Annals of Pure and Applied Mathematics
Vol. 7, No. 1, 2014, 104-109
ISSN: 2279-087X (P), 2279-0888(online)
Published on 9 September 2014
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Fuzzy Generalized γ-Closed set in Fuzzy Topological Space

Dipankar De

Department of Mathematics, Maharaja Bir Bikram College
Agartala, Tripura, India
E mail: dipankardee@yahoo.co.in

Received 30 July 2014; Accepted 21 August 2014

Abstract. The purpose of the paper is to introduce and study the concepts of fuzzy generalized γ closed sets and study their basic properties in fuzzy topological spaces. Moreover, in this paper we define fuzzy γT½ spaces in which every fuzzy generalized γ continuous is fuzzy γ continuous. In addition, we have also introduced and studied fuzzy generalized γ connectedness and fuzzy generalized γ compactness.

Keywords: Fuzzy generalized closed set, Fuzzy generalized continuous function, Fuzzy generalized compact, Fuzzy generalized γ closed set, Fuzzy generalized γ continuous, Fuzzy $\gamma T_{½}$-space and fuzzy generalized γ connectedness.

***AMS Mathematics Subject Classification* (2010):** 54A50

1. Introduction

A fuzzy topologicl space using the concept of fuzzy sets was introduced by C.L.Chang, Known as Chang's fts in 1968. The concept of fuzzy sets were introduced by Prof. L.A.Zadeh in 1965. In 1981 K.K.Azad investigate the concept of fuzzy semi open sets, fuzzy semi continuous, fuzzy regular sets and its related important properties. In this paper generalization of fuzzy closed sets are based on topological notions which were introduced and studied by Norman Levine,1970. In 1997,G.Balasubramanium and P. Sundaram introduced the concept of fuzzy generalized closed set and $T_{½}$ spaces. Recently M.E.El-Shafei et al, 2007 introduced and study semi generalized continuous functions in fts .

In 1999, Hanafy et al introduced fuzzy γ open sets and fuzzy γ continuity.In 2013, Author study and introduced fuzzy generalized γ continuity in fuzzy topological space.In the present paper, we investigate fuzzy generalized γ connectedness and fuzzy generalized γ compactness in fts. In section 2 and 3 using the concept of generalized fuzzy open (closed) set, we introduce and study the notions of fuzzy generalized γ connectedness and fuzzy generalized γ compactness respectively.

Recall the following definitions of sets which will be used often throughout this paper.

Definition 1.01. [4] A fuzzy subset A of (X,τ) is called fuzzy generalized closed if Cl(A)≤ H, A≤H, H is open set of X.
A fuzzy subset A of (X,τ) is called fuzzy generalized open (in short fgo) iff 1-A is fgc.

Remarks: i) The intersection of two fgc sets is not generally a fgc set and the union of two fgc sets is fgc set.
ii) The union of two fgo ses is not generally fgo, but intersection of any two fgo sets is fgo.

Theorem 1.2. (a) [5] Let (X,τ) be a fuzzy compact (Lindelof,countably compact) space and suppose that λis a fgc set of X. Then λ is fuzzy compact(Lindelof, countably compact.).
(b) If (X,τ) is fuzzy regular [3] and λis fuzzy compact of X , then λ is fgc set.

Definition 1.3. [4] The infimum of all fuzzy generalized closed sets containing the set A is called fuzzy generalized closure of A and is defined by
gcl(A)= inf{B: B≥ A, B is fuzzy generalized closed set}
gint(A) =sup{B: B≤ A, B is fuzzy generalized open set}

Definition 1.4. [10] A fuzzy subset A of fuzzy topological space (X,τ) is said to be fuzzy γ open set (res. Fuzzy γ closed set) if A≤ clintA V intclA (res. A≥clintA Λ intclA)

Remarks i) The union of fγo sets is fγo set, but the intersection of two fγo sets need not fγo.
ii) The intersection of a fuzzy open set which is a crisp subset and a fuzzy γ open set is fuzzy γ open.
iii) The union of fuzzy closed set which is a crisp subset and fuzzy γ closed set is fuzzy γ closed.

Definition 1.5. A function f: X→Y is said to be

(a) [4] Fuzzy generalized continuous iff $f^{-1}(A)$ is fuzzy generalized closed in X , for every fuzzy closed set A of Y.
(b) Fuzzy complete continuous iff $f^{-1}(A)$ is fuzzy regular open (res. frc) of X, for every fuzzy open (res. fc) of A in Y.

Definition 1.6. [8] A fts (X,τ) is said to be fuzzy $T_{½}$ space if every fuzzy generalized closed set in X is fuzzy closed.

Theorem 1.7. [4] If λ is a fg closed set in X and if f:X→ Y is fuzzy continuous and fuzzy closed , then f(λ) is fg closed in Y.

Definition 1.8. [6] A fuzzy subset A of (X,τ) is called fuzzy generalized γ closed set if cl(A)≤ H, A≤ H and H is fuzzy γ open set of X.

