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Modal Operator $F_{\alpha,\beta}$ in Intuitionistic Fuzzy Groups

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Abstract. In this paper, we study modal operator $F_{\alpha,\beta}$ in intuitionistic fuzzy subgroup of a group and derive some results.

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1. Introduction

The idea of intuitionistic fuzzy sets (IFSs) was given by [1, 2] to generalize the notion of fuzzy sets (FSs). Intuitionistic fuzzy modal operator was defined by Atanassov in [3]. The modal operators have been known to be important tools for IFSs where the operators are defined on the contrary to the FSs. Intuitionistic fuzzy operators and some properties of these operators were examined by several authors [4,5,6]. Recently modal operators in intuitionistic fuzzy matrices has been studied in [7]. Here in this paper, we study the impact of modal operator $F_{\alpha,\beta}$on intuitionistic fuzzy groups.

2. Preliminaries

Here we recall some definitions and results which will be used later.

Definition 2.1. [3**]** Let X be a fixed non-empty set. **An intuitionistic fuzzy set** (IFS) A of X is an object of the following form A = { < x , $\mu_A(x)$, $\nu_A(x)$ > : x $\in$ X}, whereμ_A : X $\to$ [0,1] and ν_A: X $\to$ [0, 1] define the degree of membership and degree of non-membership of the element x$\in$ X respectively and for any x $\in$ X, we have $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Remark 2.2. (i) When $\mu_A(x) + \nu_A(x) = 1$, i.e. when $\nu_A(x) = 1 - \mu_A(x) = \mu_A{}^c(x)$. Then A is called **fuzzy set.**
(ii) For convenience, we write the IFS A = { < x, $\mu_A(x)$, $\nu_A(x)$ > : x $\in$ X} by A= (μ_A , ν_A).
(iii) The set of all IFS's of X is denoted by IFS(X).

Definition 2.3. [3**]** Let A = (μ_A , ν_A) and B = (μ_B , ν_B) be any two IFS's of X , then
(i) A $\subseteq$ B if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all x$\in$ X

(ii) A = B if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$

(iii) $A \cap B = (\mu_{A \cap B}, \nu_{A \cap B})$, where

$(\mu_{A \cap B})(x) = \text{Min}\{\mu_A(x), \mu_B(x)\}$ and $(\nu_{A \cap B})(x) = \text{Max}\{\nu_A(x), \nu_B(x)\}$

(iv) $A \cup B = (\mu_{A \cup B}, \nu_{A \cup B})$, where

$(\mu_A \cup \mu_B)(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cup \nu_B)(x) = \text{Min}\{\nu_A(x), \nu_B(x)\}$

Definition 2.4. [3**]** For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ of X, if
$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, for all $x \in X$.
Then $\pi_A(x)$ is called the degree of indeterminacy of x in A.

Definition 2.5. [3**]** For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ of X and $\alpha \in [0,1]$ the operators $\Box$: IFS(X)→IFS(X), $\Diamond$: IFS(X)→IFS(X) and D_α: IFS(X)→IFS(X) are defined as

(i) $\Box A = \{< x, \mu_A(x), 1 - \mu_A(x) > : x \in X\}$ is called Necessity Operator

(ii) $\Diamond A = \{< x, 1 - \nu_A(x), \nu_A(x) > : x \in X\}$ is called Possibility Operator

(iii) $D_\alpha(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x) >: x \in X\}$is called α-Modal operator.

Remark 2.6. (i) Clearly, $\Box A \subseteq A \subseteq \Diamond A$ and the equality hold, when A is a fuzzy set (ii) Notice that $D_0(A) = \Box A$ and $D_1(A) = \Diamond A$. Thus α-modal operator $D_\alpha(A)$ is an extension of necessary operator $\Box A$ and possibility operator $\Diamond A$.

Definition 2.7. [3**]** For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ of X and for any $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$, the (α, β)**-modal operator** $F_{\alpha,\beta}$: IFS(X)→ IFS(X) is defined as $F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x)> : x \in X\}$, where $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. Therefore, we can write $F_{\alpha,\beta}(A)$ as $F_{\alpha,\beta}(A)(x) = \left(\mu_{F_{\alpha,\beta}(A)}(x),\ \nu_{F_{\alpha,\beta}(A)}(x)\right)$, where
$\mu_{F_{\alpha,\beta}(A)}(x) = \mu_A(x) + \alpha.\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x) = \nu_A(x) + \beta.\pi_A(x)$

Remark 2.8. (i) Clearly, $F_{0,1}(A) = \Box A$, $F_{1,0}(A) = \Diamond A$ and $F_{\alpha,1-\alpha}(A) = D_\alpha(A)$.

Definition 2.9. [8**]** Let A be Intuitionistic fuzzy set of a universe set X. Then (α, β)-cut of A is a crisp subset $C_{\alpha,\beta}(A)$ of the IFS A is given by
$C_{\alpha,\beta}(A) = \{x : x \in X$ such that $\mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$, where $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$.

