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Interval-valued Fuzzy Soft Matrix Theory

P.Rajarajeswari [1] **and** ***P.Dhanalakshmi*** [2]

[1]Department of Mathematics, Chikkanna Govt Arts College,Tirupur.
p.rajarajeswari29@gmail.com
[2]Department of Mathematics, Tiruppur Kumaran College for Women,Tirupur.
viga_dhanasekar@yahoo.co.in

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Abstract. Today is a world of uncertainty with its associated problems, which can be well handled by soft set theory. In this paper, we propose Interval valued fuzzy soft matrix ,its types with examples and some new operators on the basis of weights . We also study their properties .

Keywords Soft set, fuzzy soft set, , interval valued fuzzy soft set, interval valued fuzzy soft matrix.

AMS Mathematics subject classification (2010): 08A72

1. Introduction

In real life scenario, we face so many uncertainties, in all walks of life. Zadeh's classical concept of fuzzy set[20] is strong to deal with such type of problems. Since the initiation of fuzzy set theory, there are suggestions for higher order fuzzy sets for different applications in many fields. Among higher fuzzy sets intuitionistic fuzzy set introduced by Atanassov [1,2,3] have been found to be very useful and applicable.

Soft set theory has received much attention since its introduction by Molodtsov [11].The concept and basic properties of soft set theory are presented in [11,6]. Later on Maji et al. [7] have proposed the theory of fuzzy soft set. Majumdar et al. [9] have further generalised the concept of fuzzy soft sets. Maji et al. [8] extended fuzzy soft sets to intuitionistic fuzzy soft sets which is based on the combination of the intuitionistic fuzzy set [1] and soft set.

Yang et al. [18] presented the concept of the interval valued fuzzy soft sets by combining the interval valued fuzzy sets and soft set. They have also given an algorithm to solve interval valued fuzzy soft set based decision making problems.

Matrices play an important role in the broad area of science and engineering. However ,the classical matrix theory cannot solve the problems involving various types of uncertainties. Mondal et al. [12] and Shyamal et al. [17] initiated a matrix representation of a soft set and interval valued fuzzy set respectively. In [19], Yong et al initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. Borah et al. [4] extended fuzzy soft matrix theory and its application. In [5], Chetia et al and in [13,15] Rajarajeswari et al. defined intuitionistic fuzzy soft matrix and interval valued intuitionistic fuzzy soft matrix and its types. Also extended some operations and an algorithm for medical diagnosis in [14,16].In[10], Mitra Basu et al. described interval valued fuzzy soft matrix and its types.

In this paper, we extended the concept of Interval valued fuzzy soft matrix, and defined different types of matrices along with examples. We also studied some operators on the basis of weights and their properties .

2. Preliminaries

2.1. Soft set [6]

Suppose that U is an initial Universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair(F,E) is called a soft set over U where F is a mapping given by F: E→P(U).Clearly a soft set is a mapping from parameters to P(U)and it is not a set, but a parameterized family of subsets of the Universe.

2.2. Fuzzy soft set [7]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: $A \to I^U$,where I^U denotes the collection of all fuzzy subsets of U.

2.3. Fuzzy Soft Matrices [19]

Let $U=\{c_1,c_2,c_3\ldots c_m\}$ be the Universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3\ldots e_n\}$. Let $A \subseteq E$ and (F,A) be a fuzzy soft set in the fuzzy soft class (U,E). Then fuzzy soft set (F,A) in a matrix form as $A_{mxn}=[a_{ij}]_{mxn}$ or $A=[a_{ij}]$ $i=1,2,\ldots m, j=1,2,3,\ldots..n$

where $a_{ij} = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases}$ $\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$.

2.4. Interval valued fuzzy soft set [18]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F,A) is called interval valued fuzzy soft set over U where F is a mapping given by F: $A \to I^U$, where I^U denotes the collection of all interval valued fuzzy subsets of U.

