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Atanassov's Intuitionistic Fuzzy Generalized Bi-ideals of Γ-Semigroups

M.Mandal[1], S.K.Sardar[2] and S.K.Majumder[3]

[1,2]Department of Mathematics, Jadavpur University
Jadavpur, Kolkata – 700032, India
E-mail: manasi_ju@yahoo.in ; sksardarjumath@gmail.com

[3]Mahipal High School, Mahipal, Dakshin Dinajpur, West Bengal-733121, India
E-mail: samitfuzzy@gmail.com

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Abstract. In this paper we introduce the concept of Atanassov's intuitionistic fuzzy generalized bi-ideals of Γ-semigroups in order to extend the concept of Atanassov's intuitionistic fuzzy bi-ideal of a Γ-semigroup. Here we characterize regular Γ-semigroups in terms of Atanassov's intuitionistic fuzzy generalized bi-ideals.

Keywords: Γ-semigroup, Regular Γ-semigroup, Atanassov's intuitionistic fuzzy ideal, fuzzy ideal, fuzzy bi-ideal, fuzzy generalized bi-ideal.

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1. Introduction

Atanassov's intuitionistic fuzzy sets[1,2] are intuitively straightforward extension of Zadeh's[12] fuzzy sets; while a fuzzy set gives the degree of membership of an element in a given set, an Atanassov's intuitionistic fuzzy set gives both a degree of membership and a degree of non-membership. Kuroki[3, 4, 5, 6] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki[3, 4]. In [4], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. The notion of a Γ-semigroup was introduced by Sen and Saha[10] as a generalization of semigroups and ternary semigroups. S.K. Majumder and M. Mandal[7] studied fuzzy generalized bi-ideals in Γ-semigroups. We have initiated the study of Γ-semigroups in terms of Atanassov's intuitionistic fuzzy subsets[8, 9]. The purpose of this paper is as mentioned in the abstract.

2. Preliminaries

Definition 2.1. [1] Let X be a nonempty set. A mapping $A=(\mu_A,\nu_A):X\to I\times I$ is called an intuitionistic fuzzy set in X if $\mu_A(x)+\nu_A(x)\leq 1$ for each $x\in X$, where the mappings $\mu_A:X\to I$ and $\nu_A:X\to I$ denote respectively the degree of membership and the degree of non-membership of each $x\in X$ to *A*, *I* is the unit interval [0,1].

In this paper we shall use the symbol $A=(\mu_A,\nu_A)$ for the intuitionistic fuzzy subset $A=\{<x,\mu_A(x),\nu_A(x)>: x\in X\}$ of X.

Definition 2.2. [10] Let $S=\{x,y,z,\ldots\}$ and $\Gamma=\{\alpha,\beta,\gamma,\ldots\}$ be two non-empty sets. Then S is called a Γ-semigroup if there exists a mapping $S\times\Gamma\times S\to S$(images to be denoted by $a\alpha b$) satisfying (1) $x\gamma y\in S$ $\forall x,y\in S$, $\gamma\in\Gamma$, (2) $(x\beta y)\gamma z=x\beta(y\gamma z)$, $\forall x,y,z\in S$, $\forall\beta,\gamma\in\Gamma$.

Definition 2.3. [8] A non-empty intuitionistic fuzzy subsemigroup $A=(\mu_A,\nu_A)$ of a Γ-semigroup S is called an intuitionistic fuzzy bi-ideal of S if it satisfies:

(1) $\mu_A(x\alpha y\beta z)\geq\min\{\mu_A(x),\mu_A(z)\}$ $\forall x,y,z\in S$ and $\forall\alpha,\beta\in\Gamma$,

(2) $\nu_A(x\alpha y\beta z)\leq\max\{\nu_A(x),\nu_A(z)\}$ $\forall x,y,z\in S$ and $\forall\alpha,\beta\in\Gamma$.

For further preliminaries we refer the readers to [8, 11].

3. Intuitionistic fuzzy generalized bi-ideal

Definition 3.1. [7] Let S be a Γ-semigroup. A non-empty subset I of S is called a generalized bi-ideal of S if $I\Gamma S\Gamma I\subseteq I$.