Definition 2.10. [3**]** Let $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ be an IFS of a universe set X, then support of A is denoted by $Supp_X(A)$ and is defined

$$Supp_X(A) = \{x \in X :\ \mu_A(x) > 0 \text{ and } \nu_A(x) < 1\}$$

Remark 2.11. Clearly,

$$\text{Supp}_X(A) = \bigcup\{C_{\alpha,\beta}(A) :\ \text{for all } \alpha, \beta \in (0,1] \text{ such that } 0 < \alpha + \beta \leq 1\}$$

Definition 2.12. [8**]** An IFS $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in G\}$ of a group G is said to be intuitionistic fuzzy subgroup of G (in short IFSG) of G if
$\mu_A(xy^{-1}) \geq \text{Min}\ \{\mu_A(x), \mu_A(y)\}$ and $\nu_A(xy^{-1}) \leq \text{Max}\ \{\nu_A(x), \nu_A(y)\}$, for all $x , y \in G$

Definition 2.13. [8**]** An IFSG $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in G\}$ of a group G said to be intuitionistic fuzzy normal subgroup of G (in short IFNSG) of G if

$\mu_A(xy) = \mu_A(yx)$ and $\nu_A(xy) = \nu_A(yx)$, for all $x , y \in G$

Or equivalently, $\mu_A(xyx^{-1}) = \mu_A(y)$ and $\nu_A(xyx^{-1}) = \nu_A(y)$, for all $x , y \in G$

Definition 2.14. [10**,**11**]** An IFSG A of a group G is called an intuitionistic fuzzy abelian subgroup (IFASG) of G if and only if $Supp_G(A)$ is abelian subgroup of G.

Definition 2.15. [10,11**]** An IFSG A of a group G is called an intuitionistic fuzzy cyclic subgroup (IFCSG) of G if and only if $Supp_G(A)$ is cyclic subgroup of G.

Proposition 2.16. [8**]** If A be IFSG of a group G and e be the identity element of G, then

(i) $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$ for all $x \in G$
(ii) $\mu_A(e) \geq \mu_A(x)$ and $\nu_A(e) \leq \nu_A(x)$ for all $x \in G$
(iii) $\mu_A(xy^{-1}) = \mu_A(e)$ and $\nu_A(xy^{-1}) = \nu_A(e)$ then $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$.

Definition 2.17. [9] Let X and Y be two non-empty sets and $f ; X \to Y$ be a mapping. Let A and B be IFS's of X and Y respectively . Then the image of A under the map f is denoted by $f(A)$ and is defined as $f(A)(y) = \left(\mu_{f(A)}(y) , \nu_{f(A)}(y) \right)$, where

$$\mu_{f(A)}(y) = \begin{cases} \vee\{ \mu_A(x) : x \in f^{-1}(y)\} \\ 0 \quad ; \quad \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{f(A)}(y) = \begin{cases} \wedge\{ \nu_A(x) : x \in f^{-1}(y)\} \\ 1 \quad ; \quad \text{otherwise} \end{cases}$$

Also the pre-image of B under f is denoted by $f^{-1}(B)$ and is defined as

$$f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x) , \nu_{f^{-1}(B)}(x)) = (\mu_B(f(x)) , \nu_B(f(x))) \; ; \forall x \in X$$

Remark(2.18) Note that $\mu_A(x) \leq \mu_{f(A)}(f(x))$ and $\nu_A(x) \geq \nu_{f(A)}(f(x))$; $\forall x \in X$, however eqality hold when the map f is bijective.

3. Modal operator $F_{\alpha,\beta}$ in intuitionistic fuzzy groups

In this section, we study the relationship between the intuitionistic fuzzy subgroups and the modal operator on these intuitionistic fuzzy subgroups. We also establish a relationship between the support of IFS A and the support of IFS under modal operator.

Theorem 3.1. If A is IFSG of a group G, then $F_{\alpha,\beta}(A)$ is also IFSG of G.

Proof. Let x , y be any element of G , then $F_{\alpha,\beta}(A)(xy^{-1}) = \left(\mu_{F_{\alpha,\beta}}(xy^{-1}) , \nu_{F_{\alpha,\beta}}(xy^{-1})\right)$,
where $\mu_{F_{\alpha,\beta}}(xy^{-1}) = \mu_A(xy^{-1}) + \alpha.\pi_A(xy^{-1})$ and $\nu_{F_{\alpha,\beta}}(xy^{-1}) = \nu_A(xy^{-1}) + \beta.\pi_A(xy^{-1})$

Now, $\mu_{F_{\alpha,\beta}}(xy^{-1}) = \mu_A(xy^{-1}) + \alpha.\pi_A(xy^{-1}) = \mu_A(xy^{-1}) + \alpha[1 - \mu_A(xy^{-1}) - \nu_A(xy^{-1})]$