3. Interval valued fuzzy soft matrix theory

3.1. Interval valued fuzzy soft matrix

Let $U=\{c_1,c_2,c_3\ldots c_m\}$ be an Universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3\ldots e_n\}$.Let $A \subseteq E$ and (F,A) be a interval valued fuzzy soft set over U, where F is a mapping given by F: $A \to I^U$, where I^U denotes the collection of all interval valued fuzzy subsets of U. Then the interval valued fuzzy soft set can be expressed in matrix form as

$\tilde{A}_{mxn}=[a_{ij}]_{mxn}$ or $\tilde{A}=[a_{ij}]$ $i=1,2,\ldots m, j=1,2,3,\ldots..n$

where $a_{ij} = \begin{cases} \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right] & \text{if } e_j \in A \\ [0,0] & \text{if } e_j \notin A \end{cases}$

$\left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$ represents the membership of c_i in the interval valued fuzzy set $F(e_j)$.

Note that if $\mu_{jU}(c_i) = \mu_{jL}(c_i)$ then the interval-valued fuzzy soft matrix (**IVFSM)** reduces to an **FSM.**

Example 3.1. Suppose that there are four houses under consideration, namely the universes U={ h_1,h_2,h_3,h_4 },and the parameter set E={e_1,e_2,e_3, e_4 } where e_i stands for "beautiful", "large", "cheap", and "in green surroundings" respectively. Consider the mapping F from parameter set $A = \{e_1, e_2\} \subseteq E$ to the set of all interval valued fuzzy subsets of power set U. Consider an interval valued fuzzy soft set (F,A) which describes the "attractiveness of houses" that is considering for purchase. Then interval valued fuzzy soft set (F,A) is

(F,A) = { $F(e_1)$ = {(h_1,[0.6, 0.8]), (h_2,[0.8, 0.9]), (h_3,[0.6, 0.7]), (h_4,[0.5, 0.6])}
$F(e_2)$ = {(h_1,[0.7, 0.8]), (h_2,[0.6, 0.7]),(h_3,[0.5, 0.7]), (h_4,[0.8, 0.9])}}

We would represent this interval valued fuzzy soft set in matrix form as

$$\begin{pmatrix} [0.6,0.8] & [0.7,0.8] & [0.0,0.0] & [0.0,0.0] \\ [0.8,0.9] & [0.6,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.6,0.7] & [0.5,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.5,0.6] & [0.8,0.9] & [0.0,0.0] & [0.0,0.0] \end{pmatrix}$$

3.2. Interval valued fuzzy soft sub matrix

Let $\tilde{A}$ = $[a_{ij}] \in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = $[b_{ij}] \in$ **IVFSM** $_{mxn}$, Then $\tilde{A}$ is a **Interval valued Fuzzy Soft sub matrix** of $\tilde{B}$, denoted by $\tilde{A} \subseteq \tilde{B}$ if

$\mu_{\tilde{A}L} \le \mu_{\tilde{B}L}$,$\mu_{\tilde{A}U} \le \mu_{\tilde{B}U}$ for all i and j.

3.3. Interval valued fuzzy soft null (zero) matrix

An interval valued fuzzy soft matrix of order mxn is called interval valued fuzzy soft null (zero)matrix if all its elements are [0,0].It is denoted by $\tilde{\Phi}$.

3.4. Interval valued fuzzy soft universal matrix

An interval valued fuzzy soft matrix of order mxn is called interval valued fuzzy soft universal matrix if all its elements are [1,1].It is denoted by $\tilde{U}$.

Proposition 3.1. Let $\tilde{A}$ = $[a_{ij}] \in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = $[b_{ij}] \in$ **IVFSM** $_{mxn}$, $\tilde{C}$ = $[c_{ij}] \in$ **IVFSM** $_{mxn}$ then

$i)\ \tilde{\Phi} \subseteq \tilde{A}$

ii) $\tilde{A} \subseteq \tilde{U}$

$iii)\ \tilde{A} \subseteq \tilde{A}$

$iv)\ \tilde{A} \subseteq \tilde{B},\ \tilde{B} \subseteq \tilde{C} \Rightarrow \tilde{A} \subseteq \tilde{C}$

Proof: It follows from the definition**.**

3.5. Interval valued fuzzy soft equal matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] $\in$ **IVFSM** $_{mxn}$, Then $\tilde{A}$ is equal to $\tilde{B}$,denoted by $\tilde{A} = \tilde{B}$ if $\mu_{AL} = \mu_{BL}, \mu_{AU} = \mu_{BU}$ for all i and j.

3.6. Interval valued fuzzy soft transpose matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$ where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$.