Definition 3.2. A non-empty intuitionistic fuzzy subset $A=(\mu_A,\nu_A)$ of a Γ-semigroup S is called an intuitionistic fuzzy generalized bi-ideal of S if it satisfies:

(1) $\mu_A(x\alpha y\beta z)\geq\min\{\mu_A(x),\mu_A(z)\}$ $\forall x,y,z\in S$, $\forall\alpha,\beta\in\Gamma$,

(2) $\nu_A(x\alpha y\beta z)\leq\max\{\nu_A(x),\nu_A(z)\}$ $\forall x,y,z\in S$, $\forall\alpha,\beta\in\Gamma$.

Remark 1. It is clear that every intuitionistic fuzzy bi-ideal of S is an intuitionistic fuzzy generalized bi-ideal of S. But in general the converse does not hold which will be clear from the following example. For a restricted converse we refer to Proposition 3.1.

Example 1. Let $S=\{x,y,z,r\}$ and $\Gamma=\{\gamma\}$, where γ is defined on S with the following cayley table:

γ	x	y	z	r
x	x	x	x	x
y	x	x	x	x
z	x	x	y	x
r	x	x	y	y

Then S is a Γ-semigroup. We define an intuitionistic fuzzy subset $A=(\mu_A,\nu_A)$ of S as $\mu_A(x)=0.5$, $\mu_A(y)=0$, $\mu_A(z)=0.2$, $\mu_A(r)=0.$ and $\nu_A(x)=0.4$, $\nu_A(y)=1$,

$\nu_A(z) = 0.7, \nu_A(r) = 1.$ Then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of S but $A = (\mu_A, \nu_A)$ is not an intuitionistic fuzzy bi-ideal of S.

Definition 3.3. [8] For any $t \in [0,1]$ and a fuzzy subset μ of S, the set

$$U(\mu;t) = \{x \in S : \mu(x) \geq t\}(\text{resp.}L(\mu;t) = \{x \in S : \mu(x) \leq t\})$$

is called an upper (resp. lower) t-level cut of μ.

We omit the proofs of the following theorems because it is a matter of routine verification.

Theorem 3.1. Suppose $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of a Γ-semigroup S. Then the upper and lower level cuts $U(\mu_A;t)$ and $L(\mu_A;t)$ are generalized bi-ideals of S, for every $t \in Im(\mu_A) \cap Im(\nu_A)$.

Theorem 3.2. Suppose $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of a Γ-semigroup S such that the sets $U(\mu_A;t)$ and $L(\nu_A;t)$ are generalized bi-ideals of S whenever $t \in [0,1]$ and the sets are nonempty. Then the intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of S.

Theorem 3.3. If a non-empty subset I of a Γ-semigroup S is a generalized bi-ideal of S then (χ_I, χ_I^C) is an intuitionistic fuzzy generalized bi-ideal of S, where χ_I is the characteristic function of I.

Definition 3.4.[10] A Γ-semigroup S is called regular if for each element $x \in S$, there exist $y \in S$ and $\alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x$.

Proposition 3.1. Let S be a regular Γ-semigroup. Then every intuitionistic fuzzy generalized bi-ideal of S is intuitionistic fuzzy bi-ideal of S.

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy generalized bi-ideal of S. Let $a, b \in S$. Since S is regular, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $b = b\alpha x\beta b$. Then for any $\gamma \in \Gamma$,

$\mu_A(a\gamma b) \geq \min\{\mu_A(a), \mu_A(b)\}$ and $\nu_A(a\gamma b) \leq \max\{\nu_A(a), \nu_A(b)\}$.

Hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subsemigroup of S and consequently $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy bi-ideal of S.

Remark 2. In view of above proposition and Remark 1 we can say that in a regular Γ-semigroup the concepts of intuitionistic fuzzy generalized bi-ideal and intuitionistic fuzzy bi-ideal coincide.

Definition 3.5. Let S be a Γ-semigroup. Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two intuitionistic fuzzy subsets of a Γ-semigroup S. Then the product $A\circ B=(\mu_{A\circ B},\nu_{A\circ B})$ of A and B is defined as

$$(\mu_{A\circ B})(x)=\begin{cases}\sup\limits_{x=u\gamma v}[\min\{\mu_A(u),\mu_B(v)\}:u,v\in S;\gamma\in\Gamma]\\ 0, \text{if for any } u,v\in S \text{ and for any } \gamma\in\Gamma, x\neq u\gamma v\end{cases}$$

and

$$(\nu_{A\circ B})(x)=\begin{cases}\inf\limits_{x=u\gamma v}[\max\{\nu_A(u),\nu_B(v)\}:u,v\in S;\gamma\in\Gamma]\\ 1, \text{if for any } u,v\in S \text{ and for any } \gamma\in\Gamma, x\neq u\gamma v\ .\end{cases}$$

Lemma 3.1. Let S be a Γ-semigroup and $A=(\mu_A,\nu_A)$ be a non-empty intuitionistic fuzzy subset of S. Then $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of S if and only if $A\circ S\circ A\subseteq A,$ where $S=(\chi_S,\chi_S^c)$ and χ_S is the characteristic function of S.

