Annals of Pure and Applied Mathematics
Vol. 7, No. 1, 2014, 7-12
ISSN: 2279-087X (P), 2279-0888(online)
Published on 9 September 2014
www.researchmathsci.org

Annals of Pure and Applied Mathematics

Parameterized Topological Space Induced by an Intuitionistic Fuzzy Soft Multi Topological Space

***A.Mukherjee*[1] and *A.K.Das*[2]**

[1]Department of Mathematics, Tripura University, Agartala-799022, Tripura, India
E-mail: anjan2002_m@yahoo.co.in

[2]Department of Mathematics, ICV- Collegel, Belonia -799155, Tripura, India
E-mail: ajoykantidas@gmail.com

Received 15 July 2014; accepted 20 August 2014

Abstract. Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models.The aim of this paper is to construct intuitionistic fuzzy soft multi topological space and it is shown that an intuitionistic fuzzy soft multi topological space gives a family of parameterized topological spaces and some basic properties regarding with this concepts are investigated. We also introduced the notion of intuitionistic fuzzy soft multi open sets, intuitionistic fuzzy soft multi closed sets, interior and closure of intuitionistic fuzzy soft multisets and studied their basic properties.

***Keywords*:** Intuitionistic fuzzy soft multi set, intuitionistic fuzzy soft multi topological space, parameterized topological space, intuitionistic fuzzy soft multi closed set.

AMS Mathematics Subject Classification (2010): 03E72, 03E02

1. Introduction and preliminaries

Theory of fuzzy sets was initiated by Zadeh [7] in 1965 and thereafter, 1986 Atanassov [3] presented the intuitionistic fuzzy sets as a generalization of fuzzy sets which are very effective to ideal with vagueness & several authors have considered it further.

Theory of soft sets and multisets are important mathematical tool to handle uncertainties. The concept of soft set theory initiated by Molodstov [6] as a mathematical tool for dealing with uncertainties. It has recently received wide attention in real life applications and theoretical research. Combining soft sets with fuzzy sets [7] and intuitionistic fuzzy sets [3], Maji et al. [4,5] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems. Alkhazaleh et al. [1] as a generalization of Molodtsov's soft set, presented the definition of a soft multiset and its basic operations such as complement, union, and intersection etc. In 2012 Alkhazaleh and Salleh [2] introduced the concept of fuzzy soft multi set theory and studied the application of these sets.

In this paper we have introduced IFSM-topological spaces which are defined over an initial universe with a fixed set of parameter. The notion of IFSM- open sets, IFSM-closed sets; interior and closure of an IFSM- set are introduced. Also we introduced the concept of parameterized topological space induced by an IFSMs- topological space and studied some of its properties.

We now give some ready references for further discussion.

Definition 1.1. [7] Let X be a non empty set. Then a *fuzzy set* A is a set having the form A={(x, μ_A(x)): x$\in$X}, where the functions μ_A: X$\to$[0,1] represents the degree of membership of each element x$\in$X.

Definition 1.2. [3] Let U be a non empty set. Then an *intuitionistic fuzzy set* (in short *IFS*) A on U is a set having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\}$, where the functions μ_A:U$\to$[0,1] and ν_A:U$\to$[0,1] represents the degree of membership and the degree of non-membership respectively of each element x$\in$U and $0 \le \mu_A(x)+\nu_A(x) \le 1$ for each x$\in$U.

Definition 1.3. [6] Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A$\subseteq$E. Then the pair (F, A) is called a *soft set* over U, where F is a mapping given by F: A$\to$ P(U).

Definition 1.4. [2] Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_i : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a fuzzy soft multi set over U, where F is a mapping given by F: A $\to$ U .

Definition 1.5. [2] For any fuzzy soft multi set (F, A), a pair $\left(e_{U_i,j}, F_{e_{U_i,j}}\right)$ is called a U_i – fuzzy soft multiset part $\forall e_{U_i,j} \in a_k$ and $F_{e_{U_i,j}} \subseteq F(A)$ is a fuzzy approximate value set, where $a_k \in A$, k$\in$ {1,2,3,..,m} , $i \in$ {1,2,3,..,n} and $j \in$ {1,2,3,..,r} .

Definition 1.6. [2] A fuzzy soft multi set (F, A) over U is called a null fuzzy soft multi set, denoted by $(F, A)_\varphi$, if all the fuzzy soft multi set parts of (F, A) equals ϕ.

Definition 1.7. [2] A fuzzy soft multi set (F, A) over U is called an absolute fuzzy soft multi set, denoted by $(F, A)_U$, if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i$, $\forall i$.