Then $\tilde{A}^T$ is interval valued fuzzy soft transpose matrix of $\tilde{A}$ if $\tilde{A}^T$ = [a_{ji}] i=1,2,..m, j= 1,2,…n. $\tilde{A}^T$ = [a_{ji}] $\in$ **IVFSM** $_{nxm}$.

3.7. Interval valued fuzzy soft rectangular matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$,where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$.

Then $\tilde{A}$ is called interval valued Fuzzy Soft rectangular Matrix if m $\neq$ n.

3.8. Interval valued fuzzy soft square matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$.

Then $\tilde{A}$ is called interval valued Fuzzy Soft square Matrix if m= n.

3.9. Interval valued fuzzy soft row matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$.

Then $\tilde{A}$ is called interval valued Fuzzy Soft row Matrix if m= 1.It means that the universal set contains only one element.

3.10. Interval valued fuzzy soft column matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then $\tilde{A}$ is called interval valued Fuzzy Soft Column Matrix if n=1. It means that the parameter set contains only one parameter.

3.11. Interval valued fuzzy soft diagonal matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$.

Then $\tilde{A}$ is called interval valued Fuzzy Soft diagonal Matrix if m= n and $a_{ij} = [0,0]$ for all i $\neq$ j.

3.12. Interval valued fuzzy soft scalar matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then $\tilde{A}$ is called interval valued Fuzzy Soft scalar Matrix if m= n and $a_{ij} = [0,0]$ for all i $\neq$ j and $a_{ij} = [\alpha_1, \alpha_2]$ $\alpha_1, \alpha_2 \in [0,1], \alpha_1 \leq \alpha_2$ $\forall$ i =j

3.13. Interval valued fuzzy soft upper triangular matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then $\tilde{A}$ is called interval valued Fuzzy Soft upper triangular Matrix if m= n and $a_{ij} = [0,0]$ for all i> j

3.14. Interval valued fuzzy soft lower triangular matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then $\tilde{A}$ is called interval valued Fuzzy Soft lower triangular Matrix if m= n and $a_{ij} = [0,0]$ for all i< j .

3.15 Interval valued fuzzy soft triangular matrix

An interval valued Fuzzy Soft Matrix is said to be triangular if it is either interval valued Fuzzy Soft lower triangular Matrix or interval valued Fuzzy Soft upper triangular Matrix.

3.16. Addition of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] $\in$ **IVFSM** $_{mxn}$, then we define $\tilde{A}+\tilde{B}$,addition of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A}+\tilde{B}$ = [c_{ij}]$_{mxn}$ = [max(μ_{AL}, μ_{BL}),max(μ_{AU}, μ_{BU})] for all i and j .

Example 3.2. Consider

$$\tilde{A} = \begin{pmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{pmatrix}_{2X2} \text{ and } \tilde{B} = \begin{pmatrix} [0.8, 0.9] & [0.6, 0.7] \\ [0.6, 0.7] & [0.5, 0.7] \end{pmatrix}_{2X2}$$

are two interval valued fuzzy soft matrices then sum of these two is

$$\tilde{A}+\tilde{B} = \begin{pmatrix} [0.8, 0.9] & [0.7, 0.8] \\ [0.6, 0.7] & [0.8, 0.9] \end{pmatrix}_{2X2}$$

Proposition 3.2. Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] $\in$ **IVFSM** $_{mxn}$, $\tilde{C}$ = [c_{ij}] $\in$ **IVFSM** $_{mxn}$ then

$i)\ \tilde{A}+\tilde{\Phi}=\tilde{A}$ ii) $\tilde{A}+\tilde{U}=\tilde{U}$

$iii)\ \tilde{A}+\tilde{B}=\tilde{B}+\tilde{A}$ $iv)\ (\tilde{A}+\tilde{B})+\tilde{C}=\tilde{A}+(\tilde{B}+\tilde{C})$

$v)\ (\tilde{A}+\tilde{B})^T=\tilde{A}^T+\tilde{B}^T$ $vi)\ (\tilde{A}+\tilde{B}+\tilde{C})^T=\tilde{A}^T+\tilde{B}^T+\tilde{C}^T$

