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$\tau_1\tau_2$ - #g Closed Sets in Bitopological Spaces

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Abstract. This paper is to introduce a new class of sets called $\tau_1\tau_2$ - #g-closed sets in topological spaces and to analyse the properties of this set.

Keywords: $\tau_1\tau_2$ - #g-closed sets, $\tau_1\tau_2$- #g-open sets

AMS Mathematics Subject Classification (2010): 54E55

1. Introduction

Levine [11] introduced semi open sets in 1963 and also Levine [12] introduced generalized closed sets in 1970. AbdEl–Monsef et al. [1] introduced β-open sets. Veerakumar[15] introduced #g-closed sets in topological spaces. Kelley [7] initiated the study of bitopological spaces in 1963. A nonempty set X equipped with two topological spaces $\tau_1$and τ_2 is called a bitopological space and is denoted by (X, $\tau_{1,}\tau_2$). Since then several topologists generalized many of the results in topological spaces to bitopological spaces. Fukutake [5] introduced generalized closed sets in bitopological spaces. Fukutake [6] introduced semi open sets in bitopologicalspaces.Rao and Mariasingam [3] defined and studied regular generalized closed sets in bitopological settings. This paper is to introduce a new class of sets called $\tau_1\tau_2$ – #g-closed sets in bitopological spaces and to study about its properties.

2. Preliminaries

Definition 2.1. A subset A of a bitopological space (X,τ_1,τ_2) is called a

1. $\tau_1\tau_2$-semi open if $A \subset \tau_2cl(\tau_1int(A))$ and it is called $\tau_1\tau_2$-semi closed if $\tau_2\, int(\tau_1cl(A)) \subset A$
2. $\tau_1\tau_2$-pre open if $A \subset \tau_2int(\tau_1cl(A))$ and $\tau_1\tau_2$-pre closed if $\tau_2cl(\tau_1int(A)) \subset A$.
3. $\tau_1\tau_2$-α-open if $A \subset \tau_1int(\tau_2cl(\tau_1int(A)))$.
4. $\tau_1\tau_2$-semi preopen if $A \subset \tau_1cl(\tau_2int(\tau_1cl(A)))$.
5. $\tau_1\tau_2$-regular open if $A = \tau_2int(\tau_1cl(A))$.
6. $\tau_1\tau_2$-regular closed if $A = \tau_2cl(\tau_1int(A))$.

Definition 2.2. A subset A of a bitopological space (X, τ_1 , τ_2) is called a

1. $\tau_1\tau_2$-g-closed set ($\tau_1\tau_2$-generallized closed set) if τ_2-cl(A) ⊂ U , whenever A ⊂ U, U is τ_1-open.
2. $\tau_1\tau_2$-sg-closed ($\tau_1\tau_2$-semi generallized closed set) if τ_2-scl(A) ⊂ U , whenever A ⊂ U, U is τ_1-semi open.

