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On Interval Valued Bi-Cubic Vague Subgroups

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Abstract. In this paper, we introduce the notion of interval valued bi-cubic vague subgroups and related properties are investigated. We study the characterizations of a interval valued bi-cubic vague groups and how the images or inverse images of interval valued bi cubic subgroups become interval valued bi-cubic vague subgroups. Arbitrary intersection of family of IVBVG is also studied.

Keywords: Interval number, fuzzy set, interval valued fuzzy set, interval valued vague set, interval valued bi-cubic vague group

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1. Introduction

We first recall some basic concepts which are used to present the paper.
An interval number on [0,1], say $\bar{a}$ is a closed subinterval of [0,1], (ie) $\bar{a} = [a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$.
For any interval numbers $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ on [0,1] , we define

(i) $\bar{a} \leq \bar{b}$ if and only if $\bar{a} \leq \bar{b}$ and $a^+ \leq b^+$
(ii) $\bar{a} = \bar{b}$ if and only if $\bar{a} = \bar{b}$ and $a^+ = b^+$
(iii) $\bar{a} + \bar{b} = [a^- + b^-, a^+ + b^+]$, whenever $\bar{a} + \bar{b} \leq 1$ and $a^+ + b^+ \leq 1$

Let X be a set. A mapping A : X → [0,1] is called a fuzzy set in X. Let A be a fuzzy set in X and $\alpha \in [0,1]$. Define L(A : α) as follows:
$L(A: \alpha) = \{ x \in X / A(x) \leq \alpha \}$. Then L (A:α) is called the lower level cut of A.
Let X be a set. A mapping $\bar{A} : X \to D[0,1]$ is called on interval-valued fuzzy set (briefly i-v fuzzy set) of X, where D[0,1] denotes the family of all closed sub intervals of [0,1], and $\bar{A}(x) = [A^-(x), A^+(x)]$, $\forall x \in X$, where $\bar{A}$ and A^+ are fuzzy sets in X.
For an i-v fuzzy set $\bar{A}$ of a set X and $(\alpha, \beta) \in D[0,1]$ define $L(\bar{A}: [\alpha, \beta])$ as follows $\bar{L}(a:[\alpha, \beta])$ which is called the level sub set of $\bar{A}$.

2. Some Definitions

In this section, we recall some basic definitions for the sake of completeness.

Definition 2.1. [1] An interval valued fuzzy set $\tilde{F}$ (over a basic set X) is specified by a function
$T_{\tilde{F}}$: X → D([0,1]) , where D([0,1]) is the set of all intervals within [0,1], i.e. for all x ∈ X, $T_{\tilde{F}}$(x) is an interval $[\mu_1, \mu_2]$,0≤ $\mu_1 \leq \mu_2$ ≤1.

Definition 2.2. [6] A vague set $\tilde{V}$, in a basic set X, is characterized by a truth membership function $t_{\tilde{V}}$, $t_{\tilde{V}}$: X → [0,1] and a false membership function $f_{\tilde{V}}$, $f_{\tilde{V}}$: X → [0,1]. If the generic element of X is denoted by x_i then the lower bound on the membership grade of x_i derived from evidence for x_i is denoted by $t_{\tilde{V}}(x_i)$ and the lower bound on the negation of x_i is denoted by $f_{\tilde{V}}(x_i,)$ and $f_{\tilde{V}}(x_i)$, both associate a real number in the interval [0,1] with each point in X, where $t_{\tilde{V}}(x_i)$ +$f_{\tilde{V}}(x_i)$ ≤ 1 .

When X is continuous, a vague set $\tilde{V}$ can be written as

$\tilde{V} = \int_X [t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)] / x_i,\ x_i$ ∈X.

When X is discrete a vague set $\tilde{V}$ can be written as

$\tilde{V} = \sum_{i=1}^{n}[t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)]/ x_i,\ x_i$ ∈X.

Definition 2.3. [2] Let A =$[a_{1,}\ a_2$] and B=$[b_{1,}\ b_2$] be two arbitrary intervals then the minimum of A and B is represents by "MIN [A,B]" and is defined by
MIN ($[a_{1,}\ a_2$]; $[b_{1,}\ b_2$])=[min($a_{1,}\ b_1$) ($a_{2,}\ b_2$)].

