Annals of Pure and Applied Mathematics
Vol. 6, No. 2, 2014, 160-169
ISSN: 2279-087X (P), 2279-0888(online)
Published on 26 June 2014
www.researchmathsci.org

Annals of
Pure and Applied
Mathematics

Some Adjacent Edge Graceful Graphs

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Received 17 June 2014; accepted 22 June 2014

Abstract. Let $G(V,E)$ be a graph with p vertices and q edges. A (p,q) graph $G(V,E)$ is said to be an adjacent edge graceful graph if there exists a bijection $f: E(G) \to \{1,2,3,....., q\}$ such that the induced mapping f^* from $V(G)$ by $f^*(u) = \sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u is an injection. The function f is called an adjacent edge graceful labeling of G .In this paper, we prove the graphs $C_m \cup C_n (m \geq 5, n \geq 5)$, $P_m \cup C_n (m \geq 5, n \geq 5)$, $C_n^{(2)}, mK_3$ and sunflower graph $SF(n)$ (n is odd) are the adjacent edge graceful graphs and we also prove graphs $B_{n,n}{}^2$, $K_2 + mK_1$ and mK_2 are not the adjacent edge graceful graphs.

Keywords: adjacent edge graceful graph**,** adjacent edge graceful labeling.

AMS Mathematical Subject Classification (2010): 05C78

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Let $G(p,q)$ be a graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges. Graph labeling, where the vertices and edges are assigned real values or subsets of a set are subject to certain conditions. A detailed survey of graph labeling can be found in [2].Terms not defined here are used in the sense of Harary in [5].The concept of adjacent edge graceful labeling was first introduced in [18].Some results on adjacent edge graceful labeling of graphs and some non- adjacent edge graceful graphs are discussed in [18].

In this paper, we discussed about adjacent edge graceful labeling for a few more graphs.We use the following definitions in the subsequent sections.

Definition 1.1. [18] A (p,q) graph $G(V,E)$ is said to be an adjacent edge graceful graph if there exists a bijection $f: E(G) \to \{1,2,3,....., q\}$ such that the induced mapping f^* from $V(G)$ by $f^*(u)=\sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u is an injection. The function f is called an adjacent edge graceful labeling of G.

Definition 1.2. [2] The bistar graph $B_{n,n}$ is the graph obtained from two copies of star $K_{1,n}$ by joining the vertices of maximum degree by an edge.

Definition 1.3. [2] A sunflower graph SF(n) as the graph obtained by starting with an n-cycle with consecutive vertices $v_1, v_2, \ldots\ldots, v_n$ and creating new vertices $w_1, w_2, \ldots\ldots, w_n$ with w_i connected to v_i and v_{i+1} (v_{n+1} *is* v_1).

Definition 1.4. [15] For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

2. Main results

Theorem 2.1. The graph $C_m \cup C_n (m \geq 5, n \geq 5)$ is an adjacent edge graceful graph.

Proof: Let $C_m = u_1 u_2 \ldots\ldots u_m u_1$ and $C_n = v_1 v_2 \ldots\ldots v_n v_1$ be two cycles.

Let $E(C_m \cup C_n) = \begin{cases} e_i = u_i u_{i+1} & : 1 \leq i \leq m-1 \ ; \ e_m = u_1 u_m \\ e_{m+i} = v_i v_{i+1} & : 1 \leq i \leq n-1 \ ; \ e_{m+n} = v_1 v_n \end{cases}$

Define $f : E(C_m \cup C_n) \rightarrow \{1,2,3,\ldots\ldots, m+n\}$ by

Case (i) (m and n are both odd)

$f(e_i) = i \ \ if \ \ 1 \leq i \leq m+n$. Let f^* be the induced vertex labeling of f. In this case the induced vertex labels are as follows: $f^*(u_1) = 2m+2$; $f^*(u_2) = m+6$;
$f^*(u_{i+2}) = 4i+6 \ \ if \ \ 1 \leq i \leq m-3$; $f^*(u_m) = 3m-2$; $f^*(v_1) = 4m+2n+2$;
$f^*(v_2) = 4m+n+6$; $f^*(v_n) = 4m+3n-2$;
$f^*(v_{i+2}) = 4m+4i+6 \ \ if \ \ 1 \leq i \leq n-3$.