$vii)(\tilde{A}^T)^T=\tilde{A}$

Proof: It follows from the definition**.**

3.17. Subtraction of interval valued fuzzy soft matrices

If $\tilde{A}$ = $[a_{ij}]$ $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = $[b_{ij}]$ $\in$ **IVFSM** $_{mxn}$, then we define $\tilde{A}$ $\tilde{B}$,subtraction of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A}$ - $\tilde{B}$ = $[c_{ij}]_{mxn}$
= $[\min(\mu_{AL},\mu_{BL}),\min(\mu_{AU},\mu_{BU})]$ for all i and j

Example 3.2. Consider

$\tilde{A}=\begin{pmatrix}[0.6,0.8] & [0.7,0.8]\\ [0.5,0.6] & [0.8,0.9]\end{pmatrix}_{2X2}$ and $\tilde{B}=\begin{pmatrix}[0.8,0.9] & [0.6,0.7]\\ [0.6,0.7] & [0.5,0.7]\end{pmatrix}_{2X2}$

are two interval valued fuzzy soft matrices then subtraction of these two is

$$\tilde{A}-\tilde{B}=\begin{pmatrix}[0.6,0.8] & [0.6,0.7]\\ [0.5,0.6] & [0.5,0.7]\end{pmatrix}_{2X2}$$

Proposition 3.3.

Let $\tilde{A}$ = $[a_{ij}]$ $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = $[b_{ij}]$ $\in$ **IVFSM** $_{mxn}$, $\tilde{C}$ = $[c_{ij}]$ $\in$ **IVFSM** $_{mxn}$ then

$i)\ \tilde{A}-\tilde{\Phi}=\tilde{\Phi}$ ii) $\tilde{A}-\tilde{U}=\tilde{A}$

$iii)\ \tilde{A}-\tilde{B}=\tilde{B}-\tilde{A}$ $iv)\ (\tilde{A}-\tilde{B})-\tilde{C}=\tilde{A}-(\tilde{B}-\tilde{C})$

$v)(\tilde{A}+\tilde{B})-\tilde{C}=(\tilde{A}-\tilde{C})+(\tilde{B}-\tilde{C})$ $vi)(\tilde{A}-\tilde{B})+\tilde{C}=(\tilde{A}+\tilde{C})-(\tilde{B}+\tilde{C})$

$vii)\tilde{A}-(\tilde{B}+\tilde{C})=(\tilde{A}-\tilde{B})+(\tilde{A}-\tilde{C})$

Proof: It follows from the definition**.**

3.18. Product of interval valued fuzzy soft matrices

If $\tilde{A}$ = $[a_{ij}]$ $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = $[b_{jk}]$ $\in$ **IVFSM** $_{nxp}$, then we define $\tilde{A}*\tilde{B}$, multiplication of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A}*\tilde{B}$ = $[c_{ik}]_{mxp}$ = $[\max\min(\mu_{ALj},\mu_{BLj}),\max\min(\mu_{AUj},\mu_{BUj})]$, $\forall$i,j,k

Example 3.2. Consider

$$\tilde{A}=\begin{pmatrix}[0.6,0.8] & [0.7,0.8]\\ [0.5,0.6] & [0.8,0.9]\end{pmatrix}_{2X2}$$ and

$$\tilde{B}=\begin{pmatrix}[0.8,0.9] & [0.6,0.7]\\ [0.6,0.7] & [0.5,0.7]\end{pmatrix}_{2X2}$$

are two interval valued fuzzy soft matrices then product of these two matrices is

$$\tilde{A}*\tilde{B}=\begin{pmatrix}[0.6,0.8] & [0.6,0.7]\\ [0.6,0.7] & [0.5,0.7]\end{pmatrix}_{2X2}$$

Remark: $\tilde{A}*\tilde{B} \neq \tilde{B}*\tilde{A}$.

3.18. Interval valued fuzzy soft complement matrix

Let $\tilde{A}$ = $[a_{ij}]$ $\in$ **IVFSM** $_{mxn}$, where $a_{ij}=\left[\mu_{jL}(c_i),\mu_{jU}(c_i)\right]$.
Then $\tilde{A}^C$ is called interval valued Fuzzy Soft Complement Matrix if $\tilde{A}^C$ =$[b_{ij}]_{mxn}$ where $b_{ij}=\left[1\text{-}\mu_{jU}(c_i), 1-\mu_{jL}(c_i)\right]$, $\forall$i,j.

Example 3.3.