2. Intuitionistic fuzzy soft multi set

Definition 2.1. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_i : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} IFS(U_i)$ where $IFS(U_i)$ denotes the set of all intuitionistic fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called an intuitionistic fuzzy soft multi set (in short IFSM-set) over U, where F is a mapping given by F: A $\to$ U .

Definition 2.2. An IFSM-set (F, A) over U is called an absolute IFSM-set, denoted by $(F, A)_U$, if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i$, $\forall i$.

Definition 2.3. Union of two IFSM-sets (F, *A*) and (G, *B*) over *U* is an IFSM-set (H, D) where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcup(F(e), G(e)), & if\ e \in A \cap B \end{cases}$$

where $\bigcup(F(e), G(e)) = (F_{e_{U_i,j}} \tilde{\cup} G_{e_{U_i,j}})$, $\forall i \in \{1,2,3,..,m\}$, $j \in \{1,2,3,..,n\}$ with $\tilde{\cup}$ as an intuitionistic fuzzy union and is written as $(F, A) \tilde{\cup} (G, B) = (H, D)$.

Definition 2.4. Intersection of two IFSM-sets (F, *A*) and (G, *B*) over *U* is an IFSM-set (H, D) where $D = A \cap B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcap(F(e), G(e)), & if\ e \in A \cap B \end{cases}$$

where $\bigcap(F(e), G(e)) = (F_{e_{U_i,j}} \tilde{\cap} G_{e_{U_i,j}})$, $\forall i \in \{1,2,3,...,m\}$, $j \in \{1,2,3,..,n\}$ with $\tilde{\cap}$ as an intuitionistic fuzzy intersection and is written as $(F, A) \tilde{\cap} (G, B) = (H, D)$.

Definition 2.5. The complement of an IFSM-set (F, A) over U is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to U$ is a mapping given by $F^c(\alpha) = c(F(\alpha))$, $\forall \alpha \in A$, where c is an intuitionistic fuzzy complement.

3. Intuitionistic fuzzy soft multi topological spaces

We consider an absolute IFSM-set $(\mathfrak{F}, A)$ over U and $IFSMs_A(\mathfrak{F}, A)$ denote the family of all intuitionistic fuzzy soft multi sub sets of $(\mathfrak{F}, A)$ in which all the parameter set A are same. Throughout this paper, $(\mathfrak{F}, A)$ refers to an initial universal IFSM-set with fixed parameter set A.

Definition 3.1. A null IFSM-set Φ_A over $(\mathfrak{F}, A)$, is an IFSM-set in which all the IFSM-set parts equals ϕ.

Definition 3.2. A sub family τ of $IFSMs_A(\mathfrak{F}, A)$ is called intuitionistic fuzzy soft multi topology (in short IFSMs-topology) on $(\mathfrak{F}, A)$, if the following axioms are satisfied:

[O_1]. Φ_A, $(\mathfrak{F}, A) \in \tau$,

[O_2]. $\{(F_i, A) : i \in I\} \subseteq \tau \Rightarrow \tilde{\cup}_{i \in I} (F_i, A) \in \tau$,

[O_3]. If $(F, A), (G, A) \in \tau$, then $(F, A) \tilde{\cap} (G, A) \in \tau$.

Then the pair $((\mathfrak{F}, A), \tau)$ is called intuitionistic fuzzy soft multi topological space (in short IFSMs-topological space). The members of τ are called intuitionistic fuzzy soft multi open sets (simply IFSM- open sets) and the conditions [O_1], [O_2] and [O_3] are called the axioms for IFSM- open sets.

Example 3.3. Let us consider there are three universes U_1 and U_2. Let $U_1=\{h_1,h_2,h_3\}$, $U_2=\{c_1,c_2\}$ and $\{E_{U_1},E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1}=\{e_{U_1,1}=\text{exp}\,ensive,\ e_{U_1,2}=cheap,\ e_{U_1,3}=wooden\}$ *and* $E_{U_1}=\{e_{U_2,1}=\text{exp}\,ensive,\ e_{U_2,2}=cheap,\ e_{U_2,3}=sporty\}$. Let $U=\prod_{i=1}^{2}P(U_i)$, $E=\prod_{i=1}^{2}E_{U_i}$ and $A=\{e_1=(e_{U_1,1},e_{U_2,1}),\ e_2=(e_{U_1,1},e_{U_2,2})\}$,

$$\Phi_A=\left\{\left(e_1,\left(\left\{\frac{h_1}{(0,1)},\frac{h_2}{(0,1)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0,1)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(0,1)},\frac{h_2}{(0,1)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0,1)}\right\}\right)\right)\right\},$$

$$(\mathfrak{F},A)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(1,0)},\frac{h_2}{(1,0)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(1,0)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(1,0)},\frac{h_2}{(1,0)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(1,0)}\right\}\right)\right)\right\},$$