Case (ii) (m is odd and n is even)

$f(e_i) = i \ \ if \ \ 1 \leq i \leq m$; $f(e_{m+i}) = m+2i-1 \ \ if \ \ 1 \leq i \leq \frac{n}{2}$;

$f(e_{\frac{2m+n+2i}{2}}) = m+n+2-2i \ \ if \ \ 1 \leq i \leq \frac{n}{2}$.

In this case the induced vertex labels are as follows:
$f^*(u_1) = 2m+2$; $f^*(u_2) = m+6$; $f^*(u_m) = 3m-2$;
$f^*(u_{i+2}) = 4i+6 \ \ if \ \ 1 \leq i \leq m-3$;
If $n = 6$, $f^*(v_1) = 4m+n+4$; $f^*(v_2) = 4m+n+5$;
$f^*(v_3) = 4m+2n+3$; $f^*(v_4) = 4m+3n$;

$f^*(v_5) = 4m + 2n + 5$; $f^*(v_6) = 4m + 2n + 1$;

If $n \ge 8$, $f^*(v_1) = 4m + 10$; $f^*(v_2) = 4m + 11$; $f^*(v_n) = 4m + 13$;

$f^*(v_{i+2}) = 4m + 8(i+1)$ *if* $1 \le i \le \frac{n-6}{2}$; $f^*(v_{\frac{n}{2}}) = 4m + 4n - 9$;

$f^*(v_{\frac{n+2}{2}}) = 4m + 4n - 6$; $f^*(v_{\frac{n+4}{2}}) = 4m + 4n - 7$;

$f^*(v_{n-i}) = 4m + 4(2i+3)$ *if* $1 \le i \le \frac{n-6}{2}$.

Case (iii) (m is even and n is odd)

$f(e_i) = 2i - 1$ *if* $1 \le i \le \frac{m}{2}$; $f(e_{\frac{m+2i}{2}}) = m + 2 - 2i$ *if* $1 \le i \le \frac{m}{2}$;

$f(e_{m+i}) = m + i$ *if* $1 \le i \le n$.

In this case the induced vertex labels are as follows:

If $m = 6$, $f^*(u_1) = 2m - 2$; $f^*(u_2) = 2m - 1$; $f^*(u_3) = 3m - 3$;

$f^*(u_4) = 3m$; $f^*(u_5) = 3m - 1$; $f^*(u_6) = 2m + 1$;

If $m \ge 8$, $f^*(u_1) = 10$; $f^*(u_2) = 11$; $f^*(u_{i+2}) = 8(i+1)$ *if* $1 \le i \le \frac{m-6}{2}$;

$f^*(u_m) = 13$; $f^*(u_{\frac{m}{2}}) = 4m - 9$; $f^*(u_{\frac{m+2}{2}}) = 4m - 6$; $f^*(u_{\frac{m+4}{2}}) = 4m - 7$;

$f^*(u_{m-i}) = 4(2i+3)$ *if* $1 \le i \le \frac{m-6}{2}$; $f^*(v_1) = 4m + 2n + 2$;

$f^*(v_2) = 4m + n + 6$; $f^*(v_n) = 4m + 3n - 2$;

$f^*(v_{i+2}) = 4m + 4i + 6$ *if* $1 \le i \le n - 3$.

Case (iv) (m and n are both even)

$f(e_i) = 2i - 1$ *if* $1 \le i \le \frac{m}{2}$; $f(e_{\frac{m+2i}{2}}) = m + 2 - 2i$ *if* $1 \le i \le \frac{m}{2}$

$f(e_{m+i}) = m + 2i - 1$ *if* $1 \le i \le \frac{n}{2}$; $f(e_{\frac{2m+n+2i}{2}}) = m + n + 2 - 2i$ *if* $1 \le i \le \frac{n}{2}$.

In this case the induced vertex labels are as follows:

If $m = 6$ and $n = 6$, $f^*(u_1) = 2m - 2$; $f^*(u_2) = 2m - 1$; $f^*(u_3) = 3m - 3$;

$f^*(u_4) = 3m$; $f^*(u_5) = 3m - 1$; $f^*(u_6) = 2m + 1$; $f^*(v_1) = 5n + 4$;

$f^*(v_2) = 5n + 5$; $f^*(v_3) = 6n + 3$; $f^*(v_4) = 7n$; $f^*(v_5) = 7n + 1$;

$f^*(v_6) = 6n + 1$.