Definition 2.4. [2] The complement of an interval A =$[a_{1,}\ a_2$] is denoted by $\bar{A}$ and is defined by $\bar{A}$= [1-$a_{2,}$1- a_1].

The definition of interval valued vague set and definitions related to interval valued vague set are introduced here.

Definition 2.5. [17] An interval valued vague set $\tilde{V}$ over a basic set X is defined as an object of the form $\tilde{V} =< [x_i; T_{\tilde{V}}(x_i); 1 - f_{\tilde{V}}(x_i)] > x_i$ ∈X , Where $T_{\tilde{V}}$: X → D[0,1] and $f_{\tilde{V}}$: X → D[0,1] are called "Truth membership function" and "False membership function" respectively and where D([0,1]) is the set of all intervals within [0,1].

Definition 2.6. [8] Let G be a non empty set. A Q-fuzzy subset μ on G is defined by
μ : G×Q → [0,1] for all x ∈ G.

Definition 2.7. [8] Let μ be a fuzzy subset in a group G. Then μ is called a Q-fuzzy subgroup of G if

(i) $\mu(xy, q)$ ≥ min { $\mu(x, q)$, $\mu(y, q)$ } for all x, y ∈ G

(ii) $\mu(x^{-1}, q) \geq \mu(x, q)$ for all x ∈ G.

Definition 2.8. [8] Let G be a set. An interval valued Q-fuzzy set A defined on G is given by A = ($x, \mu_A^-(x, q), \mu_A^+(x, q)$) for all x ∈ G. Briefly denote A by A=$[\mu_A^-, \mu_A^+]$ where μ_A^- and μ_A^+ are lower and upper fuzzy sets in G such that $\mu_A^-(x, q) \leq \mu_A^+(x, q)$ for all x ∈ G.

Definition 2.9. [17] Let G be a non empty set. An Interval Valued Bi-cubic Set (IVBS) 'A' in a set G is a structure $\mathscr{A}$ = {($x, \tilde{\mu}_A$(x), $\tilde{V}_A$(x),),x∈G}which is briefly denoted by $\mathscr{A}$=< $\tilde{\mu}_A$,$\tilde{V}_A$> where $\tilde{\mu}_A$=[μ_A^-, μ_A^+] is an IVFS is G , $\tilde{V}_A$=[t_A, 1- f_A] is an Interval Valued Vague Set (IVVS) in G.

Definition 2.10. An IVBS $\mathscr{A}$ is said to be Interval Valued Bi-cubic Vague Group (IVBVG) if

(i) $\tilde{\mu}_A(xy, q) \geq$ rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(y, q)$}

(ii) $\tilde{\mu}_A(x^{-1}, q) \geq \tilde{\mu}_A(x, q)$

(iii) $\tilde{V}_A(xy, q) \leq$ rmax { $\tilde{V}_A(x, q), \tilde{V}_A(y, q)$}

(iv) $\tilde{V}_A(x^{-1}, q) \leq \tilde{V}_A(x, q)$, for all values of $x, y \in X$, $q \in$ Q.

3. Properties of interval valued bi-cubic vague groups

Proposition 3.1. Let $\mathscr{A}$ = { $\tilde{\mu}_A$, $\tilde{V}_A$ } be a IVBVG 'A' in a group G. Then $\tilde{\mu}_A(x^{-1}, q) = \tilde{\mu}_A(x, q)$ and $\tilde{V}_A(x^{-1}, q) = \tilde{V}_A(x, q)$ for all of $x \in G$, $q \in$ Q.

Proof: For all of $x \in G$, we have

$\tilde{\mu}_A(x, q) = \tilde{\mu}_A((x^{-1})^{-1}, q) \geq \tilde{\mu}_A(x, q) \geq \tilde{\mu}_A(x, q)$ &

$\tilde{V}_A(x, q) = \tilde{V}_A((x^{-1})^{-1}, q) \leq \tilde{V}_A(x^{-1}, q) \leq \tilde{V}_A(x, q)$.

