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Solid Transportation Problem Under Budget Constraint Using Fuzzy Measure

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Abstract. Fixed charge solid transportation problems are formulated under a budget constraint at each destination. Unit transportation costs, fixed charges, sources at origins, demands at destinations, conveyances capacities are assumed to be crisp or fuzzy. Budget constraints at destinations are imposed. It is also assumed that transported units are crisp. So the problem is formulated as constraint optimization programming problem in crisp and fuzzy environments. As optimization of fuzzy objective as well as consideration of fuzzy constraint is not well defined, credibility of fuzzy event are used to transform the problem into equivalent crisp problem. The reduced crisp problem is solved following Generalized Reduced Gradient (GRG) method using lingo software. The models are illustrated with numerical examples.

Keywords: Solid Transportation Problem, Budget constraints, Credibility measure

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1. Introduction

The solid transportation problem (STP) is a generalization of the well-known transportation problem (TP) in which three kinds of constraint sets exist instead of two (source and destination) as in TP [4]. This extra constraint is mainly due to modes of transportation (conveyances). The STP was stated by Shell [7]. Haley [2] showed a comparison of the STP with the classical TP and applied Modi-method to solve the STP. In a TP when fixed charge is considered against transportation of units from a source to a destination, the problem is transformed to fixed charge TP (FCTP). The FCTP was initialized by Hirsch and Dantzig [3]. Up to now, several approaches are followed to solve FCTP [1]. In STP, fixed charge is also considered and solved by several researchers in crisp as well as uncertain environments. Yang and Liu [8] developed fuzzy fixed charge STP and developed algorithms to solve the problem using credibility measures on fuzzy sets.

From above discussions there are some lacunas in the existing STP models, which are summarized below.

• Transportation problems-TP or STP are normally formulated and solved as cost minimization problems, very few might have formulated these as fuzzy fixed charge cost minimization problems.

• In the literature, there are several research works on transportation with uncertain sources, demands, conveyances capacities, etc. None has investigated FCSTP under destination budget constraints with transportation costs, fixed charges, etc.

Overcoming the above mentioned shortcomings, here we have considered cost minimization FCSTP under destination budget constraints with crisp and fuzzy data. In this paper, crisp and fuzzy FCSTPs are formulated as cost minimization problems under fuzzy resource (budget) constraints. The fuzzy objective and constraints are transformed to equivalent crisp forms using credibility measures. The above transportation problems are solved by generalized reduced gradient (GRG) method. The methods are illustrated with numerical examples.

2. Preliminaries

2.1. Credibility measure

Credibility measure was presented by Liu and Liu [6]. For a fuzzy variable $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$ and then for any set B⊂R of real numbers, credibility measure of fuzzy event $\{\tilde{A} \in B\}$ is defined as $Cr\{\tilde{A} \in B\} = (1/2)(Pos\{\tilde{A} \in B\} + Nec\{\tilde{A} \in B\})$, where possibility and necessity measures of $\{\tilde{A} \in B\}$ are respectively defined as

$Pos\{\tilde{A} \in B\} = \sup_{x \in B} \mu_{\tilde{A}}(x)$) and

$Nes\{\tilde{A} \in B\} = 1 - \sup_{x \in B^c} \mu_{\tilde{A}}(x)$

2. 2. Optimistic and pessimistic value (Liu[5])

Let $\tilde{A}$ be a fuzzy variable and $\beta \in [0,1]$. Then β-optimistic value of $\tilde{A}$ is denoted by $\tilde{A}_{sup(\beta)}$ and is defined as

$$\tilde{A}_{\sup(\beta)} = \sup\{r : Cr\{\tilde{A} \geq r\} \geq \beta\}$$

and Similarly β- pessimistic value of $\tilde{A}$ is denoted by $\tilde{A}_{inf(\beta)}$ and is defined as

$$\tilde{A}_{\inf(\beta)} = \inf\{r : Cr\{\tilde{A} \leq r\} \geq \beta\}$$

Lemma 1. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number. Then β-optimistic value is

$$\tilde{A}_{sup(\beta)} = 2\beta\, a_2 + (1 - 2\beta)a_3 \quad if \;\; \beta \leq 0.5$$

$$\tilde{A}_{sup(\beta)} = (2\beta - 1)a_1 + 2(1 - \beta)a_2 \quad if \;\; \beta > 0.5$$