Let $\tilde{A}=\begin{pmatrix}[0.6,0.8] & [0.7,0.8]\\ [0.5,0.6] & [0.8,0.9]\end{pmatrix}_{2X2}$

be interval valued fuzzy soft matrix then complement of this matrix is

$$\tilde{A}^C=\begin{pmatrix}[0.2,0.4] & [0.2,0.3]\\ [0.4,0.5] & [0.1,0.2]\end{pmatrix}_{2X2}$$

Proposition 3.4.

$i)\ (\tilde{A}^c)^c=\tilde{A}$ ii) $\tilde{\Phi}^c=\tilde{U}$

$iii)\ (\tilde{A}+\tilde{U})^c=\tilde{\Phi}$ $iv)\ (\tilde{A}+\tilde{B})^c=(\tilde{B}+\tilde{A})^c$

$v)\ (\tilde{A}\text{-}\tilde{B})^c=(\tilde{B}-\tilde{A})^c$ $vi)(\tilde{A}+\tilde{B})^c=\tilde{A}^c-\tilde{B}^c$

$vii)(\tilde{A}\text{-}\tilde{B})^c=\tilde{A}^c+\tilde{B}^c$ $viii)(\tilde{A}^c)^T=(\tilde{A}^T)^c$

$ix)(\tilde{A}^c\text{-}\tilde{B}^c)^T=(\tilde{A}^T)^c+(\tilde{B}^T)^c$ $x)(\tilde{A}^c\text{-}\tilde{B}^c)^c=\tilde{A}+\tilde{B}$

$xi)(\tilde{A}^c+\tilde{B}^c)^c=\tilde{A}-\tilde{B}$

Proof: It follows from the definition**.**

3.19. Scalar multiple of interval valued fuzzy soft matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then scalar multiple of interval valued Fuzzy Soft Matrix A by a scalar k is defined by k$\tilde{A}$ =[ka_{ij}]$_{mxn}$ where $0 \le k \le 1$.

Example 3.4.

Let $\tilde{A} = \begin{pmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{pmatrix}_{2X2}$

be an interval valued fuzzy soft matrix then the scalar multiple of this matrix by k = 0.5 is

$$k\tilde{A} = \begin{pmatrix} [0.3, 0.4] & [0.35, 0.4] \\ [0.25, 0.3] & [0.4, 0.45] \end{pmatrix}_{2X2}$$

Proposition 3.5.

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$.

If *m,n* are two scalars such that

$0 \le m, n \le 1$,then

i) m(n$\tilde{A}$) = (mn)$\tilde{A}$ ii) m $\le$ n $\Rightarrow m\tilde{A} \le n\tilde{A}$

$iii)\tilde{A} \subseteq \tilde{B} \Rightarrow m\tilde{A} \subseteq m\tilde{B}$ iv)m(n$\tilde{A}^T$) = (mn)$\tilde{A}^T$

v)m(n$\tilde{A}^c$) = (mn)$\tilde{A}^c$

Proof: It follows from the definition**.**

3.20. Trace of interval valued fuzzy soft matrix

Let $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$,where m=n and $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then **Trace of interval valued Fuzzy Soft Matrix** $\tilde{A}$ is $tr\tilde{A} = \left[\max\left(\mu_{jL}(c_j)\right), \max\left(\mu_{jU}(c_j)\right)\right]$.

Example 3.5.

Let $\tilde{A} = \begin{pmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{pmatrix}_{2X2}$

be an interval valued fuzzy soft matrix then trace of this matrix is

$tr\tilde{A} = [0.8, 0.9]$

Proposition 3.6.

Let $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxm}$, if k is a scalar such that $0 \le k \le 1$,then

$i) tr(k\tilde{A}) = k\ tr\tilde{A}$ $\quad$ $ii) (k\tilde{A})^T = k\ \tilde{A}^T$

$iii) tr(\tilde{A} + \tilde{B}) = tr\tilde{A} + tr\tilde{B}$ $\quad$ $iv) tr(k\tilde{A}^c) = k\ tr\tilde{A}^c$

$v) (k\tilde{A}^c)^T = k\ (\tilde{A}^c)^T$ $\quad$ $vi) tr(\tilde{A}^c + \tilde{B}^c) = tr\tilde{A}^c + tr\tilde{B}^c$

Proof: It follows from the definition**.**

3.21. Interval valued fuzzy soft symmetric matrix

Let $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{nxn}$,where $a_{ij} = \left[\mu_{jL}(c_i), \mu_{jU}(c_i)\right]$. Then an interval valued fuzzy soft square matrix $\tilde{A}$ is called an **Interval valued Fuzzy Soft Symmetric Matrix** if $\tilde{A} = \tilde{A}^T$ ie., if $[a_{ij}] = [a_{ji}]$ ∀ i,j.