3. $\tau_1\tau_2$-gs-closed ($\tau_1\tau_2$- generallized semi closed set) if τ_2.scl(A) ⊂ U , whenever A ⊂ U, U is τ_1-open.
4. $\tau_1\tau_2$-αg-closed ($\tau_1\tau_2$-α- generallized closed set) if τ_2.αcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-open.
5. $\tau_1\tau_2$-gα-closed ($\tau_1\tau_2$- generallized α-closed set) if τ_2.αcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-α-open.
6. $\tau_1\tau_2$-gp-closed ($\tau_1\tau_2$- generallized pre-closed set) if τ_2.pcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-open.
7. $\tau_1\tau_2$-gsp-closed ($\tau_1\tau_2$- generalized semi preclosed set) if τ_2.spcl(A) ⊂ U , whenever A ⊂ U,U is τ_1-open.
8. $\tau_1\tau_2$-gpr-closed ($\tau_1\tau_2$- generallized pre regular closed set) if τ_2.pcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-regular open.
9. $\tau_1\tau_2$-μ-closed set if τ_2.cl(A) ⊂ U , whenever A ⊂ U, U is τ_1-gα*-open.
10. $\tau_1\tau_2$-ψ-closed set if τ_2.scl(A) ⊂ U , whenever A ⊂ U, U is τ_1-sg-open.
11. $\tau_1\tau_2$-pre semi closed set if τ_2.spcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-g-open.
12. $\tau_1\tau_2$-g*-closed set if τ_2.cl(A) ⊂ U , whenever A ⊂ U, U is τ_1-g-open.
13. $\tau_1\tau_2$-g*-pre closed set if τ_2.pcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-g-open.
14. $\tau_1\tau_2$-g^-closed set if τ_2.cl(A) ⊂ U , whenever A ⊂ U, U is τ_1-semi open.
15. $\tau_1\tau_2$-*g-closed if τ_2.cl(A) ⊂ U , whenever A ⊂ U, U is τ_1-g^-open.
16. $\tau_1\tau_2$-*g-semi closed if τ_2.scl(A) ⊂ U , whenever A ⊂ U, U is τ_1-g^-open.
17. $\tau_1\tau_2$-α*g-closed if τ_2.αcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-g^-open.
18. $\tau_1\tau_2$-μ-semi closed ($\tau_1\tau_2$-μ-closed) if τ_2.scl(A) ⊂ U , whenever A ⊂ U, U is τ_1-gα*-open.
19. $\tau_1\tau_2$-μ-pre closed ($\tau_1\tau_2$-μp-closed) if τ_2.pcl(A) ⊂ U , whenever A ⊂ U, U is τ_1-gα*-open.
20. $\tau_1\tau_2$-semi μ- closed if τ_2.scl(A) ⊂ U , whenever A ⊂ U, U is τ_1-gα**-open.
21. $\tau_1\tau_2$-$^{\#}$g-closed if τ_2.cl(A) ⊂ U , whenever A ⊂ U, U is τ_1-*g-open.

3. Basic properties of $\tau_1\tau_2$ - $^{\#}$g-closed sets

Definition 3.1. A subset A of (X,τ_1,τ_2) is called a $\tau_1\tau_2$ - $^{\#}$g-closed if τ_2– cl A ⊂ U, whenever A ⊂ U, U is τ_1 - *g-open in (X,τ_1).

Definition 3.2. The complement of $\tau_1\tau_2$ - $^{\#}$g-closed set is called $\tau_1\tau_2$ - $^{\#}$g-open set.

Example 3.3. Let X = { a, b, c }, τ_1 = { X, φ, {a} } & τ_2 = { X, φ, {a}, {a,b} }
$\tau_1\tau_2$ - $^{\#}$g-closed sets={ X, φ, {c}, {b,c} }.

Theorem 3.4. Every τ_2-closed set is $\tau_1\tau_2$ - $^{\#}$g-closed.
Proof: Let A be τ_2 - closed.Thenτ_2 - cl A = A
⇒ τ_2 – cl A ⊂ U, whenever A ⊂ U, where U is τ_1- *g-open.
⇒ A is $\tau_1\tau_2$ -$^{\#}$g-closed.

Theorem 3.5. Every $\tau_1\tau_2$ - $^{\#}$g - closed set is $\tau_1\tau_2$- g-closed.
Proof: Let A⊂ U, U is τ_1- open. Then U is τ_1-*g-open.
⇒τ_2– cl A ⊂U (∵ A is $\tau_1\tau_2$ - $^{\#}$g-closed)
⇒ A is $\tau_1\tau_2$ - g-closed.

Theorem 3.6. Every $\tau_1\tau_2$– g^*-closed set is $\tau_1\tau_2$ - $^{\#}$g-closed.
Proof: Let A $\subset$U , U is τ_1-*g-open.Then U is τ_1 -g-open.
$\Rightarrow\tau_2$–cl A $\subset$ U (By the assumption)
$\therefore$ A is $\tau_1\tau_2$ - $^{\#}$g-closed.

Theorem 3.7. Every $\tau_1\tau_2$ – $g^{\#}$-closed is $\tau_1\tau_2$ - $^{\#}$g-closed.
Proof: Let A $\subset$U , U is τ_1- *g-open.
U is τ_1- *g-open $\Rightarrow$ U is τ_1 - αg-open.
$\Rightarrow\tau_2$–cl A $\subset$ U [by our assumption]
$\therefore$ A is $\tau_1\tau_2$ - $^{\#}$g-closed.