Proof: Let $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy generalized bi-ideal of S. Then for all $x,y,p,q\in S$ and for all $\beta,\gamma\in\Gamma,$

$$\mu_A(p\beta q\gamma y)\geq\min\{\mu_A(p),\mu_A(y)\}\quad\text{and}\quad\nu_A(p\beta q\gamma y)\leq\max\{\nu_A(p),\nu_A(y)\}.$$

Hence for $a\in S$ if there exist $x,y\in S,\gamma\in\Gamma$ with $a=x\gamma y$ and $x=p\beta q$ for some $p,q\in S$ and for some $\beta\in\Gamma,$ then $(\mu_A\circ\chi_S\circ\mu_A)(a)\leq\mu_A(a)$ (by Lemma 1[7]) and

$$\begin{aligned}(\nu_A\circ\chi_S^c\circ\nu_A)(a)&=\inf_{a=x\gamma y}[\max\{(\nu_A\circ\chi_S^c)(x),\nu_A(y)\}]\\ &=\inf_{a=x\gamma y}[\max\{\inf_{x=p\beta q}\{\max\{\nu_A(p),\chi_S^c(q)\}\},\nu_A(y)\}]\\ &=\inf_{a=x\gamma y}[\max\{\inf_{x=p\beta q}\{\max\{\nu_A(p),0\}\},\nu_A(y)\}]\\ &=\inf_{a=x\gamma y}[\max\{\nu_A(p),\nu_A(y)\}]\\ &\geq\nu_A(p\beta q\gamma y)=\nu_A(x\gamma y)=\nu_A(a).\end{aligned}$$

If for $a\in S$ no such $x,y,p,q\in S$ and $\gamma,\beta\in\Gamma$ exist then $(\mu_A\circ\chi_S\circ\mu_A)(a)=0\leq\mu_A(a)$ and $(\nu_A\circ\chi_S^c\circ\nu_A)(a)=1\geq\nu_A(a).$ Hence $A\circ S\circ A\subseteq A.$ Conversely, let $A\circ S\circ A\subseteq A.$ Then $\mu_A\circ\chi_S\circ\mu_A\subseteq\mu_A$ and $\nu_A\circ\chi_S^c\circ\nu_A\supseteq\nu_A.$ Hence for $x,y,z\in S,$ and $\beta,\gamma\in\Gamma,$ we deduce by repeated use of Definition 3.5 $\mu_A(x\beta y\gamma z)\geq\min\{\mu_A(x),\mu_A(z)\}$ (by Lemma 1[7]) and

$$\nu_A(x\beta y\gamma z)\leq(\nu_A\circ\chi_S^c\circ\nu_A)(x\beta y\gamma z)\quad\leq[\max\{(\nu_A\circ\chi_S^c)(x\beta y),\nu_A(z)\}]$$
$$\leq\max[\max\{\nu_A(x),0\},\nu_A(z)]\quad=\max\{\nu_A(x),\nu_A(z)\}.$$

Hence $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of S.

In view of the above lemma we obtain the following theorem by routine verification.

Theorem 3.4. The product of any two intuitionistic fuzzy generalized bi-ideals of a Γ-semigroup S is an intuitionistic fuzzy generalized bi-ideal of S.

Theorem 3.5. A Γ-semigroup S is regular if and only if for every intuitionistic fuzzy generalized bi-ideal $A=(\mu_A,\nu_A)$ of S, $A\circ S\circ A=A$ where $S=(\chi_S,\chi_S^c)$.