Lemma 2. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number. Then β- pessimistic value is

$$\tilde{A}_{inf(\beta)} = (1 - 2\beta)a_1 + 2\beta\, a_2 \quad if \;\; \beta \leq 0.5$$

$$\tilde{A}_{inf(\beta)} = 2(1-\beta)a_2 + (2\beta - 1)\, a_3 \quad if \quad \beta > 0.5$$

3. Proposed FCSTPs with budget constraints

3.1. Assumptions and notations

In order to construct the mathematical model for the unbalanced FCSTPs under destination budget constraints, the following notations are introduced:

(i) **M** : number of origins/sources of the transportation problem. (ii) **N** : number of destinations/demands of the transportation problem. (iii) **K**: number of conveyances i.e. different modes of transporting units from sources to destinations. (iv) $\mathbf{A_i}$**:** amount of homogeneous product available at the i-th origin. (v) $\mathbf{B_j}$: demand at the j-th destination. (vi) $\mathbf{E_k}$: amount of the product which can be carried by k-th conveyance. (vii) $\mathbf{C_{ijk}}$: per unit transportation cost from i-th origin to j-th destination by k-th conveyance. (viii) $\mathbf{f_{ijk}}$**:** fixed transportation charge for transporting units from i-th origin to j-th destination by k-th conveyance. (ix) x_{ijk}: the amount transported from i-th origin to j-th destination by k-th conveyance.

Symbol ˜ is used with the above notations to represent corresponding fuzzy parameters. If the transportation activity is assigned from source i to destination j by conveyance k, then the fixed charge will be costed. This implies that if $x_{ijk} > \mathbf{0}$ we must add the fixed charge to the total transportation cost. Thus for the convenience of modelling, the following notation is introduced: $y_{ijk} = \mathbf{1}$ for $x_{ijk} \geq 0$, 0, otherwise.

3.2. Mathematical model formulation

Model-I (Crisp Model): In this model, fixed charge and unit transportation cost are crisp. Sources, destinations, conveyances capacities of transportation problem are also taken as crisp. So the problem mathematically takes the following form

$$\text{Minimize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left\{C_{ijk}^{1}\ x_{ijk} + f_{ijk}\, y_{ijk}\right\}$$

$$\text{Subject to } \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq A_i \ \ (i = 1,2,\dots,M)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq B_j \quad (j = 1,2,\dots,N)$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq E_k \quad (k = 1,2,\dots,K)$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\left\{C_{ijk}^{1}\, x_{ijk} + f_{ijk} y_{ijk}\right\} \leq Bud_j$$

$$x_{ijk} \geq 0$$

Model-II (Fuzzy Model): In this model, fixed charge and unit transportation cost are fuzzy. Sources, destinations, conveyances capacities of transportation problem are also taken as fuzzy. So the above problem mathematically takes the following form:

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left\{\tilde{C}_{ijk}^{1}\, x_{ijk} + \tilde{f}_{ijk} y_{ijk}\right\}$$

$$=(Z_1, Z_2, Z_3)$$

$$\text{Subject to } \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \tilde{A}_i \ \ (i = 1,2,\dots,M)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \widetilde{B}_j \quad (j = 1,2,\dots,N)$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \widetilde{E}_k \quad (k = 1,2,\dots,K)$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N}\{\tilde{C}_{ijk}^{1} x_{ijk} + \tilde{f}_{ijk}\, y_{ijk}\} \leq Bud_j$$
$$x_{ijk} \geq 0$$

4. Deterministic equivalent of imprecise model

Till date, for the best of our knowledge, optimization of fuzzy objectives with fuzzy constraints are not well defined. As a result Model-II can not be solved in the present form. Deterministic equivalent of model-II can be derived in the following approach. Here credibility measure is used, which is discussed below.