Theorem 3.8. Every $\tau_1\tau_2$ - $^{\#}$g-closed set is $\tau_1\tau_2$ - gs-closed.
Proof: Let A $\subset$U , U is τ_1- open.
U is τ_1 - open $\Rightarrow$ U is τ_1 - *g-open.
$\Rightarrow$ τ_2–cl A $\subset$ U (By our assumption)
But τ_2– scl A$\subset\tau_2$–cl A $\subset$ U$\Rightarrow$ A is $\tau_1\tau_2$ - gs-closed.

Theorem 3.9. Every $\tau_1\tau_2$ - $^{\#}$g-closed set is $\tau_1\tau_2$ – αg-closed.
Proof: Let A $\subset$U , U is τ_1- open.
U is τ_1-open $\Rightarrow$ U is τ_1- *g-open.
$\Rightarrow\tau_2$– cl A $\subset$ U (By our assumption)
But τ_2 –αcl A$\subset\tau_2$ –cl A $\subset$ U$\Rightarrow$A is $\tau_1\tau_2$ - αg-closed.

Theorem 3.10. Every τ_1 τ_2 - $^{\#}$g-closed set is τ_1 τ_2 - gp-closed.
Proof: Assume that A is τ_1 τ_2 - $^{\#}$g-closed.
To prove A is τ_1 τ_2 – gp-closed.
Let A $\subset$U , U is τ_1-open.
U is τ_1-open $\Rightarrow$ U is τ_1-*g-open
$\Rightarrow\tau_2$–cl A $\subset$ U (by assumption)
But τ_2–pcl A$\subset\tau_2$–cl A$\subset$U$\Rightarrow$A is τ_1 τ_2 - gp-closed.

Theorem 3.11. Every τ_1 τ_2 - $^{\#}$g-closed set is τ_1 τ_2 - gpr-closed.
Proof: Assume that A is τ_1 τ_2 - $^{\#}$g-closed.To prove A is τ_1 τ_2 - gpr-closed.
Let A $\subset$U, U is τ_1 - regular open.
U is τ_1 - regular open $\Rightarrow$ U is τ_1 - open $\Rightarrow$ U is τ_1 *g-open.
$\Rightarrow\tau_2$ –cl A $\subset$ U (by assumption) .But τ_2 – pcl A $\subset\tau_2$– cl A$\subset$ U
$\Rightarrow\tau_2$– pcl A $\subset$U , whenever A $\subset$ U , U is τ_1-regular open.
$\Rightarrow$ A is τ_1 τ_2 - gpr-closed.

Theorem 3.12. Every $\tau_1\tau_2$ - $^{\#}$g-closed set is $\tau_1\tau_2$ – gsp-closed.
Proof: Assume that A is $\tau_1\tau_2$ - $^{\#}$g-closed.
To prove A is $\tau_1\tau_2$ – gsp-closed.Let A $\subset$U , U is τ_1 –open.
τ_1 - open $\Rightarrow$ τ_1 - *g-open$\Rightarrow$ τ_2– cl A $\subset$ U (by assumption)

$\Rightarrow\tau_2$– spcl A$\subset\tau_2$– cl A$\subset$ U$\Rightarrow$ $\tau_1\tau_2$ – gsp-closed.

Theorem 3.13. The converses of the above theorems are not true as can be seen by the following examples.

Example 3.14. Let X = { a, b, c },τ_1 = { X, φ, {a}, {b,c} } & τ_2 = { X, φ, {a} }
$\tau_1\tau_2$ - $^{\#}$g-closed sets = { X, φ, {b}, {c}, {a,b}, {a,c}, {b,c} }
Here {b}, {c}, {a,b}, {a,c} are τ_1 τ_2- $^{\#}$g-closed but they are not τ_2-closed.
τ_1 τ_2 – g*-closed = { X, φ, {b,c} }Here {b}, {c}, {b,c}, {a,b} are τ_1 τ_2 - $^{\#}$g-closed sets but they are not $\tau_1\tau_2$ - g*-closed. $\tau_1\tau_2$ - g$^{\#}$-closed sets = { X, φ, {b,c} }Here {b}, {c}, {b,c}, {a,b} are τ_1 τ_2 - $^{\#}$g-closed sets but they are not $\tau_1\tau_2$ -g$^{\#}$-closed.

