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Distances in Weighted Graphs

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Abstract. The concept of *distance* is one of the basic concepts in Mathematics. How far two objects (vertices) are apart in a discrete structure is of interest, both theoretically and for its applications. Since discrete structures are naturally modeled by graphs, this leads us to studying distance in graphs. Starting from Menger, an explosion of interest in finite metric spaces occurred. Now finite distance metric have become an essential tool in many areas of Mathematics. This paper discussing about four distances in weighted graphs, namely w- distance d_w, strong geodesic distance d_{sg}, strongest strong distance d_{ss} and δ-distance δ. They are all different metrics in weighted graphs. When strength of connectedness between every pair of vertices u and v in G equals to the weight of the edge (u, v), G becomes self-centered with respect to the metrics d_w, d_{sg}, d_{ss} and δ. Also it is proved that every connected weighted graph is ss-self centered as well as δ –self centered.

Keywords: Partial tree, self-centered graph, strong path, central vertex

AMS Mathematics Subject Classification (2010): 05C12, 05C12, 05C22, 05C38, 05C40

1. Introduction

Weighted graphs are as old as that of graphs. In majority of applications related with graphs, especially in networks, weighted graph models are used. Minimum and maximum spanning tree problems are well known. Several authors including Bondy and Fan [1], Lin, Huang,Tan and Hsu [2] and Mathew and Sunitha [6-13] introduced many concepts in weighted graphs. And some related works are seen in [5] and [14].

We consider only undirected graphs without loops or multiple edges. Let $V(G)$ and $E(G)$ denote the set of vertices and edges of a graph G, respectively. For simplicity, we use V instead of $V(G)$ if no confusion occurs. G is called a weighted graph if each edge e is assigned a nonnegative weight $w(e)$ called the weight of e. For a subgraph H of G, the weight of H is defined by, $w(H) = \sum_{e \in E(H)} w(e)$. An unweighted graph can be regarded as a weighted graph in which each edge e is assigned weight $w(e) = 1$. A path in a weighted graph G (weighted path) is a sequences of vertices and edges with a nonzero weight assigned to each edge. A weighted graph G is connected, if every pair of vertices are connected by a weighted path. Two paths say P_1 and P_2 are said to be edge disjoint if they do not have any common edge and vertex disjoint or simply disjoint if they do not share any common vertex. Two $u - v$ paths are said to be internally disjoint, if they have no common vertices other than u and v. A maximum spanning tree (MST) of a weighted graph G is a spanning graph of G, which is a tree and sum of weight of its arcs, the largest among all such trees [3,4].

Let G be a weighted graph. The strength of a path P of n edges e_i, for $1 \leq i \leq n$, denoted by $s(P)$ is equal to $s(P) = min_{1\leq i\leq n} w(ei)$. The strength of connectedness of a pair of vertices $u, v \in V(G)$, denoted by $CONN_G(u, v)$, is defined as $CONN_G(u, v) = \max\{ \mathrm{s(P)} : \mathrm{P\ is\ a\ u - v\ path\ in\ G}\}$ [8]. A $u - v$ path P in a weighted graph G is called a strongest $u - v$ path if $s(P) = CONN_G(u, v)$. An edge (x, y) is strong if its weight is atleast equal to the strength of connectedness between the vertices x and y in G [8]. A path P is called strong if every arc of P is strong [8]. A connected weighted graph G is called a weighted partial tree (partial tree in short) if G has a spanning subgraph F which is a tree and for all edges (x, y) in G which are not in F, we have $CONN_G(x, y) > w(x, y)$ [7].

Let G be any connected weighted graph, u be any vertex in G and d be any metric. Then eccentricity of u with respect to d denoted by $e_d(u)$, is defined as $e_d(u) = max_{v\in V(G)} \mathrm{d(u, v)}$. Let v be a vertex in G such that $e_d(v) = min_{u\in V(G)} \mathrm{e_d}(u) = r$, then v is called a central vertex of G and r is called the radius of G with respect to the metric d. The subgraph of G induced by the central vertices of G is called the centre of G with respect to d and is denoted by $< C_d(G) >$. Let v be a vertex in G such that $e_d(u) = d(u, v)$, then v is called eccentric node of u with respect to d and is denoted by u_d^*. Let P and Q are two paths, then, $P \cup Q$ denotes the path P followed by Q.