Hence $\tilde{\mu}_A(x^{-1}, q)$= $\tilde{\mu}_A(x, q)$ & $\tilde{V}_A(x^{-1}, q)$=$\tilde{V}_A(x, q)$

Proposition 3.2. An IVBS $\mathscr{A}$ = {$\tilde{\mu}_A$,$\tilde{V}_A$} is IVBVG of G if and only if

(i) $\tilde{\mu}_A(xy^{-1}, q) \geq$ rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(y, q)$} &

(ii) $\tilde{V}_A(xy^{-1}, q) \leq$ rmax { $\tilde{V}_A(x, q), \tilde{V}_A(y, q)$} $x, y \in X$, $q \in$ Q.

Proof: Assume that $\mathscr{A}$ = {$\tilde{\mu}_A$,$\tilde{V}_A$} is a IVBVG of G and of $x, y \in G$.

Then $\tilde{\mu}_A(xy^{-1}, q) \geq$ rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(y^{-1}, q)$} (By definition)

= rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(y, q)$} (By Proposition 3.1)

Also $\tilde{V}_A(xy^{-1}, q) \leq$ rmax { $\tilde{V}_A(x, q), \tilde{V}_A(y^{-1}, q)$} (By definition)

= rmax { $\tilde{V}_A(x, q), \tilde{V}_A(y, q)$} (By Proposition 3.1)

Conversely, suppose (i) & (ii) are valid.

If we take $y = x^{-1}$ in (i) & (ii), then

$\tilde{\mu}_A(e, q)$=$\tilde{\mu}_A(xx^{-1}, q) \geq$ rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(x^{-1}, q)$}

= rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(x, q)$} (By Proposition 3.1)

$\tilde{\mu}_A(e, q) \geq \tilde{\mu}_A(x, q)$ and

$\tilde{V}_A(e, q)$=$\tilde{V}_A(xx^{-1}, q) \leq$ rmax { $\tilde{V}_A(x, q), \tilde{V}_A(x^{-1}, q)$}

= rmax { $\tilde{V}_A(x, q), \tilde{V}_A(x, q)$} (By Proposition 3.1)

$\tilde{V}_A(e, q) \geq \tilde{V}_A(x, q)$

It follows from (i) & (ii), that

$\tilde{\mu}_A(y^{-1}, q) = \tilde{\mu}_A(ey^{-1}, q) \geq$ rmin { $\tilde{\mu}_A(e, q), \tilde{\mu}_A(y^{-1}, q)$}

$\geq$ rmin { $\tilde{\mu}_A(e, q), \tilde{\mu}_A(y, q)$} (By Proposition 3.1)

$\tilde{\mu}_A(y^{-1}, q) \geq \tilde{\mu}_A(y, q)$

Also $\tilde{V}_A(y^{-1}, q) = \tilde{V}_A(ey^{-1}, q) \leq$ rmax { $\tilde{V}_A(e, q), \tilde{V}_A(y^{-1}, q)$}

$\leq$ rmax{ $\tilde{V}_A(e, q), \tilde{V}_A(y, q)$} (By Proposition 3.1)

$\therefore \tilde{V}_A(y^{-1}, q) \leq \tilde{V}_A(y, q)$

$\tilde{\mu}_A(xy, q) = \tilde{\mu}_A(x(y^{-1})^{-1}, q) \geq$ rmin { $\tilde{\mu}_A(x, q), \tilde{\mu}_A(y^{-1}, q)$ }

$\geq$ rmin $\{ \tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q) \}$

$\tilde{V}_A(xy,q) = \tilde{V}_A(x(y^{-1})^{-1},q) \leq \text{rmax } \{\tilde{V}_A(x,q), \tilde{V}_A(y^{-1},q)\}$

$\leq \text{rmax } \{\tilde{V}_A(x,q), \tilde{V}_A(y,q)\}$.

$\therefore \{\tilde{\mu}_A, \tilde{V}_A\}$ is IVBVG of G

Definition 3.1. Let $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be a IVBS 'A' in a group G.Let [α,β] & [γ,δ] ∈[0,1]. The set ∪ {$\tilde{\mathscr{A}}$: [α,β] , [γ,δ]}={x ∈ G / $\tilde{\mu}_A(x,q)$ ≥[α,β] &$\tilde{V}_A(x,q) \leq$ [γ,δ] }is called cubic level set of $\tilde{\mathscr{A}}$.

