ = {φ, X, {a}, {a,c}}. Here {b} is gs-closed but not r*g* -closed.

Example 3.18. Let X = {a, b, c} and τ = {φ, X, {a}, {b}, {a,b}}.
Let A = {a,b}. Here {a,b} is rg-closed but not r*g* -closed.

Example 3.19. Let X = {a, b, c} and τ = {φ, X, {b}, {a,b}}. Here {a} is α g- closed but not r*g*-closed.

Theorem 3.20. Let A and B be closed subsets of (X, τ), then
(i) If A is r* g*****-closed then rcl(A)–A does not contain any nonempty g-closed set.
(ii) If A is r* g***** closed and A $\subseteq$ B $\subseteq$ rcl(A) Then B is r*g***** closed.

Theorem 3.21. If A is g -open and r*g*****-closed then A is regular closed.

Proof: Since A is r*g*-closed rcl(A) $\subseteq$ U where A $\subseteq$ U, U is g –open.
Taking U = A, rcl (A) $\subseteq$ A.
But A $\subseteq$ rcl (A)
$\Rightarrow$ A = rcl (A)
Ais regular closed.

Theorem 3.22. If A is regular open and r*g***** -closed then A is regular closed and regular open.
Proof: Since A is regular open it is g –open. By the above theorem A is regular closed. Hence A is both regular open and regular closed.

Theorem 3.23. If A is r*g*****-closed and B is regular closed then A $\cap$ B is r*****g*****-closed.
Proof: Since A is r*g*****-closed rcl(A) $\subseteq$U, whenever A $\subseteq$ U ,U g – open. Let B be such that A$\cap$B $\subseteq$ U where U is g –open.
Now rcl (A$\cap$B) $\subseteq$rcl(A) $\cap$ rcl(B) $\subseteq$U$\bigcap$B $\subseteq$U

$\Rightarrow A \cap B$ is r*g*- closed.

Theorem 3.24. Let A be r*g*-closed and regular closed then A is midly g –closed.
Proof: We have rcl(A) $\subseteq$ U where U is g –open.
Also A = cl(int(A))
Let A $\subseteq$ U where U is g –open.
$\therefore$ cl(int(A)) $\subseteq$ U
$\Rightarrow$ A is midly g –closed.

Theorem 3.25. If A $\subseteq$ Y $\subseteq$ X and suppose A is r*g*-closed set in X then A is r*g*-closed set relative to Y.
Proof: A $\subseteq$ Y $\subseteq$ X and A is r*g*-closed. To prove that A is r*g*-closed set relative to Y.
Assume A $\subseteq$ U where U is g-open. Since A is r*g*-closed, rcl (A) $\subseteq$ U when A $\subseteq$ U
Y $\bigcap$ rcl (A) $\subseteq$ Y $\bigcap$ U which is g-open in Y
$\Rightarrow$ rcl_Y (A) $\subseteq$ Y $\bigcap$ U $\Rightarrow$ A is r*g*- closed set relative to Y.

4. r*g* open sets

Definition 4.1. A set A $\subset$ X is said to be r*g*-open if its complement is r*g*-closed set.

Definition 4.2. rint(A) is defined as the union of all regular open sets contained in A.

Theorem 4.3. A subset A $\subset$ X is g*-open iff there exists a g-closed set F such that F $\subset$ r int (A) where F $\subset$ A.
Proof: Let A be r*g* -open. Let F be regular closed and F $\subset$ A.
Now X – A is r*g*-closed. rcl (X –A) $\subset$ U where U is g – open.
Let U = X – F. rcl (X – A) $\subset$ X- F
But rcl (X – A) = X – rint (A) $\subset$ X – F $\Rightarrow$ F $\subset$ rint (A)
Conversely, suppose F $\subset$ rint A where F $\subset$ A
TPT: A is r*g*-open
Let X – A $\subseteq$ U where U is g – open set
X – U $\subset$ A and X – U is g-closed
By our assumption X – U $\subset$ rint (A)
$\Rightarrow$ X – rint (A) $\subset$ U
But X – rint (A) = rcl (X–A)
$\Rightarrow$ rcl (X–A) $\subset$ U
$\Rightarrow$ X–A is r*g* -closed
$\Rightarrow$ A is r*g*-open.