Model-IIA

Minimize f

Subject to $Cr((Z_1, Z_2, Z_3) \leq f) \geq \beta$

$$Cr\left(\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \widetilde{A}_i\right) \geq \beta \quad (i = 1,2,\dots,M)$$
$$Cr\left(\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \widetilde{B}_j\right) \geq \beta \quad (j = 1,2,\dots,N)$$
$$Cr\left(\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \widetilde{E}_k\right) \geq \beta \quad (k = 1,2,\dots,K)$$
$$Cr\left(Z_j \leq Bud_j\right) \geq \beta$$
$$x_{ijk} \geq 0$$

Model-IIA-1: When DM is optimistic, then he/she assumes value of $\beta \leq 0.5$ i.e. $(0 \leq \beta \leq 0.5)$ and hence using lemma-1,-2 , the **Model-IIA** reduces to

Minimize $O_1=(1-2\beta)\, Z_1+2\beta Z_2$

Subject to $\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq 2\beta A_{i2} + (1-2B)A_{i3} \quad (i = 1,2,\dots,M)$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq (1-2\beta)B_{j1} + 2\beta B_{j2} \quad (j = 1,2,\dots,N)$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq 2\beta E_{k2} + (1-2\beta)E_{k3} \quad (k = 1,2,\dots,K)$$
$$(1-2\beta)Z_{j1} + 2\beta Z_{j2} \leq Bud_j$$
$$x_{ijk} \geq 0$$

Model-IIA-2: When DM is pessimistic, then he/she assumes value of $\beta \geq 0.5$ i.e. $(0.5 \leq \beta \leq 1)$ and hence using lemmas-1,-2 , the **Model-IIA** reduces to

Minimize $O_2=2(1-\beta)\, Z_2+(2\beta-1)\, Z_3$

Subject to $\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq (2\beta-1)A_{i2} + 2(1-\beta)A_{i3} \quad (i = 1,2,\dots,M)$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq 2(1-\beta)B_{j1} + (2\beta-1)B_{j2} \quad (j = 1,2,\dots,N)$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq (2\beta-1)E_{k2} + 2(1-\beta)E_{k3} \quad (k = 1,2,\dots,K)$$
$$2(1-\beta)Z_{j1} + (2\beta-1)Z_{j2} \leq Bud_j$$
$$x_{ijk} \geq 0$$

5. Numerical experiment

For illustration of the models, flowing two transportation problems are considered.

5.1. Input data

For both models, no of origins=2 (i.e. M=2), no of destination=2 (i.e. N=2), no of conveyance=2 (i.e. K=2) are considered. Crisp and fuzzy unit transportation costs and fixed charges for Models-I and -II are given in Table-1. Parametric values for models are presented in Table-2. Credibility measure of constraints and objective, values of β are taken as 0.4 and 0.6 respectively.

Table 1: Unit transportation cost and fixed charge for models-I and -II

		k		1		2		1		2	
Models		i/j	1	2	1	2		1	2	1	1
I	C_{ijk}	1	3	6	2	5	f_{ijk}	10	9	8	7
		2	5	10	4	9		11	12	9	10
II	$\tilde{C}_{ijk}$	1	2,3,4	5,6,7	1,2,3	4,5,6	$\tilde{f}_{ijk}$	8,10,11	7,9,10	7,8,9	6,7,9
		2	4,5,7	8,10,12	3,4,5	7,9,10		10,11,12	11,12,13	8,9,10	9,10,11