Proof: Suppose S is regular. Then for an intuitionistic fuzzy generalized bi-ideal $A=(\mu_A,\nu_A)$ of S and $a\in S$, there exist $x\in S$ and $\alpha,\beta\in\Gamma$ such that $a=a\alpha x\beta a$. Hence $\mu_A\circ\chi_S\circ\mu_A=\mu_A$. (by Theorem 3[7])

Again
$$\begin{aligned}(\nu_A\circ\chi_S^c\circ\nu_A)(a)&\le\max\{(\nu_A\circ\chi_S^c)(a\alpha x),\nu_A(a)\}(cf.\text{Definition 3.5})\\&\le\max[\max\{\nu_A(a),\chi_S^c(x)\},\nu_A(a)]\\&=\max\{\nu_A(a),\chi_S^c(x),\nu_A(a)\}\\&=\max\{\nu_A(a),0,\nu_A(a)\}=\nu_A(a).\end{aligned}$$

So $\nu_A\supseteq\nu_A\circ\chi_S^c\circ\nu_A$. By Lemma 3.1 $\nu_A\circ\chi_S^c\circ\nu_A\supseteq\nu_A$. Consequently, $\nu_A\circ\chi_S^c\circ\nu_A=\nu_A$. Hence $A\circ S\circ A=A$.

Conversely suppose the given condition holds. Let R be a generalized bi-ideal of S. Then by Theorem 3.3, (χ_R,χ_R^c) is an intuitionistic fuzzy generalized bi-ideal of S. Hence by given condition $\chi_R\circ\chi_S\circ\chi_R=\chi_R$ and $\chi_R^c\circ\chi_S^c\circ\chi_R^c=\chi_R^c$. Let $a\in R$. Then $\chi_R(a)=1.$ and $\chi_R^c(a)=0$. Hence $\sup\limits_{a=b\gamma c}[\min\{\sup\limits_{b=p\delta q}\chi_R(p),\chi_R(c)\}]=1$. (By Theorem 3[7])

Also
$$\begin{aligned}&(\chi_R^c\circ\chi_S^c\circ\chi_R^c)(a)=0\\&i.e.,\inf_{a=b\gamma c}[\max\{(\chi_R^c\circ\chi_S^c)(b),\chi_R^c(c)\}]=0\\&i.e.,\inf_{a=b\gamma c}[\max\{\inf_{b=p\delta q}\max\{\chi_R^c(p),\chi_S^c(q)\},\chi_R^c(c)\}]=0\\&i.e.,\inf_{a=b\gamma c}[\max\{\inf_{b=p\delta q}\max\{\chi_R^c(p),0\},\chi_R^c(c)\}]=0\\&i.e.,\inf_{a=b\gamma c}[\max\{\inf_{b=p\delta q}\chi_R^c(p),\chi_R^c(c)\}]=0.\end{aligned}$$

Thus we get $p,c\in S$ such that $a=b\gamma c$ and $b=p\delta q$ with $\chi_R(p)=\chi_R(c)=1$ and $\chi_R^c(p)=\chi_R^c(c)=0$ whence $p,c\in R$. So $a=b\gamma c=p\delta q\gamma c\in R\Gamma S\Gamma R$. Consequently, $R\subseteq R\Gamma S\Gamma R$. Since R is a generalized bi-ideal of S so $R\Gamma S\Gamma R\subseteq R$. Hence $R=R\Gamma S\Gamma R$ and so S is regular.

Using Lemma 3.1, Theorem 3.16[8] and Theorem 3.5 we can have the following theorem.

Theorem 3.6. A Γ-semigroup S is regular if and only if for each intuitionistic fuzzy generalized bi-ideal $A=(\mu_A,\nu_A)$ of S and each intuitionistic fuzzy ideal $B=(\mu_B,\nu_B)$ of S, $A\cap B = A\circ B\circ A$.

To conclude the paper we obtain the following result that characterizes regular Γ-semigroups in terms of intutionistic fuzzy generalized bi-ideals.

Theorem 3.7. Let S be a Γ-semigroup. then the following are equivalent:

(1) S is regular,

(2) $A\cap B\subseteq A\circ B$ for each intuitionistic fuzzy bi-ideal $A=(\mu_A,\nu_A)$ of S and for each intuitionistic fuzzy left ideal $B=(\mu_B,\nu_B)$ of S,

(3) $A\cap B\subseteq A\circ B$ for each intuitionistic fuzzy generalized bi-ideal $A=(\mu_A,\nu_A)$ of S and for each intuitionistic fuzzy left ideal $B=(\mu_B,\nu_B)$ of S,

(4) $C\cap A\cap B\subseteq C\circ A\circ B$ for each intuitionistic fuzzy bi-ideal $A=(\mu_A,\nu_A)$ of S, for each intuitionistic fuzzy left ideal $B=(\mu_B,\nu_B)$ of S, and for each intuitionistic fuzzy right ideal $C=(\mu_C,\nu_C)$ of S,

(5) $C\cap A\cap B\subseteq C\circ A\circ B$ for each intuitionistic fuzzy generalized bi-ideal $A=(\mu_A,\nu_A)$ of S, for each intuitionistic fuzzy left ideal $B=(\mu_B,\nu_B)$ of S, and for each intuitionistic fuzzy right ideal $C=(\mu_C,\nu_C)$ of S.