In everyday life *distance* usually means some degree of closeness between two physical objects or ideas, i.e., length, time interval, gap, rank difference, etc. The term metric is often used as a standard for a measurement. But here we consider the mathematical meaning of these terms.

Now a days finite distance metrics have become an essential tool in many areas of Mathematics and its applications include Geometry, Probability, Statistics, Coding Theory, Graph Theory, Clustering, Data Analysis, Pattern Recognition, Networks, Engineering, Computer Graphics/Vision, Astronomy, Cosmology, Molecular Biology, and many other areas of science. Devising the most suitable distance metrics has become a standard task for many researchers. Especially intense ongoing searches for such distances occur, for example, in Genetics, Image Analysis, Speech Recognition, Information Retrieval. Often the same distance metric appears independently in several different areas; for example, the edit distance between words, the evolutionary distance in Biology, the Levenstein distance in Coding Theory, and the Hamming+Gap or shuffle-Hamming distance.

2. Some metrics in weighted graphs

In this section, we introduce some metrics in weighted graphs and prove that they are indeed metrics on their vertex sets.

Definition 2.1. Let G be a weighted graph. The w-distance between two distinct vertices u and v in G, denoted by $d_w(u, v)$, is defined as the smallest w-length of any $u - v$ path, where w-length of a path $P = u_0, u_1, u_2, \dots, u_n$ is denoted as $l_w(P) = \sum_{i=1}^{n} \frac{1}{w(u_{i-1,} u_i)}$. Also $d_w(u, u) = 0$ for every vertex u in G. If u and v are not connected by a path, then $d_w(u, v) = \infty$.

In the following theorem we prove that d_w is a metric.

Theorem 2.2. Let G be a weighted graph with vertex set V then d_w is a metric on V.

Proof: Let $P = u_0, u_1, u_2, \dots, u_n$ be any path in G. Then $w(u_{i-1,} u_i) > 0$ for $i = 1, 2, 3, \dots, n$. So w-length of $P > 0$. Therefore w-distance, $d_w(u,v) \geq 0$ for every pair of vertices u and v in G. Also, from the definition of d_w, we get $d_w(u,v) = 0$, if and only if $u = v$. The reversal of a path from u to v is a path from v to u and vice versa. So $d_w(u,v) = d_w(v,u)$, $\forall\, u, v \in V$.

Suppose $d_w(u,v) > d_w(u,w) + d_w(w,v)$ for some vertices $u, v, w \in V$. Then there exist a path P from u to w and a path Q from w to v such that the $u - v$ path contained in $P \cup Q$ has w-length strictly less than the minimum w-length of all $u - v$ paths, which is a contradiction. Therefore $d_w(u,v) \leq d_w(u,w) + d_w(w,v)$, $\forall u, v, w \in V$. Hence d_w is a metric.

Definition 2.3. The strong geodesic distance or sg-distance between two vertices u and v in a weighted graph G denoted by $d_{sg}(u,v)$, is defined as the length of the shortest $u - v$ strong path. If u and v are not connected by a path, then $d_{sg}(u,v) = \infty$.

In the following theorem, we prove that d_{sg} is a metric.

Theorem 2.4. Let G be a weighted graph with vertex set V. Then $d_{sg}(u,v)$ is a metric on V.

Proof: Clearly $d_{sg}(u,v) \geq 0$, $\forall\, u, v \in V$. Also $d_{sg}(u,v) = 0$ if and only if $u = v$. Since, the reversal of a path from u to v is a path from v to u and vice versa $d_{sg}(u,v) = d_{sg}(v,u)$, $\forall\, u, v \in V$.

Suppose $d_{sg}(u,v) > d_{sg}(u,w) + d_{sg}(w,v)$ for some vertices $u, v, w \in V$. Let P be a shortest $u - w$ strong path, Q be a shortest $w - v$ strong path and R be a shortest $u - v$ strong path. Then, since $P \cup Q$ contains a $u - v$ strong path, the above inequality leads to a contradiction. So $d_{sg}(u,v) \leq d_{sg}(u,w) + d_{sg}(w,v)$, $\forall\, u, v, w \in V$. Hence $d_{sg}(u,v)$ is a metric on V.