$$(F_1,A)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(.2,.7)},\frac{h_2}{(.4,.5)},\frac{h_3}{(.8,.1)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.5,.5)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(.7,.2)},\frac{h_2}{(.7,.2)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.6,.3)}\right\}\right)\right)\right\},$$

$$(F_2,A)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(.3,.6)},\frac{h_2}{(.3,.6)},\frac{h_3}{(.7,.2)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.6,.3)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(.8,.1)},\frac{h_2}{(.9,.1)},\frac{h_3}{1,0)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.8,.1)}\right\}\right)\right)\right\},$$

$$(F_3,A)=(F_1,A)\tilde{\cup}(F_2,A)=$$
$$\left\{\left(e_1,\left(\left\{\frac{h_1}{(.3,.6)},\frac{h_2}{(.4,.5)},\frac{h_3}{(.8,.1)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.6,.3)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(.8,.1)},\frac{h_2}{(.9,.1)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.8,.1)}\right\}\right)\right)\right\},$$

$$(F_4,A)=(F_1,A)\tilde{\cap}(F_2,A)=$$
$$\left\{\left(e_1,\left(\left\{\frac{h_1}{(.2,.7)},\frac{h_2}{(.3,.6)},\frac{h_3}{(.7,.2)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.5,.5)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(.7,.2)},\frac{h_2}{(.7,.2)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(.8,.1)},\frac{c_2}{(.6,.3)}\right\}\right)\right)\right\},$$

Then we observe that the sub family $\tau_1=\{\Phi_A,\ (\mathfrak{F},A),\ (F_1,A),\ (F_2,A),\ (F_3,A),\ (F_4,A)\}$ of $IFSMs_A(\mathfrak{F},A)$ is an IFSMs- topology on $(\mathfrak{F},A)$, since it satisfies the necessary three axioms $[O_1],\ [O_2]$ *and* $[O_3]$ and $((\mathfrak{F},A),\tau_1)$ is an IFSMs- topological space.

Definition 3.4. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space over $(\mathfrak{F},A)$. An intuitionistic fuzzy soft multi subset (F,A) of $(\mathfrak{F},A)$ is called intuitionistic fuzzy soft multi closed (in short IFSM-closed set) if its complement $(F,A)^c$ is a member of τ.

Example 3.5. Let us consider example: 3.3., then the IFSM-closed sets in $((\mathfrak{F},A),\tau_1)$ are

$$(F_1,A)^c=\left\{\left(e_1,\left(\left\{\frac{h_1}{(.7,.2)},\frac{h_2}{(.5,.4)},\frac{h_3}{(.1,.8)}\right\},\left\{\frac{c_1}{(.1,.8)},\frac{c_2}{(.5,.5)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(.2,.7)},\frac{h_2}{(.2,.7)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(.1,.8)},\frac{c_2}{(.3,.6)}\right\}\right)\right)\right\},$$

$$(F_2,A)^c=\left\{\left(e_1,\left(\left\{\frac{h_1}{(.6,.3)},\frac{h_2}{(.6,.3)},\frac{h_3}{(.2,.7)}\right\},\left\{\frac{c_1}{(.1,.8)},\frac{c_2}{(.3,.6)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(.1,.8)},\frac{h_2}{(.1,.9)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(.1,.8)},\frac{c_2}{(.1,.8)}\right\}\right)\right)\right\},\text{etc.}$$

Definition 3.6. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space on $(\mathfrak{F},A)$ and (F,A) be an IFSM-set in $IFSMs_A(\mathfrak{F},A)$. Then,

1. **Interior** of (F,A) is denoted by $\text{int}(F,A)$ and defined as

$\operatorname{int}(F,A)=\tilde{\cup}\{(G,A)\,|\,(G,A)$ is an IFSM- open set contained in $(F,A)\}$
2. **Closure** of (F,A) is denoted by $cl(F,A)$ and defined as
$cl(F,A)=\tilde{\cap}\{(G,A)\,|\,(G,A)$ is an IFSM- closed set containing $(F,A)\}$.

Theorem 3.7. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space on $(\mathfrak{F},A)$ and let (F,A) be an IFSM-set in $IFSMs_A(\mathfrak{F},A)$. Then

(i) $(cl(F,A))^c=\operatorname{int}((F,A)^c)$ and (ii) $(\operatorname{int}(F,A))^c=cl((F,A)^c)$

Proof. Straight forward.

4. Parameterized topological space

Proposition 4.1. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space over $(\mathfrak{F},A)$. Then the collection $\tau_e=\{F(e):(F,A)\in\tau\}$ for each $e\in A$, defines a topology on $\mathfrak{F}(e)$.