If $m = 6$ and $n \ge 8$, $f^*(u_1) = 2m - 2$; $f^*(u_2) = 2m - 1$;

$f^*(u_3) = 3m - 3$; $f^*(u_4) = 3m$; $f^*(u_5) = 3m - 1$;

$f^*(u_6) = 2m+1$; $f^*(v_1) = 4m+10$; $f^*(v_2) = 4m+11$;

$f^*(v_n) = 4m+13$; $f^*(v_{i+2}) = 4m+8(i+1)$ *if* $1 \le i \le \frac{n-6}{2}$;

$f^*(v_{\frac{n}{2}}) = 4m+4n-9$ $f^*(v_{\frac{n+2}{2}}) = 4m+4n-6$; $f^*(v_{\frac{n+4}{2}}) = 4m+4n-7$;

$f^*(v_{n-i}) = 4m+4(2i+3)$ *if* $1 \le i \le \frac{n-6}{2}$.

If $m \ge 8$ and $n \ge 8$, $f^*(u_1) = 10$; $f^*(u_2) = 11$;

$f^*(u_{i+2}) = 8(i+1)$ *if* $1 \le i \le \frac{m-6}{2}$; $f^*(u_m) = 13$; $f^*(u_{\frac{m}{2}}) = 4m-9$;

$f^*(u_{\frac{m+2}{2}}) = 4m-6$; $f^*(u_{\frac{m+4}{2}}) = 4m-7$; $f^*(u_{m-i}) = 4(2i+3)$ *if* $1 \le i \le \frac{m-6}{2}$;

$f^*(v_1) = 4m+10$; $f^*(v_2) = 4m+11$; $f^*(v_n) = 4m+13$;

$f^*(v_{i+2}) = 4m+8+8i$ *if* $1 \le i \le \frac{n-6}{2}$;

$f^*(v_{n-i}) = 4m+12+8i$ *if* $1 \le i \le \frac{n-6}{2}$; $f^*(v_{\frac{n}{2}}) = 4m+4n-9$;

$f^*(v_{\frac{n+2}{2}}) = 4m+4n-6$; $f^*(v_{\frac{n+4}{2}}) = 4m+4n-7$.

If $m \ge 8$ and $n = 6$, $f^*(u_1) = 10$; $f^*(u_2) = 11$;

$f^*(u_{i+2}) = 8(i+1)$ *if* $1 \le i \le \frac{m-6}{2}$; $f^*(u_m) = 13$; $f^*(u_{\frac{m}{2}}) = 4m-9$;

$f^*(u_{\frac{m+2}{2}}) = 4m-6$; $f^*(u_{\frac{m+4}{2}}) = 4m-7$; $f^*(u_{m-i}) = 4(2i+3)$ *if* $1 \le i \le \frac{m-6}{2}$;

$f^*(v_1) = 4m+n+4$; $f^*(v_2) = 4m+n+5$; $f^*(v_3) = 4m+2n+3$;

$f^*(v_4) = 4m+3n$; $f^*(v_5) = 4m+2n+5$; $f^*(v_6) = 4m+2n+1$.

Example 2.2. An adjacent edge graceful labeling of $C_{12} \cup C_{10}$ is shown in the Fig.1.

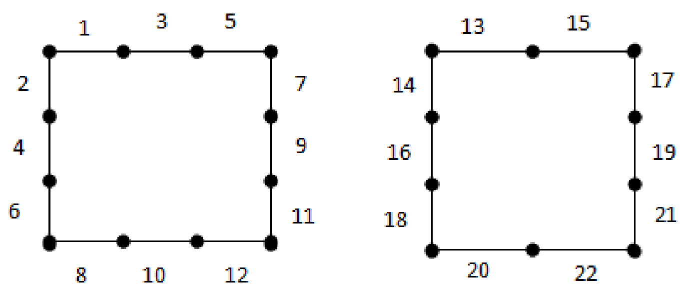

Figure 1:

Theorem 2.3. The graph $P_m \cup C_n (m \geq 5, n \geq 5)$ is an adjacent edge graceful graph.

Proof: Let $P_m = u_1 u_2 ....u_m$ be a path and $C_n = v_1 v_2 ....v_n v_1$ be a cycle .