Example 3.15. Let X = { a, b, c },τ_1 = { X, φ, {a}},τ_2 = { X, φ, {a}, {a,b} } τ_1 τ_2 - $^{\#}$g-closed sets = { X, φ, {c}, {b,c} }
τ_1 τ_2 - g-closed sets = { X, φ, {b}, {c}, {a,b}, {b,c}, {a,c} }
Here {b}, {a,b}, {a,c} are $\tau_1\tau_2$-g-closed sets but they are not $\tau_1\tau_2$ - $^{\#}$g-closed.
$\tau_1\tau_2$-gs-closed= { X, φ, {b}, {c}, {a,b}, {b,c}, {a,c} }
Here {b}, {a,b}, {a,c} are $\tau_1\tau_2$-gs-closed sets but they are not $\tau_1\tau_2$ - $^{\#}$g-closed.
$\tau_1\tau_2$-αg-closed={ X, φ, {b}, {c}, {a,b}, {b,c}, {a,c} }= $\tau_1\tau_2$-gp-closed= $\tau_1\tau_2$-gsp-closed.
Here {b}, {a,b}, {a,c} are $\tau_1\tau_2$-gs-closed, $\tau_1\tau_2$-gp-closed, $\tau_1\tau_2$-gsp-closed sets but they are not $\tau_1\tau_2$ - $^{\#}$g-closed.
$\tau_1\tau_2$-gpr-closed sets= { X, φ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c} }
Here {a},{b},{a,b},{a,c} are $\tau_1\tau_2$-gpr-closed but they are not $\tau_1\tau_2$-$^{\#}$g-closed.

Theorem 3.16. $\tau_1\tau_2$-$^{\#}$g-closedness is independent of $\tau_1\tau_2$-αclosedness, $\tau_1\tau_{2-}$semi closedness,$\tau_1\tau_2$-semipreclosedness and $\tau_1\tau_{2-}$preclosedness
Proof: Itcanbe seen from the following examples.

Example 3.17. Let X = { a, b, c },τ_1 = { X, φ, {a}{a c}},τ_2 = { X, φ, {a}, {a,b}}
$\tau_1\tau_2$ - $^{\#}$g-closed sets = { X, φ, {b}, {c}, {a,b}, {b,c} }
$\tau_1\tau_2$-α-closed sets={ X, φ, {b}, {c}, {b,c} }= $\tau_1\tau_{2-}$ semiclosed sets = $\tau_1\tau_{2-}$ preclosed sets= $\tau_1\tau_2$-semipreclosed sets.Here {a,b} is $\tau_1\tau_2$ - $^{\#}$g-closed set but it is not a $\tau_1\tau_2$-α-closed set, $\tau_1\tau_{2-}$ semiclosed set, $\tau_1\tau_{2-}$ preclosed set, $\tau_1\tau_2$-semipreclosed set.

Example 3.18. Let X = { a, b, c },τ_1 = { X, φ, {a}},τ_2 = { X, φ, {a}, {a,b} } τ_1 τ_2 - $^{\#}$g-closed sets = { X, φ, {c}, {b,c}}
$\tau_1\tau_2$-α-closed sets={ X, φ, {b}, {c}, {b,c} }= $\tau_1\tau_{2-}$ semiclosed sets = $\tau_1\tau_{2-}$ preclosed sets= $\tau_1\tau_2$-semipreclosedset.Here {b} is not a $\tau_1\tau_2$ - $^{\#}$g-closed set but it is a $\tau_1\tau_2$-α-closed set, $\tau_1\tau_{2-}$ semiclosed set, $\tau_1\tau_{2-}$ preclosed set, $\tau_1\tau_2$-semipreclosed set.

Theorem 3.19. τ_1 τ_2 - $^{\#}$g-closedness is independent of $\tau_1\tau_{2-}$ ψ closedness, $\tau_1\tau_{2-}$ gα closedness,$\tau_1\tau_{2-}$sgclosedness, $\tau_1\tau_{2-}$-*gclosedness, $\tau_1\tau_{2-}$-*gsclosedness$\tau_1\tau_{2-}$ α-*g closedness, $\tau_1\tau_{2-}$ μ closedness, $\tau_1\tau_{2-}$μs-closedness, $\tau_1\tau_{2-}$μp-closedness.
Proof: It follows from the following example