Proposition 3.3. Let $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be an IVBVG of G, then the following conditions one equivalent:

(i) $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be a IVBVG of G,

(ii) The non empty cubic level set of $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is a subgroup of G.

Proof: Assume that $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is a IVBVG of G,

Let x, y ∈ ∪ {$\tilde{\mathscr{A}}$: [α,β] , [γ,δ]} for all [α,β] & [γ,δ] ∈ D[0,1]

Then $\tilde{\mu}_A(x,q) \geq$ [α,β] , $\tilde{V}_A(x,q) \leq$[γ,δ]

$\tilde{\mu}_A(y,q) \geq$[α,β] , $\tilde{V}_A(y,q) \leq$ [γ,δ]

It follows that

$\tilde{\mu}_A(xy^{-1},q) \geq$ r min { $\tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q)$} ≥ [α,β]

$\tilde{V}_A(xy^{-1},q) \leq$ r max { $\tilde{V}_A(x,q), \tilde{V}_A(y,q)$} ≤ [γ,δ]

So that xy^{-1} ∈∪ {$\tilde{\mathscr{A}}$: [α,β] , [γ,δ]}

The non empty cubic level set $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is IVBVG of G.

Conversely, [α,β] & [γ,δ] ∈ D[0,1] such that

∪ ($\tilde{\mathscr{A}}$: [α,β] , [γ,δ]) ≠ ∅ & ∪ ($\tilde{\mathscr{A}}$: [α,β] , [γ,δ]) is a subgroup of G.

Suppose that Proposion 3. 2 (I) is not true & Proposion 3. 2 (II) is valid. Then there exist $[\alpha_0, \beta_0]$ ∈ D [0,1] & a,b ∈ G. such that

$\tilde{\mu}_A(ab^{-1},q) \leq [\alpha_0,\beta_0]$ ≤ r min { $\tilde{\mu}_A(a,q), \tilde{\mu}_A(b,q)$}

$\tilde{V}_A(ab^{-1},q) \geq [\gamma_0,\delta_0]$ ≥ r max { $\tilde{V}_A(a,q), \tilde{V}_A(b,q)$}

Proposition 3.4. Let f: G→ G^{l} is a homomorphism of groups.If $\tilde{\mathscr{A}} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is an IVBVG of G^{l}, then $\tilde{\mathscr{A}}^{f} = \{ \tilde{\mu}_A{}^{f}, \tilde{V}_A{}^{f}\}$ is IVBVG of G.

Proof:

(i) $\tilde{\mu}_A{}^{f}(xy,q) = \tilde{\mu}_A(f(xy,q)) = \tilde{\mu}_A(f(x,q), f(y,q))$ [f *is homo*]

$\geq$ rmin { $\tilde{\mu}_A(f(x,q), \tilde{\mu}_A(f(y,q))$}

= rmin { $\tilde{\mu}_A{}^{f}(x,q)$, $\tilde{\mu}_A{}^{f}(y,q)$}

(ii) $\tilde{\mu}_A{}^{f}(x^{-1},q) = \tilde{\mu}_A(f(x^{-1},q))$

$\geq \tilde{\mu}_A(f(x,q))$

$= \tilde{\mu}_A{}^{f}(x,q)$

(iii) $\tilde{V}_A{}^{f}(xy,q) = \tilde{V}_A(f(xy,q)) = \tilde{V}_A(f(x,q), f(y,q))$ [f *is homo*]

≤ r max { $\tilde{V}_A(f(x,q), \tilde{v}_A(f(y,q))$}

= r max { $\tilde{V}_A{}^{f}(x,q)$, $\tilde{V}_A{}^{f}(y,q)$}

(iv) $\tilde{V}_A{}^f(x^{-1},q) = \tilde{V}_A\big(f(x^{-1},q)\big)$
$\leq \tilde{V}_A(f(x,q))$
$= \tilde{V}_A{}^f(x,q)$

Proposition 3.5. Let $\mathscr{A}$ be an IVBVG of G. And A(e)=1 is normal defined by $\tilde{A}^{+}(x) = \tilde{A}(x) + 1 - \tilde{A}(e), \forall$ x, e $\in G$ then $\tilde{A}^{+}$ is IVBVG of G.

