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Variance of Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits Having Correlated Inter-Decision Times

***A. Devi*[1] and *A. Srinivasan*[2]**

[1]PG & Research Department of Mathematics, Bishop Heber College
Tiruchirappalli-620017, Tamil Nadu, India. E-mail: devi150187@gmail.com
[2]PG & Research Department of Mathematics, Bishop Heber College
Tiruchirappalli-620017, Tamil Nadu, India. E-mail: mathsrinivas@gmail.com

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Abstract. In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment for a single grade manpower system in which attrition takes places due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times are exchangeable and constantly correlated exponential random variables and inter- exit times form an ordinary renewal process. The analytical results are numerical illustrated and the effect of the nodal parameters on the performance measure is studied.

***Keywords*:** Single grade manpower system; decision and exit epochs; correlated inter-decision times; univariate policy of recruitment; ordinary renewal process; variance of the time to recruitment

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1. Introduction

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1, 2] the authors have discussed manpower planning models using different kinds of wastage and different types of distributions by Markovian and renewal theoretic approach. In [17] this problem is studied for a single grade manpower system using a univariate recruitment policy based on shock model approach for replacement of systems in reliability theory. In [10] the author has considered a single grade manpower system and obtained system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequence by employing a univariate cum policy of recruitment. The optimum cost of recruitment for a single grade manpower system is obtained in [19] by using several univariate and bivariate policies of recruitment. For the study of this problem corresponding to correlated inter-decision times under different policies of recruitment, one can refer to [9, 12, 13, 14, 15, 21]. In [5], the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. Variance of the time to recruitment for a single grade manpower system with optional and mandatory thresholds is obtained in [20] when inter-decision times form on order statistics. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. This aspect is taken into account for the first time in [3] and the variance of the time to recruitment is obtained when the inter-decision times and exit times are independent and non-identically distributed exponential random variables using a univariate policy for recruitment and Laplace transform in the analysis. This problem is studied in [4] by using a different probabilistic analysis. The present paper extends the research work in [3] for exchangeable and constantly correlated exponential inter-decision times.

2. Model description

Consider an organization taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) due to i^{th} exit point with probability distribution function M(.), density function m(.) and mean $\frac{1}{\alpha}$ ($\alpha>0$). Let A_i the time between $(i-1)^{th}$ and i^{th} exits, be exchangeable and constantly correlated exponential random variables with probability distribution F(.), density function f(.) and mean u. Let R be the correlation between A_i and A_j, $i\neq j$ and $v=u(1-R)$. Let B_i, a continuous random variable representing the time between the $(i-1)^{th}$ and i^{th} inter-exit times be independent and identically distributed random variables with probability distribution function G(.), density function g(.) and mean $\frac{1}{\delta}$ ($\delta>0$). Let D_{i+1} be the waiting time upto (i+1) exits. Let Y be the independent threshold level for the depletion of manpower in the organization with probability distribution function H(.) and density function h(.). Let q be the probability that every policy decision has exit of personnel. As q=0 corresponds to the case where exits are impossible, it is assumed that $q\neq 0$. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T). The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the threshold for the loss of man hours in this organization.

3. Main result

In this section the distribution and the variance of time to recruitment are obtained. By the recruitment policy, recruitment is done whenever the cumulative loss of manpower exceeds the threshold Y. When the first decision is taken, recruitment would not have been done for B_1 units of time. If the loss of manpower $X_1(=S_1)$ due to the first policy of decision is greater than Y, then the recruitment is done and in this case $T=B_1=D_1$. However, if $S_1 \leq Y$, the non-recruitment period will continue till the next policy decision is taken. If the cumulative sum S_2 of the loss of manpower the first two decision exceeds Y, then recruitment is done and $T=B_1+B_2=D_2$. If $S_2 \leq Y$, then the non-recruitment period will continue till the next policy decision is taken and depending on $S_3>Y$ or $S_3 \leq Y$, recruitment is done or the non-recruitment period continues and so on. Hence

$$T = \sum_{i=0}^{\infty} D_{i+1}\, \chi(S_i \leq Y < S_{i+1}) \quad (1)$$

From (1) and from the definition of $D_{i+1,}$ we get

$$E(T) = \sum_{i=0}^{\infty} (i+1)\, E(B)\, P(S_i \leq Y < S_{i+1}) \quad (2)$$

By using law of total probability we get

$$P(S_i \leq Y < S_{i+1}) = \int_0^{\infty}\int_0^{t} \overline{M}(t-x)\, m_i(x)\, h(t)\, dx\, dt \quad (3)$$

Case (i) 3.1. $H(x) = 1-e^{-\theta x}$

In this case from (2) and (3) and from [6] we get on simplification that

$$E(T)=\frac{v(\alpha+\theta)}{(1-R)\theta q} \quad (4)$$

and

$$V(T)=\frac{2(\alpha+\theta)[(1-R)^2\theta+v^2(R^2\,\overline{q}\theta+\alpha)]-v^2(\alpha+\theta)^2}{(1-R)^2(\theta q)^2} \quad (5)$$

(4) and (5) give the mean and variance of the time to recruitment for case(i).