A fuzzy subset A of (X,τ) is called fuzzy generalized γ open set(in short fgγo) iff 1-A is fgγc set.

Definition 1.9. [6] A fuzzy topological space (X,τ) is said to be fuzzy γ $T_{½}$ space if every fuzzy generalized γ closed set in X is fuzzy closed.

Theorem 1.10. [6] A fts(X,τ) is fuzzy γ compact iff each open cover of family of fuzzy γ open set of τ has finite sub cover.

Theorem 1.11. [6] Let (X,τ) be a fuzzy γ compact space and suppose that λ is a fgγc set of X. Then λ is fuzzy γ compact.

Theorem 1.12. [6] Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g: (Y,\sigma)\rightarrow(Z,\rho)$ be functions and Y is f$\gamma T_{½}$. If f and g are fgγ continuous, the g.f is fgγ continuous.
The converse of the theorem is not valid if Y is not f$\gamma T_{½}$.

Definition 1.13. [6] A function $f: X\rightarrow Y$ is called fuzzy generalized γ continuous if the inverse of every fuzzy γ closed set in Y is fgγ-closed in X.
If $f: X\rightarrow Y$ is fuzzy γ continuous then it is fgγ-continuous.

2. Fuzzy Generalized γ Connectedness

Definition 2.1. A fuzzy topological space X is said to be fgγ connected iff the only fuzzy space which fgγ-open and fgγ-closed are 0 and 1.

Theorem 2.2. Every fgγ connected space is fuzzy γ connected.
Proof: Let X be a fgγ connected space. If possible,let X be not a fuzzy γ connected. By definition we can write that there exits a proper subset μ such that $\mu= 0,\mu=1$.
And μ is both fγ open and fγ closed.
Also we know that fγ open $\Rightarrow$ fgγ open.
It follows that μ is not fgγ connected set which contradicts our assumption.
$\therefore$ μ is fγ connected.
The converse of the theorem is not true.

The support of the above statement an example are given below;

Example 2.3. Let X = {a,b,c} and $\tau =\{0,1,\lambda\}$
where $\lambda: X\rightarrow[0,1]$ is such that $\lambda(a)=1$, $\lambda(b)=0$, $\lambda(c)= 0$
It can be easily shown that (X,τ) is fγ connected, but it is not fgγ connected.
If we take for,$\eta: X\rightarrow [0,1]$ is such that
$\eta(a)=0,\eta(b)=0$, $\eta(c)=1$ where $\tau=\{0,1,\eta\}$
Then (X,τ) are fγ connected and fgγ connected.

Theorem 2.4. If X is fuzzy T½ space,then X is f γ connected iff X is fgγ connected.

Proof: Let X is $fT_{1/2}$ and f γ connected set. Suppose X is not fgγ-connected. It implies that there exist a proper fuzzy set μ such that μ is both fg open and fg-closed set. Finally we have μ is both fgγ-open and fgγ-closed.
Since X is $fT_{1/2}$ space $\Rightarrow$ μ is both fγ-open and fγ closed.
$\Rightarrow$ X is not fγ connected It contradict our assumption.
$\therefore$ X is fgγ connected.
The converse of the theorem follows directly from theorem 2.02.

Definition 2.5. Let λ is fuzzy set in X and we denote
$g\gamma cl(\lambda) = \vee\{\mu \mid \mu \leq \lambda$ and μ is fgγ-open$\}$ and
$g\gamma int(\lambda) = \wedge\{\mu \mid \mu \geq \lambda$ and μ is fgγ-closed$\}$

Definition 2.6. A fgγ closed set λis called regular fgγ closed if $\lambda = g\gamma cl(g\gamma int(\lambda))$
The fuzzy compliment of regular fgγ closed set is called regular fgγ open.

Definition 2.7. A fuzzy topological space X is called fgγ super connected if there is no proper regular fgγopen set in X.
Generalized fuzzy super connectedness $\Rightarrow$ fuzzy connectedness.

Theorem 2.8. Let X be fts. The following are equivalent:
(i) X is regular fgγ super connected.
(ii) $g\gamma cl(\lambda) = 1$ for every non zero fgγ open set λ.
(iii) $g\gamma int(\nu) = 0$, for every $fg\gamma cl\lambda \neq 1$.
(iv) X does not have non-zero fgγ open set μ and λ such that $\mu + \lambda \leq 1$.
(v) X does not have non zero fuzzy sets λ and μ such that $g\gamma cl\lambda + \mu = \lambda + g\gamma cl\mu = 1$.

3. Fuzzy generalized γ compact spaces

Definition 3.1. A collection $\{\lambda_i\}_{i\in\Gamma}$ of fgγ open sets in X is called fgγ cover of fuzzy sets if $\mu \leq \vee_{i\in\Gamma} \lambda_i$.

Definition 3.2. A fts X is called fgγ compact if every fgγ open cover has finite sub cover.