Proof: Let $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is IVBVG for x, y $\in G$ & $e \in G$ such that $\tilde{A}^{+}(e) = \tilde{A}(e)+1-\tilde{A}(e)=1$

Now (i) $\tilde{\mu}_A{}^{+}(xy,q) = \tilde{\mu}_A(xy,q) + 1 - \tilde{\mu}_A(e,q)$
$\geq$ rmin $\{\tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q)\} + 1 - \tilde{\mu}_A(e,q)$
$\geq$ rmin $\{\tilde{\mu}_A(x,q) + 1 - \tilde{\mu}_A(e,q), \tilde{\mu}_A(y,q) + 1 - \tilde{\mu}_A(e,q)\}$
$=$ rmin $\{\tilde{\mu}_A{}^{+}(x,q), \tilde{\mu}_A{}^{+}(y,q)\}$

(ii) $\tilde{\mu}_A{}^{+}(x^{-1},q) = \tilde{\mu}_A(x^{-1},q) + 1 - \tilde{\mu}_A(e,q)$
$\geq \tilde{\mu}_A(x,q) + 1 - \tilde{\mu}_A(e,q)$
$\geq \tilde{\mu}_A{}^{+}(x,q)$

(iii) $\tilde{V}_A{}^{+}(xy,q) = \tilde{V}_A(xy,q) + 1 - \tilde{V}_A(e,q)$
$\leq$ rmax $\{\tilde{V}_A(x,q), \tilde{\mu}_A(y,q)\} + 1 - \tilde{V}_A(e,q)$
$\leq$ rmax $\{\tilde{V}_A(x,q) + 1 - \tilde{V}_A(e,q), \tilde{V}_A(y,q) + 1 - \tilde{V}_A(e,q)\}$
$=$ rmax $\{\tilde{V}_A{}^{+}(x,q), \tilde{V}_A{}^{+}(y,q)\}$

(iv) $\tilde{V}_A{}^{+}(y^{-1},q) = \tilde{V}_A(y^{-1},q) + 1 - \tilde{V}_A(e,q)$
$\leq$ rmax $\{\tilde{V}_A(y,q) + 1 - \tilde{V}_A(e,q)$
$\leq \tilde{V}_A{}^{+}(y,q)$.

Definition 3.2. Let θ be a mapping from X to Y. If A & B are IVBVG's in X & Y respectively, then the inverse image of B under θ denoted by $\theta^{-1}(B)$ is defined by $\theta^{-1}(B) = \tilde{\mu}_\theta{}^{-1}(B)$ where $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x,q) = \tilde{\mu}_B(\theta(x,q))$ and $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x^{-1},q) = \tilde{\mu}_B(\theta(x,q))$ $\forall\, x \epsilon Y, q \epsilon Q$.

Proposition 3.6. The inverse image of an IVBVG is also IVBVG.

Proof: Let G and $\bar{G}$ be two groups and $\theta: G \to \bar{G}$ a homomorphism. Let B is IVBVG of $\bar{G}$.

We have to prove that $\theta^{-1}(B)$ is IVBVG of G.

Let $x, y \in G, q \in Q$.

(i) $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(xy,q) = \tilde{\mu}_B\big(\theta(xy,q)\big)$
$= \tilde{\mu}_B\big(\theta(x)\theta(y),q)\big)$
$\geq$ rmin $\{\tilde{\mu}_B(\theta(x),q), \tilde{\mu}_B(\theta(y),q)\}$
$\geq$ rmin $\{\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x,q), \tilde{\mu}_\theta{}^{-1}{}_{(B)}(y,q)\}$

(ii) $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x^{-1},q) = \tilde{\mu}_B(\theta(x^{-1},q))$
$= \tilde{\mu}_B(\theta x^{-1},q)$
$= \tilde{\mu}_B(\theta x,q)$
$= \tilde{\mu}_\theta{}^{-1}{}_{(B)}(x,q)$

(iii) $\tilde{V}_\theta{}^{-1}{}_{(B)}(xy,q) = \tilde{V}_B\big(\theta(xy,q)\big)$