$= \alpha + (1\text{-}\alpha)\mu_A(xy^{-1}) - \alpha.\nu_A(xy^{-1})$

$\geq \alpha + (1\text{-}\alpha)\text{Min}\{\mu_A(x), \mu_A(y)\} - \alpha.Max\{\nu_A(x),\ \nu_A(y)\}$

$= \alpha[1 - Max\{\nu_A(x),\ \nu_A(y)\}] + (1\text{-}\alpha)\text{Min}\{\mu_A(x), \mu_A(y)\}$

$= \alpha[Min\{1 - \nu_A(x),\ 1\text{-}\ \nu_A(y)\}] + (1\text{-}\alpha)\text{Min}\{\mu_A(x), \mu_A(y)\}$

$= \alpha.\ \text{Min}\{1 - \nu_A(x),\ 1\text{-}\ \nu_A(y)\} + (1\text{-}\alpha)\text{Min}\{\mu_A(x), \mu_A(y)\}$

$= \text{Min}\{\ \alpha.(1 - \nu_A(x)) + (1\text{-}\alpha)\mu_A(x)\ ,\ \alpha.(1 - \nu_A(y)) + (1\text{-}\alpha)\mu_A(y)\}$

$= \text{Min}\{\mu_A(x) + \alpha(1 - \mu_A(x) - \nu_A(x))\ ,\ \ \mu_A(y) + \alpha(1 - \mu_A(y) - \nu_A(y))$

$= \text{Min}\{\mu_A(x) + \alpha..\pi_A(x)\ ,\ \mu_A(y) + \alpha..\pi_A(y)\}$

$= \text{Min}\{\mu_{F_{\alpha,\beta}}(x)\ ,\ \mu_{F_{\alpha,\beta}}(y)\}$

Thus, $\mu_{F_{\alpha,\beta}}(xy^{-1}) \geq \text{Min}\{\mu_{F_{\alpha,\beta}}(x)\ ,\ \mu_{F_{\alpha,\beta}}(y)\}$

Similiarly, we can show that $\nu_{F_{\alpha,\beta}}(xy^{-1}) \leq \text{Max}\{\ \nu_{F_{\alpha,\beta}}(x)\ ,\ \nu_{F_{\alpha,\beta}}(y)\}$

Hence $F_{\alpha,\beta}(A)$ is IFSG of G.

Remark 3.2. The converse of the above theorem need not be true

Example 3.3. Let G be the Klein 4-group { e , a , b , ab}, where $a^2 = b^2 = e$ and ab = ba. Define A = { < e , 0.1, 0.1 > , < a , 0.21 ,0.39 > , < b , 0.3, 0.4 > , < ab, 0.2, 0.1 >} be IFS in G. It can be easily verified that A is not IFSG of G, for

$\mu_A(ab) = 0.2 < Min\{0.21, 0.3\} = Min\{\mu_A(a), \mu_A(b)\}$

Now take α = 0.2 , β = 0.2, then it can be checked that

$F_{\alpha,\beta}(A)$ = {< e , 0.26, 0.26 > , < a , 0.29 , 0.47 > , < b , 0.36, 0.46 > , < ab, 0.34, 0.24 > }

Clearly, we have

$\mu_{F_{\alpha,\beta}}(ab) = 0.34 \geq Min\{0.29, 0.36\} = Min\{\mu_{F_{\alpha,\beta}}(a), \mu_{F_{\alpha,\beta}}(b)\}$ and

$\nu_{F_{\alpha,\beta}}(ab) = 0.24 \leq Max\{0.47, 0.46\} = Max\{\nu_{F_{\alpha,\beta}}(a), \nu_{F_{\alpha,\beta}}(b)\}$.

Therefore, $F_{0.2,0.2}(A)$ is IFSG of G.

Theorem 3.4. If A is IFSNG of a group G, then $F_{\alpha,\beta}(A)$ is also IFNSG of G.

Proof. Let x , y be any element of G, then

$$\mu_{F_{\alpha,\beta}}(xy)=\mu_A(xy)+\alpha.\pi_A(xy)=\mu_A(xy)+\alpha[1-\mu_A(xy)-\nu_A(xy)]=\mu_A(yx)+\alpha[1-\mu_A(yx)-\nu_A(yx)]$$
$$=\mu_A(yx)+\alpha.\pi_A(yx)=\mu_{F_{\alpha,\beta}}(yx)$$

Similarly, we can show that $\nu_{F_{\alpha,\beta}}(xy)=\nu_{F_{\alpha,\beta}}(yx)$.

Hence $F_{\alpha,\beta}(A)$ is IFNSG of G.

