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Medical Diagnosis Based on Interval Valued Fuzzy Number Matrices

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Abstract. A major task of the medical science is to diagnose diseases. Hence medical diagnosis is an art for determining a person's pathological conditions of the body, from all the available symptoms. In real world situations, the representation of the uncertain and imprecise knowledge of medical documentation, gives rise to fuzzy environment. Recently, there are varieties of models of medical diagnosis, under the general framework of fuzzy set theory, involving fuzzy matrices to deal with the different complicating aspects of medical diagnosis. In this paper, we have used the interval valued fuzzy number matrices to represent the medical knowledge between the symptoms/diseases and patients/symptoms. Two different mathematical procedures are presented in modeling the diagnostic process, which helps to draw different types of diagnostic conclusions. Finally, examples are illustrated with the simulation result, to verify the proposed approach in detail. Comparisons are made with the existing earlier results, which reveal that, the method presented in this paper is an alternative way to diagnose the diseases.

***Keywords*:** Medical diagnosis, interval valued fuzzy number matrices, membership function, fuzzy model, network, decision maker

AMS Mathematics Subject Classification (2010): 03B52, 92C50, 91B06.

1. Introduction

Most of our real life problems in medical sciences often involve data which are not necessarily crisp, precise and deterministic in character due to various uncertainties associated with these problems. Uncertainty arises due to several factors. For example, the physician understands the medical status of the patient from his past history, physical examination and laboratory test results etc. The past history told by the patient may be incomplete; the laboratory test results need not be accurate. Thus the state and symptoms of the patients can be viewed by the physician only with a limited degree of accuracy. Such uncertainties are usually being handled with the help of fuzzy numbers and fuzzy matrices. The applications of fuzzy numbers and fuzzy matrices in medicine include medical diagnosis and fuzzy decision making for determining appropriate treatments for the respective diseases. There are varieties of models involving fuzzy matrices to deal with different complicated aspects of medical diagnosis. Sanchez formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases [6, 9, 10]. Esogbue and Elder [5] utilized fuzzy cluster analysis to model medical diagnostic. Meenakshi and Kaliraja [7] have extended Sanchez's approach for medical diagnosis using the representation of an interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it. Baruah [1,2] used the definition of complement of a fuzzy soft set proposed by Neog and Sut [8] and put forward a matrix representation of fuzzy soft set and extended Sanchez's approach for medical diagnosis. Edward Samuel and Balamurugan [3] studied Sanchez's approach for medical diagnosis using Intuitionistic fuzzy set. Elizabeth and Sujatha [4] presented the procedures for fuzzy medical diagnosis and for fuzzy decision model based on triangular fuzzy number matrices. Thus numerous papers have been published under fuzzy medical diagnosis problem.

2. Pre-requisites

Definition 2.1. Interval valued fuzzy number
Interval valued fuzzy number is denoted as $A = [a_1, a_3], a_1, a_3 \in \Re, a_1 < a_3$.

Definition 2.2. Interval valued fuzzy number matrix
Interval valued fuzzy number matrix of order m x n is defined as $A = (a_{ij})_{m\times n}$ where $a_{ij} = [a_{ijL}, a_{ijU}]$ (interval valued fuzzy number) is the ij^{th} element of A.

Definition 2.3. New membership function of interval valued fuzzy number
Membership function of $a_{ij} = [a_{ijL}, a_{ijU}]$ is defined as

$$\mu_{a_{ij}} = \left[\frac{a_{ijL}}{10}, \frac{a_{ijU}}{10}\right], if\, 0 \le a_{ijL} \le a_{ijU} \le 10 \quad \text{where} \quad 0 \le \frac{a_{ijL}}{10} \le \frac{a_{ijU}}{10} \le 1.$$

Definition 2.4. Defuzzification formula of interval valued fuzzy number
The defuzzification value t of the interval valued fuzzy number A = [a, b] is calculated as follows: For $a < t < b,\ (t-a)(1) = (b-t)(1) \Rightarrow t - a = b - t \Rightarrow 2t = a + b \Rightarrow t = \frac{a+b}{2}.$ This is the Arithmetic Mean of (A) denoted by AM(A).
The same condition holds for the membership function of interval valued fuzzy numbers.