Proof: $(1)\Rightarrow(2)$: Let S be regular, $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy bi-ideal of S and $B=(\mu_B,\nu_B)$ be an intuitionistic fuzzy left ideal of S. Let $a\in S$. Then there exist $x\in S$ and $\alpha,\beta\in\Gamma$ such that $a=a\alpha x\beta a=a\alpha x\beta a\alpha x\beta a.$ Then $\mu_A\circ\mu_B\supseteq\mu_A\cap\mu_B$ (cf. Theorem 6[7]). Again since A is a intuitionistic fuzzy bi-ideal and B is a intuitionistic fuzzy left ideal,

$(\nu_A\circ\nu_B)(a)=\inf_{a=y\rho z}[\max\{\nu_A(y),\nu_B(z)\}]$

$\leq\max\{\nu_A(a\alpha x\beta a),\nu_B(x\beta a)\}(\text{as } a=a\alpha x\beta a\alpha x\beta a)$

$\leq\max\{\nu_A(a),\nu_B(a)\}=(\nu_A\cup\nu_B)(a).$

So $\nu_A\circ\nu_B\subseteq\nu_A\cup\nu_B$. Hence $A\cap B\subseteq A\circ B.$

Similarly we can prove that (1) implies (3).

$(2)\Rightarrow(1)$: Let (2) hold. Let A be an intuitionistic fuzzy right ideal and B be an intuitionistic fuzzy left ideal of S. Then since every intuitionistic fuzzy right ideal of S is intuitionistic fuzzy quasi ideal of S and every intuitionistic fuzzy quasi ideal of S is intuitionistic fuzzy bi-ideal of S, so A is an intuitionistic fuzzy bi-ideal of S. Hence by (2), $A\cap B\subseteq A\circ B.$ Also $A\circ B\subseteq A\cap B$ always holds. Hence $A\circ B=A\cap B$ and consequently, by Theorem 3.20 [8], S is regular.

$(3)\Rightarrow(1)$: Suppose (3) holds. Let T be a generalized bi-ideal of S, L be a left ideal of S and $a\in T\cap L.$ Then $a\in T$ and $a\in L.$ Since T is a generalized bi-ideal

of S, so by Theorem 3.3, (χ_T, χ_T^c) is an intuitionistic fuzzy generalized bi-ideal of S. By Corollary 3.13 [8], (χ_L, χ_L^c) is an intuitionistic fuzzy left ideal of S. Hence by (3), $\chi_T \cap \chi_L \subseteq \chi_T \circ \chi_L$ and $\chi_T^c \cup \chi_L^c \supseteq \chi_T^c \circ \chi_L^c$. Then
$(\chi_T \circ \chi_L)(a) \geq (\chi_T \cap \chi_L)(a) = \min\{\chi_T(a), \chi_L(a)\} = 1.$
and $(\chi_T^c \circ \chi_L^c)(a) \leq (\chi_T^c \cup \chi_L^c)(a) = \max\{\chi_T^c(a), \chi_L^c(a)\} = 0.$
Hence $\chi_{T\circ L}(a) = 1$ and $\chi_{T\circ L}^c(a) = 0.$

Hence in view of Definition 3.5, there exist $b, c \in S$ and $\delta \in \Gamma$ such that $a = b\delta c$ and $\chi_T(b) = \chi_L(c) = 1$ and $\chi_T^c(b) = \chi_L^c(c) = 0$, whence, $b \in T$ and $c \in L$. Hence $a = b\delta c \in T\Gamma L$. Thus $T \cap L \subseteq T\Gamma L$. Hence by Theorem 5[7]. S is regular.