Result 3.5. If A be IFSG of a group G and e be the identity element of G, then

(i) $\mu_{F_{\alpha,\beta}}(x^{-1})=\mu_{F_{\alpha,\beta}}(x)$ and $\nu_{F_{\alpha,\beta}}(x^{-1})=\nu_{F_{\alpha,\beta}}(x)$

(*ii*) $\mu_{F_{\alpha,\beta}}(e)\geq\mu_{F_{\alpha,\beta}}(x)$ and $\nu_{F_{\alpha,\beta}}(e)\leq\nu_{F_{\alpha,\beta}}(x)$, for all $x\in G$

Pr*oof*.(*i*) Now, $\mu_{F_{\alpha,\beta}}(x^{-1})=\mu_A(x^{-1})+\alpha.\pi_A(x^{-1})=\mu_A(x^{-1})+\alpha[1-\mu_A(x^{-1})-\nu_A(x^{-1})]$
$$=\mu_A(x)+\alpha[1-\mu_A(x)-\nu_A(x)]=\mu_A(x)+\alpha.\pi_A(x)$$
$$=\mu_{F_{\alpha,\beta}}(x)$$

Similarly, we can show that $\nu_{F_{\alpha,\beta}}(x^{-1})=\nu_{F_{\alpha,\beta}}(x)$

(*ii*) Now, $\mu_{F_{\alpha,\beta}}(e)=\mu_{F_{\alpha,\beta}}(xx^{-1})\geq Min\{\mu_{F_{\alpha,\beta}}(x),\ \mu_{F_{\alpha,\beta}}(x^{-1})\}=\mu_{F_{\alpha,\beta}}(x)$ [using (i)]

Similarly, $\nu_{F_{\alpha,\beta}}(e)=\nu_{F_{\alpha,\beta}}(xx^{-1})\leq Max\{\nu_{F_{\alpha,\beta}}(x),\ \nu_{F_{\alpha,\beta}}(x^{-1})\}=\nu_{F_{\alpha,\beta}}(x)$ [using (i)]

Proposition 3.6. Let f : X→ Y be a mapping and A∈ IFS(X) and B∈ IFS(Y). Then

(i) $f^{-1}(F_{\alpha,\beta}(B))=F_{\alpha,\beta}(f^{-1}(B))$ (ii) $f(F_{\alpha,\beta}(A))\subseteq F_{\alpha,\beta}(f(A))$

Pr*oof*.(*i*) Now, $f^{-1}(F_{\alpha,\beta}(B))(x)=\left(\mu_{f^{-1}(F_{\alpha,\beta}(B))}(x)\ ,\ \nu_{f^{-1}(F_{\alpha,\beta}(B))}(x)\right)=\left(\mu_{F_{\alpha,\beta}(B)}(f(x))\ ,\ \nu_{F_{\alpha,\beta}(B)}(f(x))\right)$

But $\mu_{F_{\alpha,\beta}(B)}(f(x))=\mu_B(f(x))+\alpha.\pi_B(f(x))=\mu_B(f(x))+\alpha.[1-\mu_B(f(x))-\nu_B(f(x))]$
$$=\mu_{f^{-1}(B)}(x)+\alpha.[1-\mu_{f^{-1}(B)}(x)-\nu_{f^{-1}(B)}(x)]=\mu_{f^{-1}(B)}(x)+\alpha.\pi_{f^{-1}(B)}(x)$$
$$=\mu_{F_{\alpha,\beta}(f^{-1}(B))}(x)$$

Similarly, we can show that $\nu_{F_{\alpha,\beta}(B)}(f(x))=\nu_{F_{\alpha,\beta}(f^{-1}(B))}(x)$

Thus $f^{-1}(F_{\alpha,\beta}(B))(x)=\left(\mu_{F_{\alpha,\beta}(f^{-1}(B))}(x)\ ,\ \nu_{F_{\alpha,\beta}(f^{-1}(B))}(x)\right)=F_{\alpha,\beta}(f^{-1}(B))(x)$, for every $x\in X$

(*ii*) Now, $f(F_{\alpha,\beta}(A))(y)=\left(\mu_{f(F_{\alpha,\beta}(A))}(y)\ ,\ \nu_{f(F_{\alpha,\beta}(A))}(y)\right)$

But $\mu_{f(F_{\alpha,\beta}(A))}(y)=Sup\{\mu_{F_{\alpha,\beta}(A)}(x):\ f(x)=y\}$
$$=\text{Sup}\{\ \mu_A(x)+\alpha.\pi_A(x):f(x)=y\}$$
$$=\text{Sup}\ \{\mu_A(x)+\alpha[1-\mu_A(x)-\nu_A(x)]\}:\ f(x)=y\}$$
$$=\text{Sup}\{\ \alpha+(1\text{-}\alpha)\mu_A(x)-\alpha.\nu_A(x):f(x)=y\}$$

$\leq \{ \alpha + (1\text{-}\alpha)\mu_{f(A)}(y) - \alpha.\nu_{f(A)}(y)\}$ [using (1...)]