Definition 3.3. A fuzzy setλin X is said to be fgγ compact relative to X is for every collection $\{\lambda_i\}_{i\in\Gamma}$ of fgγ open sets such that $\lambda \leq \vee_{i\in\Gamma}\lambda_i$,there exists a finite subset Γ_0 of Γ such that $\lambda \leq \vee_{i\in\Gamma_0}\lambda_i$.

Definition 3.4. A fuzzy set λ of X is said to be fgγ compact if it is fgγ compact relative to X.

Theorem 3.5. Let (x,τ) be a fuzzy γ compact and a fuzzy set λ in X is a fgγ closed set. Then λ is fuzzy γ-compact.

Proof. Let $\{\mu_\alpha\}_{\alpha\in\Gamma}$ be a family of fγ open sets in X such that $\lambda\leq \vee_{\alpha\in\Gamma}\mu_\alpha$. Since λ is fgγ closed set and $\vee_{\alpha\in\Gamma}\mu_\alpha$ is fuzzy γ open, it follws that

$Cl(\lambda)\leq\vee_{\alpha\in\Gamma}\mu_\alpha$ But $cl(\lambda)$ is f γ-compact and therefore,

It follows that $\lambda\leq cl(\lambda)\leq\vee_{\alpha\in\Gamma}\mu_{\alpha n}$ for some natural number n.

Theorem 3.6. Let X be a fgγ compact and λ be a fg closed set in X. Then λ is fgγ compact.

Proof. Obvious.

Theorem 3.7. If (x,τ) is fuzzy regular and λ is fγ-compact, then λ is fgγ closed set.

Theorem 3.8. A fg continuous image of a fgγ compact space is fγ compact.

Proof. Let X be a fgγ compact space and let f: $X\rightarrow Y$ a fg continuous bijective mapping. If $\{f_i\}$ is a f γ open cover of Y, then $\vee_i f^{-1}(f_i) = I^X$.

Since f is 1-1 and onto, $i\in I$

We have $\{f^{-1}(f_j)\}_{j\in J}$ is a fuzzy open γ cover of X.

Since X is a fgγ compact, there exist a finite subsets $F\subseteq J$ such that

$\vee_{j\in F}[f^{-1}(f_j)] = I^{X.}$. From continuity of f , We deduce

$I_Y = f(X) = f(\vee_{j\in F}f^{-1}(f_j)) = \vee(f\, f^{-1}(f_j)) \leq \vee\, f\,(f^{-1}(f_j)) \leq\vee_{J\in F}f_j \leq I_Y$

Hence $\vee_{j\in F}f_j = I_Y$

$\Rightarrow$ Y is fγcompact.

Hence Proved.

4. Conclusion

In the present paper we introduced and studied a new concept-fuzzy generalized γ-closed set in fuzzy topological space. This new concept and the characteristic properties of the discussed fuzzy generalized γ-connectedness and fuzzy generalized γ-compactness will play an important part in studying fuzzy generalized γ-closed sets. And also it may be a useful tool for studying the theory of fuzzy generalized γ-regular closed set and some other part of this field.

REFERENCES

1. K.K.Azad, On fuzzy semicontinuity, semi almost continuity and fuzzy weakly continuity, *J.Math. Anal. Appl.*, 82 (1981) 14-32 .
2. K.M .Abd El-Hakeim, Generalized semi-continuous mappings in fuzzy topological spaces, *The Journal of Fuzzy Mathematics*, 7 (1999) 577-589.
3. G.Balsubramanian and E.Roja, On fuzzy β-$T_{½}$ spaces and its generalizations. *Bull. Cal. Math. Soc.*, 94 (6) (2002) 413-420.
4. G.Balsubramanian and P.Sundaram, On some generalization of fuzzy continuous functions, *Fuzzy Sets and Systems*, 86(1997) 93-100.
5. C.L.Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24 (1968) 182-190.
6. D.De, Fuzzy generalized γ closed set, *Rev. Bull. Cal. Math. Soc.*, 21(2) (2013) 171-180.

7. T.Fukutake, H.Harada, M.Kojima and H.Maki, Generalized closed sets in fuzzy topological spaces, *Meeting on Topological Space Theory and Application*,(1998) 23-26.
8. T.Fukutake, H.Harada, M.Kojima, H.Maki and F.Tumari, Degrees and fuzzy generalized closed sets, *The Journal of Fuzzy Mathematics*, 9(1) (2001) 159-172.
9. U.V.Fatteh and D.S. Bassan, Fuzzy connectedness and its stronger forms, *J. Math. Anal. Appl*., 111 (1985) 449-454.
10. I.M.Hanafy, Fuzzy γ-open Sets and Fuzzy γ-continuity, *The Journal of Fuzzy Mathematics*, 7(2) (1999) 419-430.
11. Norman Levine, Generalized closed sets in topology, *Rend Circ. Matem. Palermo*, (1970) 89-96.
12. EL.M.E. Shafei and A.Zakari, Semi-generalized continuous mappings in fuzzy topological spaces, *The Journal of Fuzzy Mathematics*, 15(10) (2007) 109-120.
13. L.A.Zadeh, Fuzzy sets, *Inform. Control* 8 (1965) 338-353.