Let $E(P_m \cup C_n) = \begin{cases} e_i = u_i u_{i+1} \quad : 1 \leq i \leq m-1 \;; \\ e_{m-1+i} = v_i v_{i+1} : 1 \leq i \leq n-1 \;;\; e_{m+n-1} = v_1 v_n \end{cases}$

Define $f : E(P_m \cup C_n) \to \{1,2,3,.....,m+n-1\}$ by

Case (i) (m and n are both odd)

$f(e_i) = i \;\; if \;\; 1 \leq i \leq m+n-1$.

In this case the induced vertex labels are as follows:

$f^*(u_1) = 3$; $f^*(u_2) = 6$; $f^*(u_{i+2}) = 4i+6 \;\; if \;\; 1 \leq i \leq m-4$; $f^*(u_{m-1}) = 3m-6$;

$f^*(u_m) = 2m-3$; $f^*(v_1) = 4m+2n-2$; $f^*(v_2) = 4m+n+2$;

$f^*(v_n) = 4m+3n-3$; $f^*(v_{i+2}) = 4m+4i+2 \;\; if \;\; 1 \leq i \leq n-3$.

Case (ii) (m is odd and n is even)

$f(e_i) = i \;\; if \;\; 1 \leq i \leq m-1$; $f(e_{m-1+i}) = m+2i-2 \;\; if \;\; 1 \leq i \leq \frac{n}{2}$;

$f(e_{\frac{2m+n-2+2i}{2}}) = m+n+1-2i \;\; if \;\; 1 \leq i \leq \frac{n}{2}$.

In this case the induced vertex labels are as follows:

$f^*(u_1) = 3$; $f^*(u_2) = 6$; $f^*(u_{m-1}) = 3m-6$;

$f^*(u_{i+2}) = 4i+6 \;\; if \;\; 1 \leq i \leq m-4$; $f^*(u_m) = 2m-3$.

If $n = 6$, $f^*(v_1) = 4m+n$; $f^*(v_2) = 4m+n+1$; $f^*(v_3) = 4m+n+5$;

$f^*(v_4) = 4m+2n+2$; $f^*(v_5) = 4m+2n+1$; $f^*(v_6) = 4m+n+3$.

If $n \geq 8$, $f^*(v_1) = 4m+6$; $f^*(v_2) = 4m+7$;

$f^*(v_{i+2}) = 4m+4+8i \;\; if \;\; 1 \leq i \leq \frac{n-6}{2}$; $f^*(v_{\frac{n}{2}}) = 4m+4n-13$;

$f^*(v_{\frac{n+2}{2}}) = 4m+4n-10$; $f^*(v_{\frac{n+4}{2}}) = 4m+4n-11$;

$f^*(v_{\frac{n+4+2i}{2}}) = 4m+4n-8-8i \;\; if \;\; 1 \leq i \leq \frac{n-6}{2}$; $f^*(v_n) = 4m+9$.

Case (iii) (m is even and n is odd)

$f(e_i) = 2i-1 \;\; if \;\; 1 \leq i \leq \frac{m}{2}$; $f(e_{\frac{m+2i}{2}}) = m-2i \;\; if \;\; 1 \leq i \leq \frac{m-2}{2}$;

$f(e_{m-1+i}) = m-1+i \;\; if \;\; 1 \leq i \leq n$.

In this case the induced vertex labels are as follows:

If $m = 6$, $f^*(u_1) = m-2$; $f^*(u_2) = m+3$; $f^*(u_3) = 2m+1$;

$f^*(u_4) = 2m$; $f^*(u_5) = 2m-5$; $f^*(u_6) = 6$.

If $m=8$, $f^*(u_1)=m-4$; $f^*(u_2)=m+1$; $f^*(u_3)=2m$; $f^*(u_4)=2m+5$; $f^*(u_5)=3m$; $f^*(u_6)=2m+3$; $f^*(u_7)=2m+2$ $f^*(u_8)=m-2$.

If $m\geq 10$, $f^*(u_1)=4$; $f^*(u_2)=9$; $f^*(u_{i+2})=8i+8$ *if* $1\leq i\leq \frac{m-6}{2}$;

$f^*(u_{\frac{m}{2}})=4m-11$; $f^*(u_{\frac{m+2}{2}})=4m-10$; $f^*(u_{\frac{m+4}{2}})=4m-13$;

$f^*(u_{\frac{m+4+2i}{2}})=4m-12-8i$ *if* $1\leq i\leq \frac{m-8}{2}$; $f^*(u_{m-1})=12$; $f^*(u_m)=6$

$f^*(v_1)=4m+2n-2$; $f^*(v_2)=4m+n+2$; $f^*(v_n)=4m+3n-6$;

$f^*(v_{i+2})=4m+4i+2$ *if* $1\leq i\leq n-3$.