Example 3.6.

Let $\tilde{A} = \begin{pmatrix} [0.8, 0.9] & [0.4, 0.5] \\ [0.4, 0.5] & [0.7, 0.8] \end{pmatrix}_{2X2}$

be an interval valued fuzzy soft matrix, then the transpose of this matrix is $\tilde{A}^T = \begin{pmatrix} [0.8, 0.9] & [0.4, 0.5] \\ [0.4, 0.5] & [0.7, 0.8] \end{pmatrix}_{2X2}$

since $\tilde{A} = \tilde{A}^T$, $\tilde{A}$ is called an interval valued Fuzzy Soft Symmetric Matrix of order 2.

Special operators of interval valued fuzzy soft matrices

3.19. Arithmetic mean of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] ∈ **IVFSM** $_{mxn}$, then we define $\tilde{A} \& \tilde{B}$,arithmetic mean of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A} \& \tilde{B}$ = [c_{ij}]$_{mxn}$

$$= \left(\left[\left(\frac{\mu_{\tilde{A}L} + \mu_{\tilde{B}L}}{2}\right), \left(\frac{\mu_{\tilde{A}U} + \mu_{\tilde{B}U}}{2}\right)\right]\right) \text{for all i and j}$$

3.20. Weighted arithmetic mean of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] ∈ **IVFSM** $_{mxn}$, then we define $\tilde{A} \&_w \tilde{B}$, weighted arithmetic mean of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A} \&_w \tilde{B}$ = [c_{ij}]$_{mxn}$

$$= \left(\left[\left(\frac{w_{1L}\mu_{\tilde{A}L} + w_{2L}\mu_{\tilde{B}L}}{w_{1L} + w_{2L}} \right), \left(\frac{w_{1U}\mu_{\tilde{A}U} + w_{2U}\mu_{\tilde{B}U}}{w_{1U} + w_{2U}} \right) \right] \right)$$

$w_{1L}, w_{1U}, w_{2L}, w_{2U} \in [0,1]$

$w_{1L} + w_{1U} = 1, w_{2L} + w_{2U} = 1$

3.21. Geometric mean of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] ∈ **IVFSM** $_{mxn}$, then we define $\tilde{A}\$\tilde{B}$,geometric mean of $\hat{A}$ and $\tilde{B}$ as

$\tilde{A}\$\tilde{B}$ = [c_{ij}]$_{mxn}$

$$= \left(\left[\sqrt{\mu_{\tilde{A}L}.\mu_{\tilde{B}L}}, \sqrt{\mu_{\tilde{A}U}.\mu_{\tilde{B}U}} \right] \right)$$

3.22. Weighted geometric mean of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] ∈ **IVFSM** $_{mxn}$, then we define $\tilde{A}\$_w\tilde{B}$, weighted geometric mean of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A}\$_w\tilde{B}$ = [c_{ij}]$_{mxn}$

$$= \left(\left[((\mu_{\tilde{A}L})^{W_{1L}}.(\mu_{\tilde{B}L})^{W_{2L}})^{\frac{1}{W_{1L}+W_{2L}}}, ((\mu_{\tilde{A}U})^{W_{1U}}.(\mu_{\tilde{B}U})^{W_{2U}})^{\frac{1}{W_{1U}+W_{2U}}} \right] \right)$$

$w_{1L}, w_{1U}, w_{2L}, w_{2U} \in [0,1]$

$w_{1L} + w_{1U} = 1, w_{2L} + w_{2U} = 1$

3.23. Harmonic mean of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] ∈ **IVFSM** $_{mxn}$,then we define $\tilde{A}@\tilde{B}$ harmonic mean of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A}@\tilde{B}$ = [c_{ij}]$_{mxn}$

$$= \left(\left[2\frac{\mu_{\tilde{A}L}.\mu_{\tilde{B}L}}{\mu_{\tilde{A}L}+\mu_{\tilde{B}L}}, 2\frac{\mu_{\tilde{A}U}.\mu_{\tilde{B}U}}{\mu_{\tilde{A}U}+\mu_{\tilde{B}U}} \right] \right) \text{for all i and j}.$$

