Annals of Pure and Applied Mathematics
Vol. 7, No. 1, 2014, 1-6
ISSN: 2279-087X (P), 2279-0888(online)
Published on 9 September 2014
www.researchmathsci.org

Annals of
Pure and Applied
Mathematics

Several Similarity Measures of Neutrosophic Soft Sets and its Application in Real Life Problems

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Received 15 July 2014; accepted 12 August 2014

Abstract. Main objective of this paper is to introduce the concept of similarity measures between two neutrosophic soft sets based on set theoretic approach and based on distance between two neutrosophic soft sets and to study some basic properties. A decision making method is established based on similarity measure between two neutrosophic sets. A decision making method is developed based on similarity measure between two neutrosophic soft sets and lastly an example is given to illustrate the proposed decision making method in a medical diagnosis problem.

Keywords: Soft set, neutrosophic set, neutrosophic soft set, Hamming distance, Eulidean distance, similarity measure.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

In 1965 Zadeh introduced the concept of fuzzy set theory [10]. After then several researchers have extended this concept in many directions. The traditional fuzzy sets is characterized by the membership value or the grade of membership value. In some real life problems in expert system, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Intuitionistic fuzzy set [1] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. But it does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache in 2005 introduced the concept of neutrosophic set [8] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Soft set theory [4,6] has enriched its potentiality since its introduction by Molodtsov in 1999. Using the concept of soft set theory Maji in 2013 introduced neutrosophic soft set [7]. Neutrosophic sets and neutrosophic soft sets now become the most useful mathematical tools to deal with the problems which involves the indeterminate and inconsistent informations.

Similarity measure is an important topic in the fuzzy set theory. Similarity measure of fuzzy sets is now being extensively applied in many research fields such as fuzzy clustering, image processing, fuzzy reasoning, fuzzy neural network, pattern recognition, medical diagnosis , game theory, coding theory and several problems that contain uncertainties. Similarity measure of soft sets [5], similarity measure of intuitionistic fuzzy soft sets [3], similarity measure of several fuzzy sets and soft sets have been studied by many researchers. Recently Said Broumi and Florentin Smarandache introduced the concept of several similarity measures of neutrosophic sets [2]. In this paper the Hamming distances and Euclidean distances between two neutrosophic soft sets (NSSs) are defined and similarity measures between two NSSs based on distances are proposed. Similarity measures between two NSSs based on set theoretic approach also proposed. A decision making method is established based on the proposed similarity measures. An illustrative example demonstrates the application of proposed decision making method in a real life problem (medical diagnosis).

The rest of the paper is organized as section 2: some preliminary basic definitions are given in this section. In section 3 Hamming and Euclidean distances between two NSSs are defined and similarity measures based on distances are defined with example. In section 4 similarity measures between two NSS sets based on set theoretic approach is defined with example. In section 5 a decision making method is established with an with an example in a real life problem. Finally in section 6 some remarks of the similarity measures between NSSs and the proposed decision making method are given.

2. Preliminaries

Definition 2.1. [10] Let X be a non empty collection of objects denoted by x. Then a *fuzzy set* (*FS for short*) α in X is a set of ordered pairs having the form $\alpha = \{(x, \mu_\alpha(x)) : x \in X\}$,

where the function $\mu_\alpha : X \to [0,1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in α .The interval M = $[0,1]$ is called membership space.

Definition 2.2. [4,6] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F, A) is called a *soft set* over U , where F is a mapping given by $F : A \to P(U)$.

Definition 2.3. [8] A neutrosophic set A on the universe of discourse X is defined as $A = \{(x, T_A(x), I_A(x), F_A(x)), x \in X\}$ where *T, I, F* : $X \to]^-0,1^+[$ and

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+ .$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1] that is

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3 .$$

Definition 2.4. [7] Let U be the universe set and E be the set of parameters. Also let A ⊆ E and NS(U) be the set of all neutrosophic sets of U. Then the collection (F, A) is called neutrosophic soft set(NSS) over U, where F is a mapping given by F: A $\longrightarrow$ NS(U).