Case (iv) (m and n are both even)

$f(e_i)=2i-1$ *if* $1\leq i\leq \frac{m}{2}$; $f(e_{\frac{m+2i}{2}})=m-2i$ *if* $1\leq i\leq \frac{m-2}{2}$;

$f(e_{m-1+i})=m+2i-2$ *if* $1\leq i\leq \frac{n}{2}$; $f(e_{\frac{2m+n-2+2i}{2}})=m+n+1-2i$ *if* $1\leq i\leq \frac{n}{2}$.

In this case the induced vertex labels are as follows:

If $m=6$, $f^*(u_1)=m-2$; $f^*(u_2)=m+3$; $f^*(u_3)=2m+1$;

$f^*(u_4)=2m$; $f^*(u_5)=2m-5$; $f^*(u_6)=6$.

If $m=8$, $f^*(u_1)=m-4$; $f^*(u_2)=m+1$; $f^*(u_3)=2m$; $f^*(u_4)=2m+5$; $f^*(u_5)=3m$; $f^*(u_6)=2m+3$; $f^*(u_7)=2m+2$ $f^*(u_8)=m-2$.

If $m\geq 10$, $f^*(u_1)=4$; $f^*(u_2)=9$; $f^*(u_{i+2})=8i+8$ *if* $1\leq i\leq \frac{m-6}{2}$;

$f^*(u_{\frac{m}{2}})=4m-11$; $f^*(u_{\frac{m+2}{2}})=4m-10$; $f^*(u_{\frac{m+4}{2}})=4m-13$;

$f^*(u_{\frac{m+4+2i}{2}})=4m-12-8i$ *if* $1\leq i\leq \frac{m-8}{2}$; $f^*(u_{m-1})=12$; $f^*(u_m)=6$

If $n=6$, $f^*(v_1)=4m+n$; $f^*(v_2)=4m+n+1$; $f^*(v_3)=4m+n+5$;

$f^*(v_4)=4m+2n+2$; $f^*(v_5)=4m+2n+1$; $f^*(v_6)=4m+n+3$.

If $n\geq 8$, $f^*(v_1)=4m+6$; $f^*(v_2)=4m+7$;

$f^*(v_{i+2})=4m+4+8i$ *if* $1\leq i\leq \frac{n-6}{2}$; $f^*(v_{\frac{n}{2}})=4m+4n-13$;

$f^*(v_{\frac{n+2}{2}})=4m+4n-10$; $f^*(v_{\frac{n+4}{2}})=4m+4n-11$;

$f^*(v_{\frac{n+4+2i}{2}})=4m+4n-8-8i$ *if* $1\leq i\leq \frac{n-6}{2}$; $f^*(v_n)=4m+9$.

Example 2.4. An adjacent edge graceful labeling of $C_9 \cup P_7$ is shown in the Fig. 2.

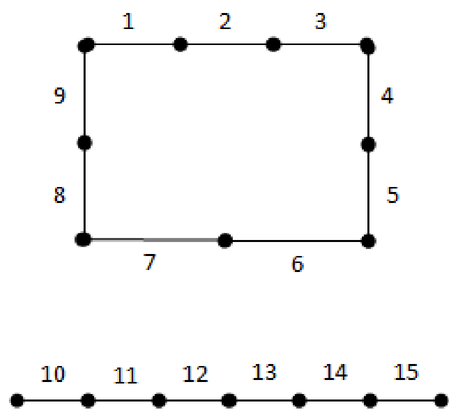

Figure 2:

Theorem 2.5. The graph $C_n^{(2)}$ is an adjacent edge graceful graph.

Proof: Let ' w ' be the central vertex of $C_n^{(2)}$.Let $\{ u_i : 1 \le i \le n\}$ be the vertices of first cycle of $C_n^{(2)}$. Let $\{ v_i : 1 \le i \le n\}$ be the vertices of second cycle of $C_n^{(2)}$.