$(1) \Rightarrow (4)$: Let S be regular. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy bi-ideal, $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy left ideal and $C = (\mu_C, \nu_C)$ be an intuitionistic fuzzy right ideal of S respectively. Let $a \in S$. Then there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a = a\alpha x\beta a\alpha x\beta a = a\alpha x\beta a\alpha x\beta a\alpha x\beta a$. Then $\mu_C \cap \mu_A \cap \mu_B \subseteq \mu_C \circ \mu_A \circ \mu_B$ (*cf.* Theorem 6[7]). Again

$$(\nu_C \circ \nu_A \circ \nu_B)(a) \leq \max\{\nu_C(a\alpha x), (\nu_A \circ \nu_B)(a\alpha x\beta a\alpha x\beta a)\}$$
$$\leq \max\{\nu_C(a), (\nu_A \circ \nu_B)(a\alpha x\beta a\alpha x\beta a)\}$$

(since C is an intuitionistic fuzzy right ideal of S)

$$\leq \max[\nu_C(a), \max\{\nu_A(a\alpha x\beta a), \nu_B(x\beta a)\}]$$
$$\leq \max[\nu_C(a), \max\{\nu_A(a), \nu_B(a)\}]$$

(since A is an intuitionistic fuzzy bi-ideal of S and B is an intuitionistic fuzzy left ideal)

$$\leq \max\{\nu_C(a), \nu_A(a), \nu_B(a)\} = (\nu_C \cup \nu_A \cup \nu_B)(a).$$

Hence $\nu_C \cup \nu_A \cup \nu_B \supseteq \nu_C \circ \nu_A \circ \nu_B$. Hence $C \cap A \cap B \subseteq C \circ A \circ B$.

Similarly we can prove that (1) implies (5).

$(4) \Rightarrow (1)$: Let (4) hold. Let $B = (\mu_B, \nu_B)$ and $C = (\mu_C, \nu_C)$ be any intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal of S. Since $S = (\chi_S, \chi_S^c)$ itself is an intuitionistic fuzzy bi-ideal of S, by (4), we have $C \cap B = C \cap S \cap B \subseteq C \circ S \circ B \subseteq C \circ B$. Also $C \circ B \subseteq C \cap B$. Therefore $C \circ B = C \cap B$. Hence by Theorem 3.20 [8], S is regular.

$(5) \Rightarrow (1)$: Suppose (5) holds. Let T be a generalized bi-ideal of S, L be a left ideal of S, R be a right ideal of S and $a \in R \cap T \cap L$. Then $a \in R$, $a \in A$ and $a \in L$. Since T is a generalized bi-ideal of S, so by Theorem 3.3, (χ_T, χ_T^c) is an intuitionistic fuzzy generalized bi-ideal of S, by Theorem 3.13 [8], (χ_L, χ_L^c) is an intuitionistic fuzzy left ideal of S and (χ_R, χ_R^c) is an intuitionistic fuzzy right ideal of

S. Hence by(5), $\chi_R \cap \chi_A \cap \chi_L \subseteq \chi_R \circ \chi_A \circ \chi_L$ and $\chi_R^c \cup \chi_A^c \cup \chi_L^c \supseteq \chi_R^c \circ \chi_A^c \circ \chi_L^c$
Then $(\chi_R \circ \chi_T \circ \chi_L)(a) \geq (\chi_R \cap \chi_T \cap \chi_L)(a) = \min\{\chi_R(a), \chi_T(a), \chi_L(a)\} = 1.$
and $(\chi_R^c \circ \chi_T^c \circ \chi_L^c)(a) \leq (\chi_R^c \cup \chi_T^c \cup \chi_L^c)(a) = \max\{\chi_R^c(a), \chi_T^c(a), \chi_L^c(a)\} = 0.$
Hence $\chi_{(R\circ T)\circ L}(a) = 1$ and $\chi^c_{(R\circ T)\circ L}(a) = 0.$

Hence in view of Definition 3.5, there exist $b, c \in S$ and $\delta \in \Gamma$ such that $a = b\delta c$ and $(\chi_R \circ \chi_T)(b) = \chi_L(c) = 1$ and $(\chi_R^c \circ \chi_T^c)(b) = \chi_L^c(c) = 0$. Hence by applying similar argument as above we see that there exist $d, e \in S$ and $\theta \in \Gamma$ such that $b = d\theta e$ and $\chi_R(d) = \chi_T(e) = 1$ and $\chi_R^c(d) = \chi_T^c(e) = 0$. Thus $c \in L$, $d \in R$ and $e \in T$, with $a = b\delta c = d\theta e\delta c \in R\Gamma T\Gamma L$. Hence $R \cap T \cap L \subseteq R\Gamma T\Gamma L$. Consequently, by Theorem 5 [7], S is regular.

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