Table 2: Parametric values for models

Models	Source (A_1, A_2)	Demand (B_1, B_2)	Capacities of Conveyances $(E_1,\ E_2)$	Budjet $\mathrm{Bud}_1, \mathrm{Bud}_2$
I	(25,24)	(14,21)	(25,22)	
II	{(24,25,26) ,(23,24,25)}	{(12,14,16), (19,21,23)}	{(23,25,27), (20,22,24)}	(105,115)

5.2. Optimum results

Table 3: Marketing decisions of Models-I and -II (using GRG)

Models	Budget constraint	β	k	1		2		Min cost (O_1, O_2)	Z Z_1, Z_2, Z_3	Utilized budget
			i/j	1	2	1	2			
I	With		1	3.2	0.0	0.6	21.	209.		
			2	9.9	0.0	0.3	0.0			
	Without		1	1.7	5.7	2.3	15.3	220.		
			2	5.6	0.0	4.3	0.0			
IIA-1	With	.4	1	3.9	0.0	0.7	20.6	196.9	164.8, 205.,253.	91.2,105.7
			2	7.9	0.0	1.1	0.0			
IIA-2		.6	1	4.3	0.0	0.7	19.7	215.	164.9, 205.1, 254.5	105., 110.
			2	9.3	0.0	0.3	0.0			
IIA-1	Without	.4	1	1.9	4.8	2.6	15.7	205.6	171.8,214.,260.2	
			2	5.0	0.0	3.9	0.0			
IIA-2		.6	1	1.7	12.4	1.7	8.9	223.0	172.2,215., 256.8	
			2	0.0	0.0	11.0	0.0			

With the above input data, transportation Models-I, IIA-1,IIA-2 are solved by GRG(LINGO 11.0 software).

6. Discussion

This paper presents fuzzy cost minimization FCSTPs under budget constraints at destinations with credibility measure. Several examples are used for the illustration. Table-3 furnish the results of crisp (Model-I) and fuzzy (Model-II) cost minimization FCSTP respectively following credibility approach. These results are without and with budget constraints. In all cases, it is observed from Table-3 that minimum cost for the models without budget constraint are more than those with budget constraints. To mention one case, in Table-3, say the minimum cost without budget constraint is 220. $ which is more than the minimum cost (209. $) with budget constraint. This is as per the usual expectation.

In Tables-3, lowest cost of all the models, IIA-1 and -2 models, give the lowest and highest minimum cost respectively. This is because both fuzzy objective function and transportation constraints have been changed by the credibility measures (average of possibility and necessity measures) in the ranges (0.0-0.5) and (0.5-1.0)for IIA-1 and IIA-2 respectively.

REFERENCES

1. J.Gottlieb and L.Paulmann, Genetic algorithms for the fixed charge transportation problems in: *Proceedings of the IEEE Conf. Evol. Comp.*, ICEC, 1998, 330-335.
2. K.B.Haley, The solid transportation problem, *Oper. Res.*, 11 (1962) 446-448.
3. W.M.Hirsch and G.B.Dantzig, The fixed charge transportation problem, *Naval Research Logistics Q.*, 15 (1968) 413-424.
4. F.L.Hitchcock, The distribution of the product from several sources to numerous localities, *Journal of Mathematical Physics*, 20 (1941) 224-230.
5. B.Liu, *Theory and Practice of Uncertain Prog.*, Physica-Verlag, Heidelberg, 2002.
6. B.Liu and Y.K.Liu, Expected value of the fuzzy variable and fuzzy expected value models, *IEEE Transactions on Fuzzy Systems*, 10 (2002) 445-450.
7. E.D.Schell, Distribution of a product by several properties, in: *Proceedings of 2nd Symposium in Linear Programming*, DCS/comptroller, HQ US Air Force, Washington DC, 1955, 615-642.
8. L.Yang and L.Liu, Fuzzy fixed charge solid transportation problem and algorithm, *Applied Soft Computing*, 7(2007) 879-889.