Let $E(C_n^{(2)}) = \begin{cases} e_i = u_i u_{i+1} & :1 \le i \le n-1 \ , \ e_n = u_1 u_n \\ e_{n+i} = v_i v_{i+1} & :1 \le i \le n-1 \ , \ e_{2n} = v_1 v_n \end{cases}$

Take $w = u_1 = v_1$ .Define $f : E(C_n^{(2)}) \to \{1,2,3,......,2n\}$ by $f(e_i) = i \ \ if \ \ 1 \le i \le n$. Let f^* be the induced vertex labeling of f.

The induced vertex labels are as follows: $f^*(u_1) = 8n + 4$; $f^*(u_2) = 4n + 7$; $f^*(u_n) = 6n - 1$; $f^*(u_{i+2}) = 4i + 6 \ \ if \ \ 1 \le i \le n-3$; $f^*(v_2) = 6n + 7$; $f^*(v_n) = 8n - 1$; $f^*(v_{i+2}) = 4n + 4i + 6 \ \ if \ \ 1 \le i \le n-3$.

Theorem 2.6. The graph $K_2 + mK_1$ is not an adjacent edge graceful graph.

Proof: Let $G = K_2 + mK_1$. Let $V(G) = \{u, v, w_i : 1 \le i \le m\}$.

Let $E(G) = \{ e_i = uw_i \ , e_{m+i} = vw_i \ :1 \le i \le m \ ; e_{2m+1} = uv\}$.

Let f be a bijection from $E(G)$ to $\{1,2,3,.......,2m+1\}$.

Let $\alpha_1, \alpha_2, ..., \alpha_m, \beta_1, \beta_2, ..., \beta_m, \alpha$ be the label of edges $e_1, e_2, ..., e_m, e_{m+1}, ..., e_{2m+1}$ respectively by f. And f induces that $f^*: V(G) \to \{1,2,3,......\}$ by $f^*(u) = \sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u. Then

$$f^*(w_i) = \sum_{i=1}^{m} f(e_i) + f(e_{2m+1}) + \sum_{i=m+1}^{2m} f(e_i) + f(e_{2m+1}) = \sum_{i=1}^{m} \alpha_i + \sum_{i=1}^{m} \beta_i + \alpha + \alpha$$

$=1+2+3+.........+(2m+1)+\alpha=(m+1)(2m+1)+\alpha$ for $1\le i\le m$. That is every vertex w_i have same vertex labels by f^*. Hence f^* is not injective.Therefore the graph K_2+mK_1 is not an adjacent edge graceful graph.

Theorem 2.7. The graph mK_2 is not an adjacent edge graceful graph.

Proof: Let the vertex set of mK_2 be $V=V_1\cup V_2\cup........\cup V_m$ where $V_i=\{v_i^1,v_i^2\}$.Let $E(mK_2)=\{e_i=v_i^1v_i^2 : 1\le i\le m\}$. Let f be a bijection from $E(mK_2)$ to $\{1,2,3,.......,m\}$. And f induces that $f^*:V(mK_2)\to\{1,2,3,....\}$ by $f^*(u)=\sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u.The vertices v_i^1 and v_i^2 are incident with same and only one edge e_i.Then $f^*(v_i^1)=f^*(v_i^2)=f(e_i)$.That is each vertex pairs (v_i^1,v_i^2) have same vertex labels by f^*. Hence f^* is not injective.

Therefore the graph mK_2 is not an adjacent edge graceful graph.

Theorem 2.8. The graph mK_3 is an adjacent edge graceful graph.

Proof: Let the vertex set of mK_3 be $V=V_1\cup V_2\cup.....\cup V_m$ where $V_i=\{v_i^1,v_i^2,v_i^3\}$.

Let $E(mK_3)=\{e_i=v_i^1v_i^2, e_{m+i}=v_i^2v_i^3, e_{2m+i}=v_i^1v_i^3 : 1\le i\le m\}$.

Define $f:E(mK_3)\to\{1,2,3,......,3m\}$ by $f(e_i)=3i-2$ if $1\le i\le m$; $f(e_{m+i})=3i-1$ if $1\le i\le m$; $f(e_{2m+i})=3i$ if $1\le i\le m$.

Let f^* be the induced vertex labeling of f.

The induced vertex labels are as follows: $f^*(v_i^1)=12i-4$ if $1\le i\le m$; $f^*(v_i^2)=12i-3$ if $1\le i\le m$; $f^*(v_i^3)=12i-5$ if $1\le i\le m$.