Case (ii) 3.2. $H(x) = [1-e^{-\theta x}]^2$ which is the extended exponential distribution with scale parameter θ and shape parameter two [7].

In this case from (2) and (3) and from [6] we get on simplification that

$$E(T)=\left(\frac{v}{(1-R)q}\right)\left(\frac{3\alpha+2\theta}{2\lambda\theta q}\right) \quad (6)$$

and

$$V(T)=\frac{4\theta[(1-R)^2+v^2R^2\overline{q}](3\alpha+2\theta)+v^2(5\alpha^2-4\theta^2)}{4(1-R)^2q^2\theta^2} \quad (7)$$

(6) and (7) give the mean and variance of the time to recruitment for case(ii)

Case (iii) 3.3. $H(x) = p_1 e^{-(\theta_1+\mu)x} + p_2 e^{-\theta_2 x}$, which is the distribution function with SCBZ property [7], where $p_1 = \frac{\theta_1-\theta_2}{\mu+\theta_1-\theta_2}$ *and* $p_2 = 1-p_1$

In this case from (2) and (3) and from [6] we get on simplification that

$$E(T) = \frac{p_1 \mathrm{v}(\alpha+\theta_1+\mu)\theta_2 + p_2 \mathrm{v}(\theta_1+\mu)(\alpha+\theta_2)}{(1-\mathrm{R})\mathrm{q}(\theta_1+\mu)\theta_2} \tag{8}$$

and

$$V(\mathrm{T}) = \frac{[2\{[(1-\mathrm{R})^2+\mathrm{v}^2\mathrm{R}^2\bar{q}]\theta_2+\alpha \mathrm{v}^2\}-\mathrm{p}_2\mathrm{v}^2(\alpha+\theta_2)-2p_1p_2v^2\theta_2(\alpha+\theta_1+\mu)(\theta_1+\mu)(\alpha+\theta_2)}{(1-R)^2(\mathrm{q}(\theta_1+\mu)\theta_2)^2} \tag{9}$$

(8) and (9) give the mean and variance of the time to recruitment for case(iii).

Note 3.4

i. When R=0, our results agree with the results in [3] for all the three cases.
ii. When q=1, our results for cases (i) and (iii) are consistent with those of [15] and [9] respectively.
iii. When R=0 and q=1, our results for cases (ii) and (iii) are consistent with those of [10] and [18] respectively.

4. Numerical illustration

The mean and variance of time to recruitment for all the three cases are numerically illustrated by varying one parameter and keeping all the other parameters fixed. The effect of the nodal parameters on the mean and variance of time to recruitment is shown in a table. In all the computations we take θ=0.01 and v=1.

α	Q	R	Case (i)		Case (ii)		Case (iii)	
			E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.5000	0.5	0.5	44.0000	1.7160×10^3	64.0000	2.1760×10^3	11.2727	5.6364×10^3
0.2500	0.5	0.5	24.0000	0.4560×10^3	34.0000	0.5860×10^3	7.6364	3.2236×10^3
0.1667	0.5	0.5	17.3333	0.2138×10^3	24.0000	0.2782×10^3	6.4242	2.5681×10^3
0.2	0.5000	0.5	20.0000	0.3000×10^3	28.0000	0.0388×10^4	6.9091	0.2821×10^4
0.2	0.2500	0.5	40.0000	1.2400×10^3	56.0000	0.1608×10^4	13.8182	1.1398×10^4
0.2	0.1667	0.5	60.0000	2.8200×10^3	84.0000	0.3660×10^4	20.7273	2.5728×10^4
0.2	0.5	0.5000	20.0000	300.0000	28.0000	388.0000	6.9091	2.8214×10^3
0.2	0.5	0.2500	13.3333	148.8889	18.6667	194.2222	4.6061	1.3597×10^3
0.2	0.5	0.1667	12.0000	127.2000	16.8000	166.5600	4.1455	1.1411×10^3

Table 1: Effect of the nodal parameters on E(T) and V(T)

5. Findings

From the above table the following observations are presented which agree with reality,

i. When α decreases and keeping all the other parameter fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment decreases for all the three cases.
ii. As q decreases, the mean and variance of time to recruitment increase for all the three cases when the other parameters are fixed.

iii. As R increases the mean and variance of time to recruitment increase for all the three cases when the other parameters are fixed.

6. Conclusion

The model discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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