3.24. Weighted Harmonic mean of interval valued fuzzy soft matrices

If $\tilde{A}$ = [a_{ij}] ∈ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] ∈ **IVFSM** $_{mxn}$, then we define $\tilde{A}@\tilde{B}$,weighted harmonic mean of $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A}@_W\tilde{B}$ = [c_{ij}]$_{mxn}$ =

$$\left(\left[\frac{(\mu_{\tilde{A}L}.\mu_{\tilde{B}L})(w_{1L}+w_{2L})}{w_{2L}\mu_{\tilde{A}L}+w_{1L}\mu_{\tilde{B}L}},\frac{(\mu_{\tilde{A}U}.\mu_{\tilde{B}U})(w_{1U}+w_{2U})}{w_{2U}\mu_{\tilde{A}U}+w_{1U}\mu_{\tilde{B}U}}\right]\right)$$

$$w_{1L}, w_{1U}, w_{2L}, w_{2U} \in [0,1]$$

$$w_{1L}+w_{1U}=1, w_{2L}+w_{2U}=1$$

Proposition 3.7.

If $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] $\in$ **IVFSM** $_{mxn}$, then

i) $\tilde{A}\&\tilde{B}=\tilde{B}\&\tilde{A}$ and $(\tilde{A}\&\tilde{B})^C=(\tilde{B}\&\tilde{A})^C$

ii) $\tilde{A}\$\tilde{B}=\tilde{B}\$\tilde{A}$ and $(\tilde{A}\$\tilde{B})^C=(\tilde{B}\$\tilde{A})^C$

iii) $\tilde{A}@\tilde{B}=\tilde{B}@\tilde{A}$ and $(\tilde{A}@\tilde{B})^C=(\tilde{B}@\tilde{A})^C$

iv) $\tilde{A}\&_w\tilde{B}=\tilde{B}\&_w\tilde{A}$ and $(\tilde{A}\&_W\tilde{B})^C=(\tilde{B}\&_W\tilde{A})^C$

v) $\tilde{A}\$_w\tilde{B}=\tilde{B}\$_w\tilde{A}$ and $(\tilde{A}\$_W\tilde{B})^C=(\tilde{B}\$_W\tilde{A})^C$

vi) $\tilde{A}@_w\tilde{B}=\tilde{B}@_w\tilde{A}$ and $(\tilde{A}@_W\tilde{B})^C=(\tilde{B}@_W\tilde{A})^C$

Proof: It follows from the definition**.**

Proposition 3.8.

If $\tilde{A}$ = [a_{ij}] $\in$ **IVFSM** $_{mxn}$, $\tilde{B}$ = [b_{ij}] $\in$ **IVFSM** $_{mxn}$, then

i) $(\tilde{A}^c\&\tilde{B}^c)^c=\tilde{A}\&\tilde{B}$ ii) $(\tilde{A}^c\$\tilde{B}^c)^c=\tilde{A}\$\tilde{B}$

iii) $(\tilde{A}^c@\tilde{B}^c)^c=\tilde{A}@\tilde{B}$ *iv*) $(\tilde{A}^c\&_w\tilde{B}^c)^c=\tilde{A}\&_w\tilde{B}$

v) $(\tilde{A}^c\$_w\tilde{B}^c)^c=\tilde{A}\$_w\tilde{B}$ *vi*) $(\tilde{A}^c@_w\tilde{B}^c)^c=\tilde{A}@_w\tilde{B}$

Proof: It follows from the definition**.**

4. Conclusion

In this paper, we proposed the concept of Interval valued fuzzy soft matrix, and defined different types of matrices in Interval valued fuzzy soft set theory along with examples. Then we have studied some new operations and properties on the basis of weights. As far as future directions are concerned, we hoped that our finding would help enhancing this study on Interval valued fuzzy soft set theory. In future, we will use special operators which are proposed in this paper in decision making problem in medical diagnosis .We will compare the results with other types of disease diagnosis.

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