Definition 2.5. Let G be a weighted graph. The strongest strong distance between two vertices u and v in G, denoted by $d_{ss}(\mathrm{u,v})$, is defined as $d_{ss}(\mathrm{u,v}) = \frac{1}{CONN_G(u,v)}$ and $d_{ss}(\mathrm{u,u}) = 0$, $\forall\, u \in V(G)$. If G is disconnected and two vertices(say) u and v of G are not connected by a path, then $CONN_G(u,v) = 0$ and $d_{ss}(\mathrm{u,v}) = \infty$.

Theorem 2.6. Let G be a weighted graph with vertex set V. Then strongest strong distance d_{ss} is a metric on V.

Proof: For any two distinct vertices u and v, $CONN_G(u,v) \geq 0$. So, $d_{ss}(\mathrm{u,v}) \geq 0$, $\forall u, v \in V$. Also $d_{ss}(\mathrm{u,v}) = 0$ if and only if $u = v$. Since, reversal of a path from u to v is a path from v to u and vice versa, $d_{ss}(\mathrm{u,v}) = d_{ss}(\mathrm{v,u})$. For any three vertices $u, v, w \in V$, $CONN_G(u,v) \geq CONN_G(u,w) \wedge CONN_G(w,v)$, where $\wedge$ represents the minimum. This gives, $\frac{1}{CONN_G(u,v)} \leq \frac{1}{CONN_G(u,w) \wedge CONN_G(w,v)} \leq \frac{1}{CONN_G(u,w)} + \frac{1}{CONN_G(w,v)}$. That is $d_{ss}(\mathrm{u,v}) \leq d_{ss}(\mathrm{u,w}) + d_{ss}(\mathrm{w,v})$, $\forall\, \mathrm{u,v,w} \in \mathrm{V}$.

Since d_{ss} satisfies all the conditions for a metric, d_{ss} is a metric on V.

Definition 2.7. The δ-distance between two vertices u and v in a connected weighted graph G denoted by $\delta(u,v)$, is defined as $\delta(u,v) = 1 + \Delta_w - CONN_G(u,v)$, where Δ_w is the maximum weight of all arcs and $\delta(u,u) = 0$, for every vertex $u \in V$.

Theorem 2.8. δ-distance in a connected weighted graph G with vertex set V is a metric on V.

Proof: $CONN_G(u,v) \leq \Delta_w$, for every pair of vertices $u, v \in V$. Also, $\Delta_w + 1 \geq 0$. Therefore, $\delta(u,v) \geq 0$, $\forall\, u, v \in V$. Also $\delta(u,v) = 0$ if and only if $u = v$. Since $CONN_G(u,v) = CONN_G(v,u)$, $\forall\, u, v \in V$, $\delta(u,v) = \delta(v,u)$, $\forall\, u, v \in V$. Also for any three vertices $u, v, w \in V$. $CONN_G(u,v) \geq CONN_G(u,w) \wedge \mathrm{CONN}_G(w,v)$, where $\wedge$ represents the minimum.

So, $1 + \Delta_w - CONN_G(u,v) \leq 1 + \Delta_w - [CONN_G(u,w) \wedge CONN_G(w,v)]$.

Therefore $\delta(u,v) \leq \delta(u,w) + \delta(w,v)$, $\forall\, u, v, w \in V$. Hence δ is a metric on V.

Example 2.9.

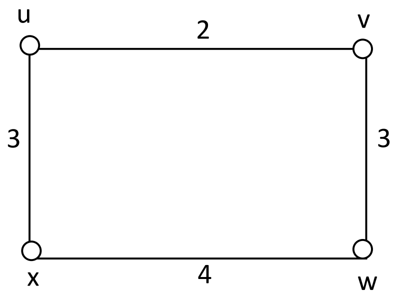

Figure 1:

In Figure 1, $d_w(\mathrm{u},\mathrm{v}) = \frac{1}{2}$, $d_{sg}(\mathrm{u},\mathrm{v}) = 3$, $d_{ss}(\mathrm{u},\mathrm{v}) = \frac{1}{3}$, $\delta(u,v) = 2$.

From this example, it can be seen that all metrics defined above are different in a weighted graph.

3. Metrics in partial trees

Partial trees are weighted graphs with unique maximum spanning tree. It was introduced by the authors of [7]. In the following section, we shall discuss the properties of the above mentioned metrics in partial trees.

Proposition 3.1. Let G be a partial tree and F be the maximum spanning tree of G. Then d_{sg}, d_{ss} in G are eguivalent to d_{sg}, d_{ss} respectively in F.