Example 3.20. Let X = { a, b, c },τ_1 = { X, φ, {a}},τ_2 = { X, φ, {a}, {a,b} } τ_1 τ_2 - $^{\#}$g-closed sets = { X, φ, {c}, {b,c}}
$\tau_1\tau_2$- ψ closed sets={ X, φ, {b}{ c}, {b,c}}= $\tau_1\tau_2$- gα closed sets=$\tau_1\tau_2$-sg closed sets. Here {b} is a $\tau_1\tau_2$- ψ closed set, $\tau_1\tau_2$- gα closed set,$\tau_1\tau_2$-sg closed set. But it is not a τ_1 τ_2 - $^{\#}$g-closed set.$\tau_1\tau_2$-*g closed sets={ X, φ, {b}{ c},{a,b},{a,c}, {b,c}}= $\tau_1\tau_2$-*gs closed sets= $\tau_1\tau_2$- α-*g closed sets=$\tau_1\tau_2$- μ closed sets=$\tau_1\tau_2$-μs-closed sets=$\tau_1\tau_2$-μp-closedsets.Here {b}{a,b},{a,c} are not τ_1 τ_2 - $^{\#}$g-closed sets.

Example 3.21. Let X = { a, b, c },τ_1 = { X, φ, {a},{b,c}},τ_2 = { X, φ, {a}} τ_1 τ_2 - $^{\#}$g-closed sets = { X, φ, {b} {c} {a,b}{a,c} {b,c}}
$\tau_1\tau_2$- ψ closed sets={ X, φ, {b}{ c}, {b,c}}= $\tau_1\tau_2$-μs-closed sets=$\tau_1\tau_2$-μp-closedsets= $\tau_1\tau_2$-*gs closed sets= $\tau_1\tau_2$- α-*g closed sets.Here {a,b},{a,c} are τ_1 τ_2 - $^{\#}$g-closed sets. But they are not $\tau_1\tau_2$- ψ closed sets, $\tau_1\tau_2$-μs-closed sets, $\tau_1\tau_2$ - μp-closedsets, $\tau_1\tau_2$-*gs closed sets, $\tau_1\tau_2$- α-*g closed sets.
$\tau_1\tau_2$- μ closed sets={ X, φ}{b.,c}}= $\tau_1\tau_2$-*g closed sets. Here {b},{c},{a,b}{{a,c} are τ_1 τ_2 - $^{\#}$g-closed sets. But they are not $\tau_1\tau_2$- μ closed sets, $\tau_1\tau_2$-*g closed sets.

Example 3.22. Let X = { a, b, c },τ_1 = { X, φ, {a},{b},{a,b}},τ_2 = { X, φ, {a}{a,b}}
τ_1 τ_2 - $^{\#}$g-closed sets = { X, φ, {c} {a,c} {b,c}}
$\tau_1\tau_2$-sg closed sets={ X, φ,{b} {c} {b,c}}= $\tau_1\tau_2$- gα closed sets. Here {a,c} is a τ_1 τ_2 - $^{\#}$g-closed set. But it is not a $\tau_1\tau_2$-sg closed set and $\tau_1\tau_2$- gα closed set. Also {b} is $\tau_1\tau_2$-sg closed set and $\tau_1\tau_2$- gα closed set. But it is not a τ_1 τ_2 - $^{\#}$g-closed set.

4. Properties of $\tau_1\tau_2$ -$^{\#}$g -closed sets and $\tau_1\tau_2$ -$^{\#}$g-open sets

Theorem 4.1. Union of two$\tau_1\tau_2$ -$^{\#}$g closed set is $\tau_1\tau_2$ -$^{\#}$g closed.
Proof: Assume that A and B are $\tau_1\tau_2$ -$^{\#}$g closed sets.
Let A∪B ⊂ U, where U is τ_1-*g -open.
Then A⊂ U and B ⊂ U.
⟹τ_2-cl(A) ⊂ U and τ_2-cl(B) ⊂ U
⟹τ_2-cl(A)∪τ_2-cl(B) ⊂ U
But τ_2-cl(A∪B) = τ_2-cl(A)∪τ_2-cl(B) ⊂ U
⟹ A∪B is $\tau_1\tau_2$ -$^{\#}$g closed set.

Theorem 4.2. Intersection of two $\tau_1\tau_2$-$^{\#}$g closed sets need not be $\tau_1\tau_2$ -$^{\#}$g -closed .
This can be seen from the following example.