Proof: $[O_1]$ Since $\Phi_A,\ (\mathfrak{F},A)\in\tau$ implies that $\varphi,\ \mathfrak{F}(e)\in\tau_e$, for each $e\in E$.
$[O_2]$ Let $\{F_i(e):i\in I\}\subseteq\tau_e$, for some $\{(F_i,A):i\in I\}\subseteq\tau$. Since $\tilde{\cup}_{i\in I}(F_i,A)\in\tau$, so $\tilde{\cup}_{i\in I}F_i(e)\in\tau_e$, for each $e\in E$.
$[O_3]$ Let $F(e),G(e)\in\tau_e$, fore some $(F,A),\ (G,A)\in\tau$. Since $(F,A)\tilde{\cap}(G,A)\in\tau$, so $F(e)\tilde{\cap}G(e)\in\tau_e$, for each $e\in E$.
Thus τ_e defines a topology on $\mathfrak{F}(e)$ for each $e\in E$.

Definition 4.2. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space over $(\mathfrak{F},A)$. Then the topology $\tau_e=\{F(e):(F,A)\in\tau\}$ for each $e\in A$, is called parameterized topology and $(\mathfrak{F}(e),\tau_e)$ is called parameterized topological space.

Example 4.3. Let us consider the IFSMs- topology $\tau_1=\{\Phi_A,\ (\mathfrak{F},A),\ (F_1,A),\ (F_2,A),\ (F_3,A),\ (F_4,A)\}$ as in the example: 3.3., It can be easily seen that $\tau_{e_1}=\{\varphi,\ \mathfrak{F}(e_1),\ F_1(e_1),\ F_2(e_1),\ F_3(e_1),\ F_4(e_1)\}$, and $\tau_{e_2}=\{\varphi,\ \mathfrak{F}(e_2),\ F_1(e_2),\ F_2(e_2),\ F_3(e_2),\ F_4(e_2)\}$ are parameterized topologies on $\mathfrak{F}(e_1)$ and $\mathfrak{F}(e_2)$ respectively.

Definition 4.4. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space on $(\mathfrak{F},A)$ and (F,A) be an IFSM- set in $IFSMs_A(\mathfrak{F},A)$. Then we defined an IFSM- set associate with (F,A) over $(\mathfrak{F},A)$ is denoted by $(cl(F),A)$ and defined by $cl(F)(e)=cl(F(e))$, where $cl(F(e))$ is the closer of $F(e)$ in τ_e for each $e\in A$.

Proposition 4.5. Let $((\mathfrak{F},A),\tau)$ be an IFSMs- topological space on $(\mathfrak{F},A)$ and (F,A) be an IFSM- set in $IFSMs_A(\mathfrak{F},A)$. Then $(cl(F),A)\tilde{\subseteq}cl(F,A)$.

Proof. For any $e \in A, cl(F(e))$ is the smallest closed set in (U, τ_e), which contains $F(e)$. Moreover if $cl(F,A) = (G,A)$, then $G(e)$ is also closed set in (U, τ_e) containing $F(e)$. This implies that $cl(F)(e) = cl(F(e)) \subseteq G(e)$. Thus $(cl(F), A) \tilde{\subseteq} cl(F,A)$.

Corollary 4.6. Let $((\mathfrak{F}, A), \tau)$ be an IFSMs- topological space on $(\mathfrak{F}, A)$ and (F,A) be an IFSM- set in $IFSMs_A(\mathfrak{F}, A)$. Then $(cl(F), A) = cl(F,A)$ if and only if $(cl(F), A)^c \in \tau$

Proof. If $(cl(F), A) = cl(F,A)$, then $(cl(F), A) = cl(F,A)$ is an IFSM- closed set and so $(cl(F), A)^c \in \tau$.

Conversely if $(cl(F), A)^c \in \tau$ then $(cl(F), A)$ is an IFSM- closed set containing (F,A). By proposition 4.6., $(cl(F), A) \tilde{\subseteq} cl(F,A)$ and by the definition of intuitionistic fuzzy soft multi closure of (F,A), any IFSM- closed set over $(\mathfrak{F}, A)$, which contains (F,A) will contain $cl(F,A)$. This implies that $cl(F,A) \tilde{\subseteq} (cl(F), A)$. Thus $(cl(F), A) = cl(F,A)$.

5. Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. In this paper we have introduced IFSMs- topological spaces which are defined over an initial universe with a fixed set of parameter. The notion of IFSM- open sets, IFSM- closed sets; interior and closure of an IFSM- set are introduced. Also we introduced the concept of parameterized topological space induced by an IFSMs- topological space and studied some of its properties.

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