Definition 2.5. Arithmetic Operation on interval valued fuzzy number matrices
Let $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ be two interval valued fuzzy number matrices of same order. Then
(i) Addition Operation: $A(+)B = (a_{ij} + b_{ij})_{n\times n}$ where $a_{ij} + b_{ij} = [a_{ijL} + b_{ijL}, a_{ijU} + b_{ijU}]$ is the ij^{th} element of $A(+)B$
(ii) Subtraction Operation: $A(-)B = (a_{ij} - b_{ij})_{n\times n}$ where $a_{ij} - b_{ij} = [a_{ijL} - b_{ijU}, a_{ijU} - b_{ijL}]$ is the ij^{th} element of $A(-)B.$

The same condition holds for interval valued fuzzy membership number.

Definition 2.6. Max – Min Composition on membership function of interval valued fuzzy numbers

Let F_{mn} denote the set of all m x n matrices over F. Elements of F_{mn} are called as fuzzy membership value matrices. For $A=(a_{ij})\in F_{mp}$ and $B=(b_{ij})\in F_{pn}$ the max- min product $A(\cdot)B=\left(\sup_k[\{\inf\{a_{ik},b_{kj}\}\}\right)\in F_{mn}$.

Definition 2.7. Maximum/Minimum Operation on interval valued fuzzy number

Let $A=(a_{ij})_{n\times n}$ where $a_{ij}=[a_{ijL},a_{ijU}]$ and $B=(b_{ij})_{n\times n}$ where $b_{ij}=[b_{ijL},b_{ijU}]$ be two interval valued fuzzy number matrices of same order. Then the maximum operation on it is given by $L_{\max}=\max(A,B)=(\sup\{a_{ij},b_{ij}\})$ where $\sup\{a_{ij},b_{ij}\}=\left(\sup[a_{ijL},b_{ijL}],\sup[a_{ijU},b_{ijU}]\right)$ is the ij^{th} element of max(A,B). The minimum operation on it is given by $L_{\min}=\min(A,B)=(\inf\{a_{ij},b_{ij}\})$ where $\inf\{a_{ij},b_{ij}\}=\left(\inf[a_{ijL},b_{ijL}],\inf[a_{ijU},b_{ijU}]\right)$ is the ij^{th} element of min(A,B). The same holds for membership function of interval valued fuzzy numbers.

3. Proposed algorithms

Algorithm 3.1.

Step 1: Construct an interval valued fuzzy matrices (F,D) over S and (F_1,S) over P using the available medical documentation. Here F is a mapping given by $F:D\to\tilde{F}(S)$. This interval valued fuzzy matrix gives the relation matrix R_1 called symptom – disease matrix. F_1 is a mapping given by $F_1:S\to\tilde{F}(P)$. This interval valued fuzzy matrix gives the relation matrix Q called patient-symptom matrix.

Step 2: Construct the compliment $(F,D)^C$ gives another relation matrix say R_2 called non-symptom disease matrix that is $R_2=J-R_1$ where $J=[10,10]$.

Step 3: Convert R_1,Q,R_2 to $(R_1)_{mem},(Q)_{mem},(R_2)_{mem}$ using definition 2.3.

Step 4: Compute the following relation matrix to diagnostic the disease

(i) $(R_3)_{mem}=(Q)_{mem}(\cdot)(R_1)_{mem}\in[0,1]$,(ii) $(R_4)_{mem}=(Q)_{mem}(\cdot)(R_2)_{mem}\in[0,1]$,(iii) $(R_5)_{mem}=AM(R_3)_{mem}-AM(R_4)_{mem}\in[-1,1]$

Find the maximum value of each row in a matrix $(R_5)_{mem}$, which gives the strong diagnostic of the disease to the patient.

Example 1. Suppose there are three patients p_1, p_2 and p_3 in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases relating to the above symptoms be viral fever and malaria. Consider the set $S=\{s_1,s_2,s_3,s_4\}$ as universal set where s_1,s_2,s_3 and s_4 represent the symptoms temperature, headache, cough and stomach problem respectively and the set $D=\{d_1,d_2\}$ where d_1 and d_2 represent the parameters viral fever and malaria respectively.