$= \tilde{V}_B(\theta(x)\theta(y), q))$

$\leq$ rmax $\{\tilde{V}_B(\theta(x), q), \tilde{V}_B(\theta(y), q)\ \}$

$\leq$ rmax $\{\ \tilde{V}_\theta{}^{-1}{}_{(B)}(x, q), \tilde{V}_\theta{}^{-1}{}_{(B)}(y, q)\}$

(iv)) $\tilde{V}_\theta{}^{-1}{}_{(B)}(x^{-1}, q) = \tilde{V}_B(\theta(x^{-1}, q))$

$= \tilde{V}_B(\theta x^{-1}, q)$

$= \tilde{V}_B(\theta x, q)$

$= \tilde{V}_\theta{}^{-1}{}_{(B)}(x, q)$

$\therefore \theta^{-1}(B)$ is IVBVG of G.

Proposition 3.6. If $\{A_i\}$i$\in A$ is a family of IVBVG's of G , then $\bigcap_{i\in A} A_i$ is IVBVG of G, where $\bigcap_{i\in A} A_i$= { ((x,q), $\tilde{\mu}_{A_i}(x, q)$) / $x \epsilon G, q \epsilon Q$ }

Proof: Let $x, y \epsilon G, q \epsilon Q$.

(i) $(\bigcap_{i\in A} \tilde{\mu}_{A_i})\ (xy, q) = \bigwedge_{i\in A} \tilde{\mu}_{A_i}\ (xy, q)$

$\geq \bigwedge_{i\in A} rmin\ \{\ \tilde{\mu}_{A_i}(x, q), \tilde{\mu}_{A_i}(y, q)\ \}$

$= rmin\ \{\ \bigwedge_{i\in A} \tilde{\mu}_{A_i}\ (x, q), \bigwedge_{i\in A} \tilde{\mu}_{A_i}\ (y, q)\ \}$

$= rmin\ \{\ (\bigcap_{i\in A} \tilde{\mu}_{A_i})\ (x, q), (\bigcap_{i\in A} \tilde{\mu}_{A_i})\ (y, q)\ \}$

(ii) $(\bigcap_{i\in A} \tilde{\mu}_{A_i})\ (x^{-1}, q) = \bigwedge_{i\in A} \tilde{\mu}_{A_i}\ (x^{-1}, q)$

$\geq \bigwedge_{i\in A} \tilde{\mu}_{A_i}\ (x, q)$

$= (\bigcap_{i\in A} \tilde{\mu}_{A_i})\ (x, q)$

(iii) $(\bigcap_{i\in A} \tilde{V}_{A_i})\ (xy, q) = \bigwedge_{i\in A} \tilde{V}_{A_i}\ (xy, q)$

$\geq \bigwedge_{i\in A} rmin\ \{\ \tilde{V}_{A_i}(x, q), \tilde{V}_{A_i}(y, q)\ \}$

$= rmax\ \{\ \bigwedge_{i\in A} \tilde{V}_{A_i}\ (x, q), \bigwedge_{i\in A} \tilde{V}_{A_i}\ (y, q)\ \}$

$= rmax\ \{\ (\bigcap_{i\in A} \tilde{V}_{A_i})\ (x, q), (\bigcap_{i\in A} \tilde{V}_{A_i})\ (y, q)\ \}$

(iv) $(\bigcap_{i\in A} \tilde{V}_{A_i})\ (x^{-1}, q) = \bigwedge_{i\in A} \tilde{V}_{A_i}\ (x^{-1}, q)$

$\geq \bigwedge_{i\in A} \tilde{V}_{A_i}\ (x, q)$

$= (\bigcap_{i\in A} \tilde{V}_{A_i})\ (x, q)$

Hence $\bigcap_{i\in A} A_i$ is IVBVG of G.

4. Conclusion

Group theory has vast and potential applications in many core areas like physics, chemistry, communication, coding theory, computer science, etc. In this paper we have studied interval valued bi-cubic vague groups and their properties. We have also proved a result on classical groups with the help of interval valued vague group theory. As mentioned in [2], we too observe that the notion of 'IVBVG" defined by Dimirci in [4] is a completely different concept and not in the context of vague set theory of Gau and Buehrer [6].

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