Definition 2.5. [9] Let U be a space of points (objects), with a generic element in U. An interval value neutrosophic set (*IVN-set*) A in U is characterized by truth membership function T_A, a indeterminacy-membership function I_A and a falsity- membership function F_A. For each point u∈ U; T_A, I_A *and* $F_A \subseteq [0,1]$.
Thus a *IVN-set* A over U is represented as

$$A = \{(T_A(u), I_A(u), F_A(u)) : u \in U\}$$

Where $0 \leq \sup(T_A(u) + \sup I_A(u) + \sup F_A(u) \leq 3$ and $(T_A(u), I_A(u), F_A(u))$ is called interval value neutrosophic number for all u ∈ U.

3. Similarity measure between two NSSs based on distances
In this section we define Hamming and Euclidean distances between two neutrosophic soft sets(NSS) and proposed similarity measures based on theses distances.

Definition 3.1. Let $U = \{x_1, x_2, x_3, ..., x_n\}$ be an initial universe and $E = \{e_1, e_2, e_3, ..., x_m\}$ be a set of parameters. Let NS(U) denotes the set of all neutrosophic sets over the universe U. Also let (N_1,E) and (N_2,E) be two neutrosophic soft sets over U, where N_1 and N_2 are mappings given by N_1,N_2: E → NS(U). We define the following distances between (N_1, E) and (N_2, E) as follows:

1. Hamming Distance:

$$L_H(N_1,N_2) = \frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left|T_{N_1}(x_i)(e_j) - T_{N_2}(x_i)(e_j)\right| + \left|I_{N_1}(x_i)(e_j) - I_{N_2}(x_i)(e_j)\right| + \left|F_{N_1}(x_i)(e_j) - F_{N_2}(x_i)(e_j)\right|\right\}$$

2. Normalized Hamming distance:

$$L_{NH}(N_1,N_2) = \frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left|T_{N_1}(x_i)(e_j) - T_{N_2}(x_i)(e_j)\right| + \left|I_{N_1}(x_i)(e_j) - I_{N_2}(x_i)(e_j)\right| + \left|F_{N_1}(x_i)(e_j) - F_{N_2}(x_i)(e_j)\right|\right\}$$

3. Euclidean distance:

$$L_E(N_1,N_2) = \left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left(T_{N_1}(x_i)(e_j) - T_{N_2}(x_i)(e_j)\right)^2 + \left(I_{N_1}(x_i)(e_j) - I_{N_2}(x_i)(e_j)\right)^2 + \left(F_{N_1}(x_i)(e_j) - F_{N_2}(x_i)(e_j)\right)^2\right)\right]^{\frac{1}{2}}$$

4. Normalized Euclidean distance:

$$L_{NE}(N_1,N_2) = \left[\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left(T_{N_1}(x_i)(e_j) - T_{N_2}(x_i)(e_j)\right)^2 + \left(I_{N_1}(x_i)(e_j) - I_{N_2}(x_i)(e_j)\right)^2 + \left(F_{N_1}(x_i)(e_j) - F_{N_2}(x_i)(e_j)\right)^2\right)\right]^{\frac{1}{2}}$$

Definition 3.2. Let U be universe and E be the set of parameters and (N_1,E) , (N_2,E) be two neutrosophic soft sets over U. Then based on the distances defined in definition 3.1 similarity measure between (N_1,E) and (N_2,E) is defined as

$$SM(N_1,N_2) = \frac{1}{1 + L(N_1, N_2)} \quad \text{..} \quad (3.1)$$

Another similarity measure of (N_1,E) and (N_2,E) can also be defined as

$$\text{SM}(\text{N}_1,\text{N}_2) = \text{e}^{-\alpha L(\text{N}_1,\text{N}_2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.2)$$

where $L(N_1,N_2)$ is the distance between the interval valued neutrosophic soft sets (N_1,E) and (N_2,E) and α is a positive real number, called steepness measure.