$F(d_1)=[<s_1,[7,10]>,<s_2,[1,4]>,<s_3,[5,6]>,<s_4,[2,4]>],F(d_2)=[<s_1,[6,9]>,<s_2,[4,6]>,<s_3,[3,6]>,<s_4,[8,10]>]$

This gives the relation matrix R_1 called symptom – disease matrix. Again we take $P = \{p_1, p_2, p_3\}$ as the universal set where p_1, p_2 and p_3 represent patients respectively and $S = \{s_1, s_2, s_3, s_4\}$ as the set of parameters. Suppose that

$F_1(s_1) = [< p_1, [6,9] >, < p_2, [3,5] >, < p_3, [6,8] >], F_1(s_2) = [< p_1, [3,5] >, < p_2, [3,7] >, < p_3, [2,6] >]$

$F_1(s_3) = [< p_1, [8,10] >, < p_2, [2,4] >, < p_3, [5,7] >], F_1(s_4) = [< p_1, [6,9] >, < p_2, [3,5] >, < p_3, [2,5] >]$

This gives the relation matrix Q called patient- symptom matrix.

$$R_1 = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}\begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} [7,10] & [6,9] \\ [1,4] & [4,6] \\ [5,6] & [3,6] \\ [2,4] & [8,10] \end{bmatrix} \end{array} \quad Q = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}\begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{bmatrix} [6,9] & [3,5] & [8,10] & [6,9] \\ [3,5] & [3,7] & [2,4] & [3,5] \\ [6,8] & [2,6] & [5,7] & [2,5] \end{bmatrix} \end{array}$$

By, applying algorithm 3.1, we get the following result. The result can also be represented in the form of a graph namely network as shown in Figure1.

$$(R_5)_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}\begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} 0.05 & 0.2 \\ -0.1 & 0 \\ 0.2 & 0.15 \end{bmatrix} \end{array} \begin{array}{c} Row_i' = Maximum \quad of \quad i^{th} \quad row \\ 0.2 \\ 0 \\ 0.2 \end{array}$$

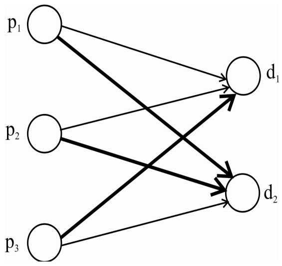


Figure 1: Fuzzy medical diagnosis network

In the above network, nodes or vertices denote the patients and diseases, lengths or edges denote the assumption of diseases to the patients. The darken edges denotes the strong confirmation of disease to the patient. Hence we conclude that patients' p_1 and p_2 are suffering from the disease d_2 and the patient p_3 is suffering from the disease d_1.

Algorithm 3.2.

Step 1: Let S be the set of symptoms, D be the set of diseases and P be the set of patients. The fuzzy occurrence and conformability relations denoted by R_0 and $R_C \in S \times D$ (symptom-disease matrix) and $R_S \in P \times S$ (patient – symptom matrix) are determined from the expert medical documentation. The following grades namely [10, 10], [5, 10], [4, 6], [1, 4] and [0, 0] are assigned for the linguistic terms always, often, unspecific, seldom and never respectively.

Step 2: Construct the compliment of relation matrix R_0, R_C and R_S namely $J - R_0, J - R_C$ and $J - R_S$ respectively where $J = [10,10]$.

Step 3: Convert R_0, R_C, R_S and $J-R_0, J-R_C, J-R_S$ to $(R_0)_{mem}, (R_C)_{mem}, (R_S)_{mem}$, $(J-R_0)_{mem}, (J-R_C)_{mem}$ and $(J-R_S)_{mem}$ using definition 2.3.

Step 4: Compute the following relation matrix to diagnostic the disease and to draw different types of diagnostic conclusions.

(i) $(R_1)_{mem} = (R_S)_{mem}(\cdot)(R_0)_{mem}$, (ii) $(R_2)_{mem} = (R_S)_{mem}(\cdot)(R_C)_{mem}$, (iii) $(R_3)_{mem} = (R_S)_{mem}(\cdot)(J-R_0)_{mem}$,
(vi) $(R_4)_{mem} = (R_S)_{mem}(\cdot)(J-R_C)_{mem}$, (v) $(R_5)_{mem} = (J-R_s)_{mem}(\cdot)(R_o)_{mem}$,
(vi) $(R_6)_{mem} = Max\,((R_1)_{mem},(R_2)_{mem}) \in [0,1]$, (vii) $(R_7)_{mem} = Max\,((R_3)_{mem},(R_4)_{mem},(R_5)_{mem}) \in [0,1]$,
(viii) $(R_8)_{mem} = Min\,((R_3)_{mem},(R_4)_{mem},(R_5)_{mem}) \in [0,1]$, (ix) $(R_9)_{mem} = (R_6)_{mem} - (R_8)_{mem} \in [0,1]$,
(x) $(R_{10})_{mem} = AM\ (R_9)_{mem} \in [-1,1]$, (xi) $(R_{11})_{mem} = AM(R_6)_{mem}$, (xii) $(R_{12})_{mem} = AM(R_7)_{mem}$.