$= \mu_{f(A)}(y) + \alpha.\pi_{f(A)}(y)$

$= \mu_{F_{\alpha,\beta}(f(A))}(y)$

Thus $\mu_{f(F_{\alpha,\beta}(A))}(y) \leq \mu_{F_{\alpha,\beta}(f(A))}(y)$, for all y$\in$ Y

Similarly, we can show that $\nu_{f(F_{\alpha,\beta}(A))}(y) \geq \nu_{F_{\alpha,\beta}(f(A))}(y)$, for all y$\in$ Y

Thus $f(F_{\alpha,\beta}(A))(y) = \left(\mu_{f(F_{\alpha,\beta}(A))}(y)\ ,\ \nu_{f(F_{\alpha,\beta}(A)}(y)\right) \subseteq \left(\mu_{F_{\alpha,\beta}(f(A))}(y)\ ,\ \nu_{F_{\alpha,\beta}(f(A))}(y)\right) = F_{\alpha,\beta}(f(A))(y)$

Hence proved.

Corollary 3.7. Let f: X→ Y be a bijective mapping. Then $f(F_{\alpha,\beta}(A)) = F_{\alpha,\beta}(f(A))$

Proposition 3.8. [10**]** Let f: X → Y be a mapping and A , B are IFS of X and Y respectively. Then the following results hold

(i) $f(\text{Supp}_X(A)) \subseteq \text{Supp}_Y(f(A))$ and equality hold when the map f is bijective

(ii) $f^{-1}(\text{Supp}_Y(B)) = \text{Supp}_X(f^{-1}(B))$

Pr*oof*.(i) Let y$\in f(\text{Supp}_X(A))$ be any element. Therefore, ∃'s x$\in \text{Supp}_X(A)$ such that $f(x) = y$.

As x$\in \text{Supp}_X(A) \Rightarrow \mu_A(\text{x}) > 0$ and $\nu_A(\text{x}) < 1$.

But $\mu_A(\text{x}) \leq \mu_{f(A)}(f(\text{x}))$ and $\nu_A(\text{x}) \geq \nu_{f(A)}(f(\text{x}))$; for all x$\in$ X

$\therefore \mu_{f(A)}(f(\text{x})) > 0$ and $\nu_{f(A)}(f(\text{x})) < 1 \Rightarrow \text{y} = f(\text{x}) \in \text{Supp}_Y(f(A))$

Hence $f(\text{Supp}_X(A)) \subseteq \text{Supp}_Y(f(A))$. The second part follow by Remark(2.18)

(ii) Let x$\in f^{-1}(\text{Supp}_Y(B))$ be any element $\Leftrightarrow f(x) \in \text{Supp}_Y(B)$

$\Leftrightarrow \mu_B(f(\text{x})) > 0$ and $\nu_B(f(\text{x})) < 1 \Leftrightarrow \mu_{f^{-1}(B)}(\text{x}) > 0$ and $\nu_{f^{-1}(B)}(\text{x}) < 1 \Leftrightarrow$ x$\in \text{Supp}_X(f^{-1}(B))$

Hence $f^{-1}(\text{Supp}_Y(B)) = \text{Supp}_X(f^{-1}(B))$.

Lemma 3.9. If A be any IFS of the universe set X, then $Supp_X(F_{\alpha,\beta}(A)) = Supp_X(A)$

Proof. Since $Supp_X(F_{\alpha,\beta}(A))$ = { $x \in$ X : $\mu_{F_{\alpha,\beta}}(x) > 0$ and $\nu_{F_{\alpha,\beta}}(x) < 1$}

But $\mu_{F_{\alpha,\beta}}(x) = \mu_A(x) + \alpha.\pi_A(x) = \mu_A(x) + \alpha[1 - \mu_A(x) - \nu_A(x)]$

$= \alpha + (1\text{-}\alpha)\mu_A(x) - \alpha.\nu_A(x)$

and $\nu_{F_{\alpha,\beta}}(x) = \nu_A(x) + \beta.\pi_A(x) = \nu_A(x) + \beta[1 - \mu_A(x) - \nu_A(x)]$

$= \beta - \beta\mu_A(x) + (1-\beta)\nu_A(x)$

Now, $\nu_A(x) < 1 \Leftrightarrow -\alpha\nu_A(x) > -\alpha$. Also, $\mu_A(x) > 0 \Leftrightarrow (1-\alpha)\mu_A(x) > 0$

Thus, $\alpha + (1\text{-}\alpha)\mu_A(x) - \alpha.\nu_A(x) > \alpha - \alpha = 0$

i.e. $\mu_{F_{\alpha,\beta}}(x) > 0$. Similarly, we can show that $\beta - \beta\mu_A(x) + (1-\beta)\nu_A(x) < 1$ i.e. $\nu_{F_{\alpha,\beta}}(x) < 1$.Therefore, $Supp_X(F_{\alpha,\beta}(A)) = Supp_X(A)$.