Example 4.3. Let X = {a,b,c}, τ_1 = {ϕ,X,{a},{b,c}}, τ_2 = {ϕ,X,{a}}.
$\tau_1\tau_2$ --$^{\#}$g closed sets={ϕ,X,{b},{c},{a,b},{a,c},{b,c}}.Here {a,b},{a,c}are $\tau_1\tau_2$ --$^{\#}$g closed, but their intersection is not $\tau_1\tau_2$ --$^{\#}$g closed.

Theorem 4.4. Let A be $\tau_1\tau_2$ -$^{\#}$g closed and A⊂ B⊂τ_2-cl(A), then B is $\tau_1\tau_2$-$^{\#}$g -closed.
Proof: Let B ⊂U , Where U is τ_1-*g -open. Then A⊂ B ⊂ U ⟹τ_2-cl(A) ⊂U.
Given B ⊂τ_2-cl(A), but τ_2-cl(B) is the smallest closed set containing B.
∴ B ⊂τ_2-cl(B) ⊂τ_2-cl(A) ⊂⟹τ_2-cl(B) ⊂ U⟹ B is $\tau_1\tau_2$-$^{\#}$g closed.

Theorem 4.5. If A is $\tau_1\tau_2$ -$^{\#}$g -closed then τ_2-cl (A) - A does not contain any non-empty τ_1-*g - closed set.

Proof: Suppose τ_2-cl(A) - A contains a non-empty τ_1*g -closed set F.That is F $\subset\tau_2$-cl(A) - A

$\Rightarrow$F $\subset\tau_2$-cl(A) but F $\not\subset$ A$\Rightarrow$ F $\subset A^c$

$\Rightarrow$ A $\subset F^c$, where F^c is τ_1 *g -open$\Rightarrow\tau_2$-cl(A) $\subset F^c\Rightarrow$ F $\subset(\tau_2\text{-cl(A)})^c$

We have F $\subset\tau_2$-cl(A) $\cap(\tau_2\text{-cl(A)})^c = \Phi$.$\tau_2$-cl(A) – A does not contain any non-empty τ_1*g -closed set.

Theorem 4.6. Let A be $\tau_1\tau_2$ #g -closed. Then A is τ_2-closed if and only if τ_2-cl(A) – A isτ_1*g - closed set.

Proof: Suppose that A is $\tau_1\tau_2$ -$^{\#}$g - closed and τ_2-closed. Then τ_2-cl(A) = A.

$\Rightarrow\tau_2$-cl(A) – A = Φ, which is τ_1-*g -closed.

Conversely assume that A is $\tau_1\tau_2$ -$^{\#}$g -closed and τ_2-cl(A) – A is τ_1-*g - closed. Since A is $\tau_1\tau_2$ -$^{\#}$g -closed ,τ_2-cl(A) – A does not contain any non-empty τ_1 -*g -closed set$\Rightarrow\tau_2$-cl(A) – A = $\Phi\Rightarrow\tau_2$-cl(A) = A $\Rightarrow$ A is τ_2-closed.

Theorem 4.7. If A is $\tau_1\tau_2$ -$^{\#}$g -closed and A $\subset$ B $\subset\tau_2$-cl(A), then τ_2-cl(B) – B contains no non-empty τ_1 -*g -closed set.

Proof: By theorem 4.4, the proof follows.

Theorem 4.8. For each x $\in$ X, the singleton {x} is either τ_1-*g -closed or its complement $\{x\}^c$is$\tau_1\tau_2$ -$^{\#}$g -closed.

Proof: Suppose {x} is not τ_1 -*g -closed, then $\{x\}^c$ will not be τ_1-*g -open.

$\Rightarrow$ X is the only τ_1-*g open set containing $\{x\}^c$.

$\Rightarrow\tau_2$-cl$\{x\}^c\subset$ X $\Rightarrow$ $\{x\}^c$ is $\tau_1\tau_2$ -$^{\#}$g -closed.

$\Rightarrow$ {x} is $\tau_1\tau_2$ -$^{\#}$g - open set.