Find the maximum value of each row in a matrix $(R_{10})_{mem}$ which gives the strong diagnostic of the disease to the patient. If $0.9 \le (R_{11})_{mem} \le 1$ then the disease D is strongly possible for the patient P. If $0.9 \le (R_{12})_{mem} \le 1$ then the disease D is not possible for the patient P.

Example 2. Let us consider the set of symptoms fever, joint pain, blood count increase denoted by $S = \{s_1, s_2, s_3\}$ and the set of diseases pneumonia and arthritis denoted by $D = \{d_1, d_2\}$. Let R_0, R_C, R_S be determined from the expert medical documentation.

$$R_0 = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array}\begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} [10,10] & [1,3] \\ [3,5] & [10,10] \\ [8,10] & [1,4] \end{bmatrix}\end{array} \quad R_S = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}\begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{bmatrix} [3,5] & [8,10] & [6,10] \\ [6,8] & [0,0] & [5,7] \\ [6,10] & [6,8] & [10,10] \end{bmatrix}\end{array} \quad R_C = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array}\begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} [6,10] & [0,0] \\ [4,6] & [10,10] \\ [5,10] & [4,6] \end{bmatrix}\end{array}$$

On applying algorithm 3.2, the required result is obtained.

$$(R_{10})_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}\begin{array}{c} \begin{array}{ccc} d_1 & d_2 & Row_i^{'} = Maximum\ \ of\ \ i^{th}\ \ row \end{array} \\ \begin{bmatrix} 0.3 & 0.65 \\ 0.6 & -0.2 \\ 0.6 & 0.4 \end{bmatrix}\begin{array}{c} 0.65 \\ 0.6 \\ 0.6 \end{array}\end{array}$$

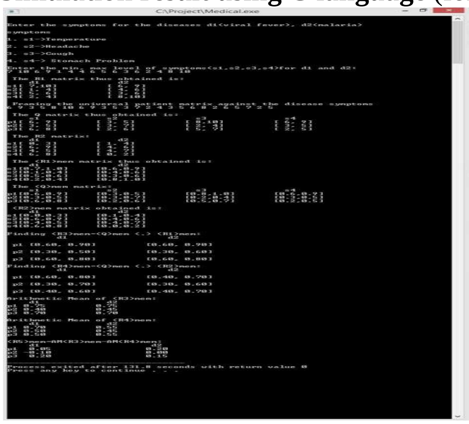
$$(R_{11})_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}\begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} 0.8 & 0.9 \\ 0.7 & 0.5 \\ 0.9 & 0.7 \end{bmatrix}\end{array} \quad (R_{12})_{mem} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}\begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.75 \\ 0.4 & 1 \\ 0.6 & 0.8 \end{bmatrix}\end{array}$$

From the above example we get the following conclusion: (i) Patient p_1 is suffering from the disease d_2 and the patients p_2 and p_3 are suffering from the disease d_1. (ii) Disease d_2 is strongly possible for the patient p_1 and disease d_1 is strongly possible for the patient p_3. (iii) Disease d_2 is not possible for the patient p_2.

4. Results and discussions

One way to verify the solution obtained is to make an exhaustive comparison. The results obtained in illustrative example 1 and 2 coincides with the existing earlier results [6,9,10] and [7] respectively. The comparison reveals that the method developed in this paper is an alternative way to solve the medical diagnosis problem under fuzzy environment.

Simulation result using C language (for illustrative example 1)



5. Conclusion

Medicine is one of the fields in which fuzzy numbers and fuzzy matrices find great applications. In the present, many researchers have focused on the medical diagnosis problem under fuzzy environment and also some case studies were done for the same. In this paper, two different mathematical procedures were developed for fuzzy medical diagnosis problem. The result of the example indicates that it is possible to draw different types of diagnostic conclusions. Hence it can be concluded that the method developed in this paper will be an efficient tool for medical diagnosis and the physicians' decisions.

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