By using the Lemma (3.9) and the definition (2.14) and (2.15), we have the following theorem.

Theorem 3.10. If A is IFSG of a group G, then A is IFASG of G if and only if $F_{\alpha,\beta}(A)$ is IFASG of G.

Theorem 3.11. If A is IFSG of a group G, then A is IFCSG of G if and only if $F_{\alpha,\beta}(A)$ is IFCSG of G.

4. Homomorphisms of modal operator in intuitionistic fuzzy groups

Theorem 4.1. Let $f : G_1 \rightarrow G_2$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFSG of G_2. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFSG of G_1.

Proof. Let $F_{\alpha,\beta}(B)$ is IFSG of G_2. By Proposition (3.6)(i), it is enough to show that $F_{\alpha,\beta}(f^{-1}(B))$ is IFSG of G_1.

Let x , y $\in G_1$ be any element, then

$$\mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy^{-1}) = \mu_{F_{\alpha,\beta}(B)}(f(xy^{-1}))$$
$$= \mu_{F_{\alpha,\beta}(B)}(f(x)(f(y))^{-1})$$
$$\geq Min\{\mu_{F_{\alpha,\beta}(B)}(f(x)) , \mu_{F_{\alpha,\beta}(B)}(f(y))\}$$
$$= \text{Min}\{ \mu_{F_{\alpha,\beta}(f^{-1}(B))}(x) , \mu_{F_{\alpha,\beta}(f^{-1}(B))}(y)\}$$

Thus, $\mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy^{-1}) \geq \text{Min}\{ \mu_{F_{\alpha,\beta}(f^{-1}(B))}(x) , \mu_{F_{\alpha,\beta}(f^{-1}(B))}(y)\}$

Similarly, we can show that $\nu_{F_{\alpha,\beta}(f^{-1}(B))}(xy^{-1}) \leq \text{Max}\{ \nu_{F_{\alpha,\beta}(f^{-1}(B))}(x) , \nu_{F_{\alpha,\beta}(f^{-1}(B))}(y)\}$

Thus $F_{\alpha,\beta}(f^{-1}(B))$ and hence $f^{-1}(F_{\alpha,\beta}(B))$ is IFSG of G_1.

Theorem 4.2. Let $f : G_1 \rightarrow G_2$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFNSG of G_2. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFNSG of G_1.

Proof. Let $F_{\alpha,\beta}(B)$ is IFNSG of G_2. By Proposition (3.6)(i), it is enough to show that $F_{\alpha,\beta}(f^{-1}(B))$ is IFNSG of G_1.

Let x , y $\in G_1$ be any element, then

$$\mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy) = \mu_{F_{\alpha,\beta}(B)}(f(xy)) = \mu_{F_{\alpha,\beta}(B)}(f(x)f(y)) = \mu_{F_{\alpha,\beta}(B)}(f(y)f(x)) = \mu_{F_{\alpha,\beta}(B)}(f(yx)) = \mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy) and$$

$$\nu_{F_{\alpha,\beta}\left(f^{-1}(B)\right)}(xy)=\nu_{F_{\alpha,\beta}(B)}(f(xy))=\nu_{F_{\alpha,\beta}(B)}(f(x)f(y))=\nu_{F_{\alpha,\beta}(B)}(f(y)f(x))=\nu_{F_{\alpha,\beta}(B)}(f(yx))=\nu_{F_{\alpha,\beta}\left(f^{-1}(B)\right)}(yx)$$

Thus $F_{\alpha,\beta}(f^{-1}(B))$ and hence $f^{-1}(F_{\alpha,\beta}(B))$ is IFNSG of G_1.

Theorem 4.3. Let $f: G_1 \rightarrow G_2$ be a group isomorphism and $F_{\alpha,\beta}(A)$ is IFSG of G_1. Then $f(F_{\alpha,\beta}(A))$ is IFSG of G_2.

Proof. Let $x_2, y_2 \in G_2$ be any elements. As f is bijective, so let $\exists$'s unique $x_1, y_1 \in G_1$ such that $f(x_1)=x_2$ and $f(y_1)=y_2$

$$f(F_{\alpha,\beta}(A))(x_2y_2^{-1})=\left(\mu_{f(F_{\alpha,\beta}(A))}(x_2y_2^{-1}),\ \nu_{f(F_{\alpha,\beta}(A))}(x_2y_2^{-1})\right)$$