Example 3.3. Let U={x_1,x_2,x_3} be the universal set and E={e_1,e_2, e_3} be the set of parameters. Let (N_1,E) and (N_2,E) be two neutrosophic soft sets over U such that their tabular representations are as follows:

(N_1,E)	e_1	e_2	e_3
x_1	(0.2,0.4,0.7)	(0.5,0.1,0.3)	(0.4,0.2,0.3)
x_2	(0.7,0.0,0.4)	(0.0,0.4,0.8)	(0.5,0.7,0.3)
x_3	(0.3,0.4,0.3)	(0.6,0.5,0.2)	(0.5,0.7,0.1)

Table 1: Tabular representation of (N_1,E)

(N_2,E)	e_1	e_2	e_3
x_1	(0.3,0.5,0.4)	(0.4,0.3,0.4)	(0.5,0.1,0.2)
x_2	(0.7,0.1,0.5)	(0.2,0.4,0.7)	(0.5,0.6,0.3)
x_3	(0.3,0.3,0.4)	(0.7,0.5,0.2)	(0.6,0.6,0.2)

Table 2: Tabular representation of (N_2,E)

Now by definition **3.1** the Hamming distance between (N_1,E) and (N_2,E) is given by $L_H(N_1,N_2)=0.40$. Therefore by equation **3.1** similarity measure between (N_1,E) and (N_2,E) is $\text{SM}(\text{N}_1,\text{N}_2)=0.714$.

4. Similarity measure between two NSSs based on setheoretic approach

Definition 4.1. Let $S=\{x_1,x_2,x_3,.......,x_n\}$ be the universe and $E=\{e_1,e_2,e_3,.......,x_m\}$ be a set of parameters. Let NS(U) denotes the set of all neutrosophic subsets of S. Also let (N_1,E) and (N_2,E) be two neutrosophic soft sets over S, where N_1 and N_2 are mappings given by N_1, N_2 : E → NS(U). We define similarity measure $SM(N_1,N_2)$ between (N_1,E) and (N_2,E) based on set theoretic approach as follows:

$$SM(N_1,N_2)=\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(T_{N_1}(x_i)(e_j)\wedge T_{N_2}(x_i)(e_j)\right)+\left(I_{N_1}(x_i)(e_j)\wedge I_{N_2}(x_i)(e_j)\right)+\left(F_{N_1}(x_i)(e_j)\wedge F_{N_2}(x_i)(e_j)\right)\right\}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(T_{N_1}(x_i)(e_j)\vee T_{N_2}(x_i)(e_j)\right)+\left(I_{N_1}(x_i)(e_j)\vee I_{N_2}(x_i)(e_j)\right)+\left(F_{N_1}(x_i)(e_j)\vee F_{N_2}(x_i)(e_j)\right)\right\}}$$

Example 4.1. Here we consider example **3.3**. Then by definition **4.1** similarity measure measure between (N_1,E) and (N_2,E) is given by $SM(N_1,N_2)$= 0.798 .

Theorem 4.2. If $SM(N_1,N_2)$ be the similarity measure between two NSSs (N_1,E) and (N_2,E) then

(i) $SM(N_1,N_2) = SM(N_2,N_1)$

(ii) $0\leq SM(N_1,N_2) \leq 1$

(iii) $SM(N_1,N_2) = 1$ if and only if $(N_1,E) = (N_2,E)$

5. Application of similarity measure of NSSs in a medical diagnosis problem

In this section we have developed an algorithm using similarity measure between two neutrosophic soft sets to determine whether an ill person having some visible symptoms suffering from a certain diseases or not. In this method we first construct an ideal NSS for a certain diseases and a NSS for the ill person and we also assume that if the similarity measure between these two NSSs is greater than or equal to **0.6** then the ill person possibly suffering from the diseases. The algorithm is as follows:

Step 1: construct an ideal NSS for illness, which can be done with the help of a medical expert person.
Step 2: construct NSS for ill person(s).
Step 3: calculate distance between the ideal NSS for illness and the NSS for ill person.
Step 4: calculate similarity measure between the ideal NSS for illness and the NSS for ill person.
Step 5: if similarity measure is greater than or equal to **0.6** then the ill person is possibly suffering from the diseases and if similarity measure is less than **0.6** then the ill person is possibly not suffering from the diseases.