This is because every strong arcs in G are in the unique maximum spanning tree F of G.

In the next theorem, we show that centre of a partial tree and the centre of the associated maximum spanning tree are isomorphic with respect to the metric d_{sg}.

Theorem 3.1. Let G be a partial tree and F be the maximum spanning tree of G. Then

$$< C_{sg}(G) > = < C_{sg}(F) >.$$

Proof: Consider a vertex v in G. Let $e_{sg}(v) = k$ in G. We want to prove that $e_{sg} = k$ in F. $e_{sg} = k$ in G implies that there exist a vertex $u = v^*$ such that there is a strong $v - u$ path P of length k in G and $d_{sg}(v, u) = \max_{a \in V} d_{sg}(v, a)$. Since G ia a partial tree, P is the unique strong $v - u$ path in G and F contains all strong edges in G. Thus F contains the path P and $e_{sg}(v) = k$ in F . So $e_{sg} = k$ in F. That is, for any vertex v in G eccentricities in G and F are the same. Therefore by the definition of center of a weighted graph $< C_{sg}(G) > = < C_{sg}(F) >$.

Remark 3.2. Let G be a partial tree and F be the maximum spanning tree of G. Then, G and F have the same set of sg-eccentric vertices, same set of ss-eccentric vertices, same set of δ-eccentric vertices.

Proposition 3.2. Let G be a partial tree and F be the maximum spanning tree of G. Then

$$(1) < C_{ss}(G) > = < C_{ss}(F) >$$
$$(2) < C_{\delta}(G) > = < C_{\delta}(F) >$$

Proof: For any pair of vertices u and v in G, $CONN_G(u, v) = CONN_F(u, v)$. So eccentricity of u in both G and F are the same with respect to metrics d_{ss} and δ.

4. Metrics in self-centered weighted graphs

Definition 4.1. A connected weighted graph G is self-centered with respect to the metric d if each node is a central node with respect to d.

Theorem 4.2. Let G be connected weighted graph and d be any one of the metric d_w, d_{sg}, d_{ss} or δ, then G is self-centered with respect to d if $CONN_G(u, v) = w(u, v)$, $\forall u, v \in V$ and

(1) $r_{d_w}(G) = \frac{1}{w_0}$, where w_0 is the minimum weight of all arcs in G.

(2) $r_{sg}(G) = 1$.

(3) $r_{ss}(G) = \frac{1}{w_0}$, where w_0 is the least among the weights of edges of G.

(4) $r_{\delta}(G) = 1 + \Delta_w - w_0$, where w_0 is the least among the weights of edges of G.

Proof: By assumption, since $CONN_G(u, v) = w(u, v)$, the underlying graph is complete. Also every edge (u, v) is a strongest $u - v$ path and every edges is strong.

1. $CONN_G(u, v) = w(u, v), \ \forall u, v \in V$. This gives that the weight of the weakest arc in any other strongest $u - v$ path is w(u,v). Hence the w-length of a strongest $u - v$ path is atleast $\frac{1}{w(u,v)}$.

Let $\rho : u = u_0, u_1, u_2, \dots, u_n = v$ be any $u - v$ path which is not strongest. Then the strength of ρ is strictly less than $w(u, v)$. So w-length of ρ is strictly greater than $\frac{1}{w(u,v)}$, and hence $d_w(u, v) = \frac{1}{w(u,v)}$. Also,

$$e_w(u, v) = \max_v d_w(u, v) = \max_v \frac{1}{w(u,v)} = \frac{1}{\min_v w(u,v)} \quad (1)$$

Claim : $e_w(v_i) = e_w(v_j), \forall\, v_i, v_j \in V$.
If not, let $e_w(v_i) < e_w(v_j)$ and let u_i and u_j are two vertices in G such that $e_w(v_i) = \frac{1}{w(v_i, u_i)}$ and $e_w(v_j) = \frac{1}{w(v_j, u_j)}$.

$$e_w(v_i) < e_w(v_j) \Rightarrow \frac{1}{w(v_i, u_i)} < \frac{1}{w(v_j, u_j)} \Rightarrow w(v_i, u_i) > w(v_j, u_j). \quad (2)$$