But $\mu_{f(F_{\alpha,\beta}(A))}(x_2y_2^{-1})=\mu_{F_{\alpha,\beta}(A)}(t)$, where $f(t)=x_2y_2^{-1}=f(x_1)f(y_1^{-1})=f(x_1y_1^{-1})$ *implies* $t=x_1y_1^{-1}$

$$\begin{aligned}
&=\mu_A(x_1y_1^{-1})+\alpha.\pi_A(x_1y_1^{-1})\\
&=\mu_A(x_1y_1^{-1})+\alpha[1-\mu_A(x_1y_1^{-1})-\nu_A(x_1y_1^{-1})]\\
&=\alpha+(1\text{-}\alpha)\mu_A(x_1y_1^{-1})-\alpha.\nu_A(x_1y_1^{-1})\\
&\geq\alpha+(1\text{-}\alpha)\mathrm{Min}\{\mu_A(x_1),\ \mu_A(y_1)\}-\alpha.Max\{\nu_A(x_1),\ \nu_A(y_1)\}\\
&=\alpha\{1-Max\{\nu_A(x_1),\ \nu_A(y_1)\}\}+(1\text{-}\alpha)\mathrm{Min}\{\mu_A(x_1),\ \mu_A(y_1)\}\\
&=\alpha.Min\{1-\nu_A(x_1),\ 1-\nu_A(y_1)\}+(1\text{-}\alpha)\mathrm{Min}\{\mu_A(x_1),\ \mu_A(y_1)\}\\
&=\mathrm{Min}\ \{\ (1\text{-}\alpha)\mu_A(x_1)+\alpha.(1-\nu_A(x_1)),\ (1\text{-}\alpha)\mu_A(y_1)+\alpha.(1-\nu_A(y_1))\}\\
&=\mathrm{Min}\{\ \mu_A(x_1)+\alpha.(1-\mu_A(x_1)-\nu_A(x_1)),\ \mu_A(y_1)+\alpha.(1-\mu_A(y_1)-\nu_A(y_1))\}\\
&=\mathrm{Min}\{\ \mu_A(x_1)+\alpha.\pi_A(x_1)\ ,\ \mu_A(y_1)+\alpha\pi_A(y_1)\}\\
&=\mathrm{Min}\{\ \mu_{F_{\alpha,\beta}(A)}(x_1)\ ,\ \mu_{F_{\alpha,\beta}(A)}(y_1)\}\\
&=\mathrm{Min}\{\ \mu_{f(F_{\alpha,\beta}(A))}(f(x_1))\ ,\ \mu_{f(F_{\alpha,\beta}(A))}(f(y_1))\}\\
&=\mathrm{Min}\{\ \mu_{f(F_{\alpha,\beta}(A))}(x_2)\ ,\ \mu_{f(F_{\alpha,\beta}(A))}(y_2)\}
\end{aligned}$$

i.e. $\mu_{f(F_{\alpha,\beta}(A))}(x_2y_2^{-1})\geq\mathrm{Min}\{\ \mu_{f(F_{\alpha,\beta}(A))}(x_2)\ ,\ \mu_{f(F_{\alpha,\beta}(A))}(y_2)\}$

Similarly, we can show that $\nu_{f(F_{\alpha,\beta}(A))}(x_2y_2^{-1})\leq\mathrm{Max}\{\ \nu_{f(F_{\alpha,\beta}(A))}(x_2)\ ,\ \nu_{f(F_{\alpha,\beta}(A))}(y_2)\}$

Hence $f(F_{\alpha,\beta}(A))$ *is* a IFSG of G_2.

Theorem 4.4. Let $f: G_1 \rightarrow G_2$ be a group isomorphism and $F_{\alpha,\beta}(A)$ is IFSNG of G_1. Then $f(F_{\alpha,\beta}(A))$ is IFNSG of G_2.

Proof. Let $x_2, y_2 \in G_2$ be any elements. As f is bijective, so let $\exists$'s unique $x_1, y_1 \in G_1$ such that $f(x_1)=x_2$ and $f(y_1)=y_2$. Now, $f(F_{\alpha,\beta}(A))(x_2y_2)=\left(\mu_{f(F_{\alpha,\beta}(A))}(x_2y_2)\ ,\ \nu_{f(F_{\alpha,\beta}(A))}(x_2y_2)\right)$

But $\mu_{f(F_{\alpha,\beta}(A))}(x_2y_2) = \mu_{F_{\alpha,\beta}(A)}(t)$, where $f(t) = x_2y_2 = f(x_1)f(y_1) = f(x_1y_1)$ *implies* $t = x_1y_1$