Theorem 4.9. Arbitrary union of $\tau_1\tau_2$ -$^{\#}$g -closed sets $\{A_i$, i $\in$ I} in a bitopological space
(X, τ_1,τ_2) is$\tau_1\tau_2$ -$^{\#}$g -closed if the family $\{A_i$, i$\in$I} is locally finite on X.

Proof: Let $\{A_i$/i $\in$ I} be locally finite in X and each A_i be $\tau_1\tau_2$ -$^{\#}$g -closed in X.

To prove $\cup A_i$ is $\tau_1\tau_2$ -$^{\#}$g -closed.Let $\cup A_i \subset$ U, where U is τ_1-*g -open.

$\Rightarrow$ $A_i\subset$ U, for every i $\in$ I$\Rightarrow\tau_2$-cl(A_i) $\subset$ U for every i $\in$ I$\Rightarrow\tau_2$ -cl(Ai) $\subset$ U.

$\Rightarrow\cup\tau_2$-cl(Ai) $\subset$U.Since { A_i } is locally finite , τ_2-cl($\cup$ A_i) = $\cup\tau_2$-cl(A_i)

$\Rightarrow\tau_2$-cl($\cup$ A_i) $\subset$ U$\Rightarrow\cup$ A_i is$\tau_1\tau_2$ -$^{\#}$g -closed.

Theorem 4.10. If A and B are $\tau_1\tau_2$ -$^{\#}$g –open sets in a bitopological space (X, τ_1,τ_2) then theirintersectionis a$\tau_1\tau_2$ -$^{\#}$g –open set.

Proof: If A and B are $\tau_1\tau_2$ -$^{\#}$g –open sets, then A^c and B^c are $\tau_1\tau_2$ -$^{\#}$g –closedsets . $A^c\cup B^c$is$\tau_1\tau_2$ -$^{\#}$g –closed by theorem 4.1. That is $(A \cap B)^c$ is $\tau_1\tau_2$ -$^{\#}$g –closed

$\Rightarrow$ A $\cap$B is $\tau_1\tau_2$ -$^{\#}$g –open set.

Theorem 4.11. The union of two $\tau_1\tau_2$ -$^{\#}$g –open sets is need not be $\tau_1\tau_2$ -$^{\#}$g –open in X. This can see from the following example.

Example 4.12. Let X = {a,b,c}, τ_1 = {ϕ,X,{a},{b,c}}, τ_2 = {ϕ,X,{a}}.
$\tau_1\tau_2$-$^{\#}$g closed sets={ϕ,X,{b},{c},{a,b},{a,c},{b,c}}.Here {b},{c}are $\tau_1\tau_2$ -$^{\#}$g open but their union is not $\tau_1\tau_2$ --$^{\#}$g open.

Theorem 4.13. If τ_2-intA ⊂B⊂A and A is $\tau_1\tau_2$ -$^{\#}$g –open in X, then B is also $\tau_1\tau_2$ -$^{\#}$g –open in X.
Proof: Suppose τ_2-intA ⊂B⊂A and A is $\tau_1\tau_2$ -$^{\#}$g –open in X.
Then $A^c \subset B^c \subset X$-$\tau_2$-intA =$\tau_2$-cl(X-A) =$\tau_2$-cl$A^c$. Since A^c is$\tau_1\tau_2$ -$^{\#}$g –closed by theorem 4.4 B^c is $\tau_1\tau_2$ -$^{\#}$g –closed ⟹ B is a $\tau_1\tau_2$ -$^{\#}$g –open set.

Theorem 4.14. A set A is $\tau_1\tau_2$ -$^{\#}$g –open if and only if F⊂τ_2-intA where F is τ_1-*g –closed and F⊂A.
Proof: If F⊂τ_2-intA, where F is τ_1-*g –closed and F⊂A.
⟹$A^c \subset F^c$=G where G is τ_1-*g –open and τ_2-clA^c ⊂ G
⟹A^c is$\tau_1\tau_2$ -$^{\#}$g closed is $\tau_1\tau_2$ -$^{\#}$g –open.
Conversely assume that A is $\tau_1\tau_2$ -$^{\#}$g –open and F⊂A, where F is τ_1-*g –closed.
Then $A^c \subset F^c$⟹τ_2-cl$A^c \subset F^c$ (Since A^c is $\tau_1\tau_2$ -$^{\#}$g closed).
⟹ F⊂X - τ_2-clA^c = τ_2-intA.

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