Theorem 4.4. If rint (A) $\subset$ B $\subset$ A and if A is r*g*-open then B is r*g*-open
Proof: B $\subset$ A $\Rightarrow$ X – A $\subset$ X – B
rint (A) $\subset$ B $\Rightarrow$ X – B $\subset$ X – rint (A)
X –A $\subset$ X –B $\subset$ rcl (X –A)

Since X –A is r*g*-closed by theorem 3.20, X – B is r*g*-closed

$\Rightarrow$ B is r*g*-open.

Theorem 4.5. A set A is r*g*-closed iff rcl(A) – A is r*g*-open.
Proof: Let A be r*g*-closed. Let F be a g-closed set such that $F \subset$ rcl(A) – A
then F = φ (by theorem 3.20).
Conversely, suppose rcl(A) – A is r*g*-open.
TPT: A is r*g*-closed
Let $A \subset U$ where U is g-open.
TPT: rcl(A) $\subset$ U
If not, rcl (A) $\cap U^c \neq \varphi$ then
rcl (A) $\cap U^c \subset$ rcl (A) $\cap A^c$
= rcl (A) – A
By theorem (3.20), rcl (A) $\cap U^c \subset$ rint (rcl (A) – A) = φ
$\therefore$ rcl (A) $\subset$ U
$\Rightarrow$ A is r*g*-closed.

Theorem 4.6. Let A, B $\subset$ X. If B is regular open and rint (B) $\subset$ A then $A \cap B$ is r*g*-open.
Proof: Now rint (B) $\subset$ A and also rint(B) $\subset$ B
rint (B) $\subset A \cap B \subset$ B
By theorem(4.4), $A \cap B$ is r*g*-open.

Defnition 4.7. For every set $A \subset X$ we define r*g*-closure of A to be the intersection of all r*g*-closed sets that contains A and is denoted by r*g*-cl(A) and r*g*-interior of A to be the union of all r*g*-open sets contained in A and is denoted by r*g*-int(A).

Definition 4.8. If a subset A of (X,τ) is r*g*-closed then A = r*g*-cl(A).

Theorem 4.9. For an element $x \in X$, $\mathbf{x} \in$ r*g*-cl (A) iff there exists an g-open set U containing **x** such that $U \cap A \neq \varphi$

Proof: Let $x \in$ r*g*-cl (A). Let U be an r*g*-open set containing **x**. Suppose $U \cap A = \varphi$ then $A \not\subset U$. $\therefore A \subset U^c$. Since U^c is an r*g*-closed set containing A, we have r*g*-cl(A) $\subset$ U

- $\Rightarrow$ $x \notin$ rcl(A) which is a contradiction.
- $\Rightarrow$ $U \cap A \neq \varphi$.

Conversely, suppose $U \cap A \neq \varphi$

TPT: $x \in$ r*g*-cl(A)

Suppose $x \notin$ r*g*cl(A), there exists an r*g*-closed set F containing A such that $x \notin F$

$\Rightarrow$ $x \in F^c$ and F^c is r*g*-open set.

Now $F^c \cap A = \varphi$ which is a contradiction to the hypothesis.

$\therefore$ $x \in$ r*g*-cl(A).

Theorem4.10.

i. [r*g*-int(A)]^c = r*g*-cl(A^c)

ii. [r*g*-cl(A)]^c = r*g*-int(A^c)

Proof: i. Let $x \in$ [r*g*-int(A)]^c then $x \notin$ r*g*-int(A).

By definition of r*g*- interior, $x \notin$ every r*g*-open set $\subset A$

If U is an r*g*-open set containing x then $U \cap A^c \neq \varphi$.

By the above theorem, $x \in$ r*g*-cl(A^c)

$\Rightarrow$ [r*g*-int(A)]^c $\subset$ r*g*-cl(A^c) (1)

Conversely, let $x \in$ r*g*-cl(A^c) then by above theorem $U \cap A \neq \varphi$ for every r*g*-open set U containing x. That is, every r*g*-open set U containing x is such that $U \not\subset A$

$\Rightarrow$ $x \notin$ r*g*-int(A) $\Rightarrow$ $x \in$ [r*g*-int(A)]^c

$\therefore$ r*g*-cl(A^c) $\subset$ [r*g*-int(A)] (2)

From (1) and (2),

[r*g*-int(A)]^c = r*g*-cl(A^c)

Similarly, we can prove (ii).

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