Example 5.1. Here we are giving a fictitious example based on above decision making method to illustrate the possible application of similarity measure of NSS in a medical diagnosis problem. In this example our proposed method is applied to determine whether an ill person having some visible symptoms is suffering from cancer or not suffering from cancer.

Let U be the universal set, which contains only two elements x_1 = severe and x_2 = mild i.e. U={x_1,x_2}. Here the set of parameters E is a set of certain visible symptoms. Let E = {e_1,e_2,e_3,e_4,e_5}, where e_1 = headache, e_2 = fatigue, e_3 = nausea and vomiting , e_4 = skin changes, e_5 = weakness.

Step 1: construct an ideal NSS (N,E) for illness (cancer) which can be done with the help of an medical expert.

(N,E)	e_1	e_2	e_3	e_4	e_5
x_1	(0.6,0.2,0.3)	(0.7,0.3,0.4)	(0.4,0.3,0.6)	(0.8,0.2,0.3)	(0.5,0.3,0.2)
x_2	(0.4,0.1,0.2)	(0.3,0.1,0.2)	(0.2,0.2,0.4)	(0.3,0.1,0.4)	(0.2,0.1,0.3)

Table 3: Tabular representation of NSS (N,E) for cancer.

Step 2: construct NSSs for ill persons (patients) X and Y.

(N_1,E)	e_1	e_2	e_3	e_4	e_5
x_1	(0.7,0.3,0.4)	(0.8,0.2,0.5)	(0.4,0.2,0.5)	(0.8,0.1,0.2)	(0.5,0.3,0.2)
x_2	(0.3,0.2,0.3)	(0.2,0.2,0.3)	(0.3,0.1,0.3)	(0.3,0.2,0.3)	(0.1,0.2,0.2)

Table 4: Tabular representation of NSS (N_1,E) for patient X.

(N_2,E)	e_1	e_2	e_3	e_4	e_5
x_1	(0.2,0.5,0.8)	(0.1,0.0,0.8)	(0.8,0.6,0.1)	(0.1,0.5,0.8)	(0.9,0.6,0.8)
x_2	(0.9,0.6,0.7)	(0.7,0.5,0.6)	(0.7,0.6,0.1)	(0.8,0.7,0.9)	(0.8,0.7,0.7)

Table 5: Tabular representation of NSS (N_2,E) for patient Y.

Step 3: Now by definition 3.1 The Hamming distance between (N,E) and (N_1,E) is given by $L_H(N, N_1) = 0.45$ and Hamming distance between (N,E) and (N_2,E) is given by $L_H(N, N_2) = 2.267$.

Step 4: By equation 3.1 similarity measure between (N,E) and (N_1,E) is given by $SM(N, N_1) = 0.69$ and similarity measure between (N,E) and (N_1,E) is given by $SM(N, N_2) = 0.31$.

Also by set theoretic approach we get $SM(N, N_1) = 0.75$ and $SM(N, N_2) = 0.33$.

Step 5: Since $SM(N, N_1) = 0.69 >$ **0.6** therefore patient X possibly suffering from cancer. Again since $SM(N, N_2) = 0.30 <$ **0.6** therefore patient Y possibly not suffering from cancer.

6. Conclusion

In this paper, we have defined several distances between two neutrosophoic soft sets and based on these distances we proposed similarity measure between two neutrosophic soft sets. We also proposed similarity measure between two neutrosophic soft sets based on set theoretic approach. A decision making method based on similarity measure is developed and a numerical example is illustrated to show the possible application of similarity measures between two neutrosophic soft sets in a medical diagnosis problem. Thus we can use the method to solve the problems that contain uncertainty such as problem in social, economic system, medical diagnosis, game theory, coding theory and so on.

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