Example 2.9. An adjacent edge graceful labeling of $5K_3$ is shown in the Fig. 3.

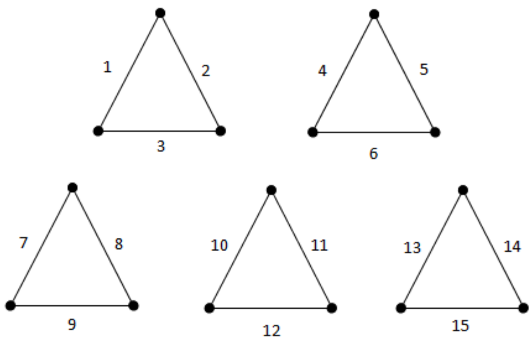

Figure 3:

Theorem 2.10. The graph $B_{n.n}{}^{2}$ is not an adjacent edge graceful graph.

Proof: Let $V(B_{n.n}{}^{2})=\{u_i\ ,\ v_i\ :\ 1\le i\le n+1\}$.

Let $E(B_{n.n}{}^{2})=\begin{cases} e_i=u_iu_{n+1}\ ,\ e_{n+i}=v_iv_{n+1} & :\ 1\le i\le n \\ e_{2n+i}=u_iv_{n+1}\ ,\ e_{3n+i}=v_iu_{n+1} & :\ 1\le i\le n\ \ ;\ e_{4n+1}=u_{n+1}v_{n+1} \end{cases}$

Let f be a bijection from $E(B_{n,n}{}^{2})$ to $\{1,2,3,......,4n+1\}$.

Let $\alpha_1,\alpha_2,....,\alpha_n,\alpha_{n+1},....,\alpha_{2n},\alpha_{2n+1},....,\alpha_{3n},\alpha_{3n+1},...,\alpha_{4n},\alpha$ be the label of edges of $e_1,e_2,....,e_n,e_{n+1},....,e_{2n},e_{2n+1},....,e_{3n},e_{3n+1},...,e_{4n+1}$ respectively by f. And f induces that $f^*:V(B_{n,n}{}^{2})\to\{1,2,3,....\}$ by $f^*(u)=\sum_i f(e_i)$ over all edges e_i incident to adjacent vertices of u. Then $f^*(u_i)=\sum_{i=1}^{4n} f(e_i)+2f(e_{4n+1})=$ $1+2+3+.........+(2n+1)+\alpha\ =\ (2n+1)(4n+1)+\alpha$ for $1\le i\le n$. That is every vertex u_i have same vertex labels by f^*. Hence f^* is not injective. Therefore the graph $B_{n.n}{}^{2}$ is not an adjacent edge graceful graph.

Theorem 2.11. The sunflower graph $SF(n)$ (n *is odd*) is an adjacent edge graceful graph.

Proof: Let $V(SF(n))=\{v_i\ ,\ w_i\ :1\le i\le n\}$. Take $v_{n+1}=v_1$.

Let $E(SF(n))=\{e_i=v_i\,v_{i+1}\ ,\ e_{n+i}=v_iw_i\ ,\ e_{2n+i}=v_{i+1}w_i\ :1\le i\le n\}$.

Define $f:E(SF(n))\to\{1,2,3,......,3n\}$ by $f(e_i)=i$ *if* $1\le i\le 3n$;

Let f^* be the induced vertex labeling of f. The induced vertex labels are as follows:

$f^*(v_1)=18n+6$; $f^*(v_2)=14n+18$; $f^*(v_n)=22n-6$;

$f^*(v_{i+2})=12n+18+12i$ *if* $1\le i\le n-3$; $f^*(w_1)=8n+8$;

$f^*(w_n)=12n$; $f^*(w_{i+1})=6n+8+8i$ *if* $1\le i\le n-2$.

Example 2.12. An adjacent edge graceful labeling of $SF(7)$ is shown in the Fig. 4.

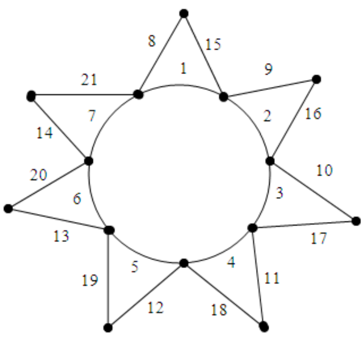

Figure 4:

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