$$\begin{aligned}
&= \mu_A(x_1y_1) + \alpha.\pi_A(x_1y_1) \\
&= \mu_A(x_1y_1) + \alpha[1 - \mu_A(x_1y_1) - \nu_A(x_1y_1)] \\
&= \alpha + (1\text{-}\alpha)\mu_A(x_1y_1) - \alpha.\nu_A(x_1y_1) \\
&= \alpha + (1\text{-}\alpha)\mu_A(y_1x_1) - \alpha.\nu_A(y_1x_1) \\
&= \alpha + (1\text{-}\alpha)\mu_{f(A)}(f(y_1x_1)) - \alpha.\nu_{f(A)}(f(y_1x_1)) \\
&= \alpha + (1\text{-}\alpha)\mu_{f(A)}(f(y_1)f(x_1)) - \alpha.\nu_{f(A)}(f(y_1)f(x_1)) \\
&= \alpha + (1\text{-}\alpha)\mu_{f(A)}(y_2x_2) - \alpha.\nu_{f(A)}(y_2x_2) \\
&= \mu_{f(A)}(y_2x_2) + \alpha.\pi_{f(A)}(y_2x_2) \\
&= \mu_{f(F_{\alpha,\beta}(A))}(y_2x_2)
\end{aligned}$$

Similarly, we can show that $\nu_{f(F_{\alpha,\beta}(A))}(x_2y_2) = \nu_{f(F_{\alpha,\beta}(A))}(y_2x_2)$

Hence $f(F_{\alpha,\beta}(A))$ is IFNSG of G_2.

Theorem 4.5. Let $f : G_1 \rightarrow G_2$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFASG of G_2. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFASG of G_1.

Proof. Let $F_{\alpha,\beta}(B)$ is IFASG of $G_2 \Rightarrow Supp_{G_2}(F_{\alpha,\beta}(B))$ is abelian subgroup of G_2 [by definition (2.14)]

$\Rightarrow Supp_{G_2}(B)$ is abelian subgroup of G_2 [by Lemma(3.9)]

$\Rightarrow f^{-1}(Supp_{G_2}(B))$ is abelian subgroup of G_1

$\Rightarrow Supp_{G_1}(f^{-1}(B))$ is abelian subgroup of G_1 [by Proposition (3.8)(ii)]

$\Rightarrow Supp_{G_1}(F_{\alpha,\beta}(f^{-1}(B))$ is abelian subgroup of G_1 [by Lemma (3.9)]

$\Rightarrow Supp_{G_1}(f^{-1}(F_{\alpha,\beta}(B))$ is abelian subgroup of G_1 [by Proposition (3.6)]

$\Rightarrow f^{-1}(F_{\alpha,\beta}(B))$ is IFASG of G_1 [by definition (2.14)]

Theorem 4.6. Let $f : G_1 \rightarrow G_2$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFCSG of G_2. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFCSG of G_1.

Proof. Follow similar to the proof of Theorem (4.5)

Theorem 4.7. Let $f : G_1 \rightarrow G_2$ be a group isomorphism and $F_{\alpha,\beta}(A)$ is IFSAG of G_1. Then $f(F_{\alpha,\beta}(A))$ is IFASG of G_2.

Proof. Since $F_{\alpha,\beta}(A)$ is IFASG of $G_1 \Rightarrow Supp_{G_1}(F_{\alpha,\beta}(A))$ is abelian subgroup of G_1 [by definition (2.14)]

$\Rightarrow f\,(Supp_{G_1}\,(F_{\alpha,\beta}(A)))$ is abelian subgroup of G_2

$\Rightarrow Supp_{G_2}\,(f(F_{\alpha,\beta}(A)))$ is abelian subgroup of G_2 [by proposition (3.8)(i)]

$\Rightarrow f(F_{\alpha,\beta}(A))$ is IFASG of G_2.

Theorem 4.8. Let $f : G_1 \rightarrow G_2$ be a group isomorphism and $F_{\alpha,\beta}(A)$ is IFCSG of G_1. Then $f\,(F_{\alpha,\beta}(A))$ is IFCSG of G_2.

Proof. Since $F_{\alpha,\beta}(A)$ is IFCAG of $G_1 \Rightarrow Supp_{G_1}\,(F_{\alpha,\beta}(A))$ is cyclic subgroup of G_1 [by definition (2.15)]

$\Rightarrow f\,(Supp_{G_1}\,(F_{\alpha,\beta}(A)))$ is cyclic subgroup of G_2

$\Rightarrow Supp_{G_2}\,(f(F_{\alpha,\beta}(A)))$ is cyclic subgroup of G_2 [by proposition (3.8)(i)]

$\Rightarrow f\,(F_{\alpha,\beta}(A))$ is IFCSG of G_2.

5. Conclusion

In this paper, we have studied the impact of modal operator $F_{\alpha,\beta}$on intuitionistic fuzzy groups and proved that many properties of intuitionistic fuzzy subgroups like normality, commutatively (abelian-ness) and cyclic groups remain invariant under the modal operator. We have also obtained the impact of these operator under homomorphism. A similar type of impact of modal operator can be realized on other algebraic structure like intuitionistic fuzzy subring, intuitionistic fuzzy submodules etc. and this work is under progress and will be published shortly

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