Consider the path ρ: v_j, v_i, u_j. Then $w(v_j, v_i) \geq w(v_i, u_i)$ and $w(v_i, u_j) \geq w(v_i, u_i)$, since $u_i = v_i^*$ and by (1).
So, $w(v_j, v_i) \wedge w(v_i, u_j) \geq w(v_i, u_i) > w(v_j, u_j)$ by (2).
That is, strength of a $v_j - u_j$ path exceeds $w(v_j, u_j)$, which contradicts our assumption that every edge is a strongest path. Interchanging i and j a similar argument holds.
Thus $e_w(v_i) = e_w(v_j)$, $\forall\, v_i, v_j \in V$. That is, G is self-centered with respect to d_w and $r_{d_w = \frac{1}{w_0}}$, where w_0 is the minimum weight of all arcs in G.

2. By assumption every arc is strong and the underlying graph is complete. So, $d_{sg}(u, v) = 1$, $\forall\, u, v \in V$. Therefore $e_{sg}(v) = 1$, $\forall\, v \in V$. So G is self-centered and $r_{sg}(G) = \min_{v \in V} e_{sg}(v) = 1$.

3. $d_{ss}(u, v) = \frac{1}{CONN(u,v)} = \frac{1}{w(u,v)}$, by assumption.

Claim: $e_{ss}(v_i) = e_{ss}(v_j)$, $\forall v_i, v_j \in V$.
Proof is same as in (**1.**). Hence G is self-centered and $r_{ss}(G) = \frac{1}{w_0}$, where w_0 is least.

4. $\delta(u, v) = 1 + \Delta_w - CONN_G(u, v)$, where Δ_w is the maximum weight of all arcs. So, $\delta(u, v) = 1 + \Delta_w - w(u, v)$, by assumption.

Claim: $e_\delta(v_i) = e_\delta(v_j)$.
Suppose not, that is, $e_\delta(v_i) < e_\delta(v_j)$.
Let $u_i = v_i^*$ and $u_j = v_j^*$. Then $e_\delta(v_i) < e_\delta(v_j) \Rightarrow 1 + \Delta_w - w(u_i, v_i) < 1 + \Delta_w - w(u_j, v_j)$.
That is, $w(u_i, v_i) > w(u_j, v_j)$. (3)
$u_i = v_i^*$ and $u_j = v_j^*$ implies that

$$\delta(u_i, v_i) = \max_v \delta(v_i, v) \text{ and } \delta(u_j, v_j) = \max_v \delta(v_j, v) \quad (4)$$

Consider the path ρ: v_j, v_i, u_j then (4) gives
$w(v_j, v_i) \geq w(v_i, u_i)$ and $w(v_i, u_j) \geq w(v_i, u_i)$, since $u_i = v_i^*$.

So, $w(v_j, v_i) \wedge w(v_i, u_j) \geq w(v_i, u_i) > w(v_j, u_j)$ by (3). This is a contradiction to the assumption that $CONN_G(u, v) = w(u, v), \ \forall u, v \in V$. So $e_\delta(v_i) = e_\delta(v_j), \forall\, v_i,\ v_j \in V$. So G is self-centered and $r_\delta(G) = 1 + \Delta_w - w_0$, where w_0 is the least among the weights of edges of G.

Proposition 4.1. If G is a self-centered graph. Then each node of G is eccentric. This property is independent of the metric defined on it.

Remark 4.3. When connectivity of u and v, $CONN_G(u, v) = w(u, v), \forall\, u, v \in V$. Then the metrics d_w and d_{ss} coincide.

Remark 4.4. The condition in the above theorem is not necessary for a weighted graph to be self-centered as seen from the following example.

Example 4.5

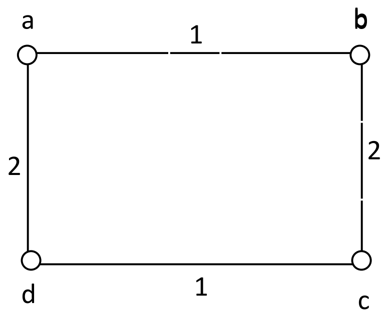

Figure 2:

In Figure 2,

$CONN_G(a, c) = 1 \neq w(a, c)$. But $e_w(a) = e_w(b) = e_w(c) = e_w(d) = \frac{3}{2}$. Therefore G is self-centered with respect to the metric d_w.

Since every arc is strong, $e_{sg}(a) = e_{sg}(b) = e_{sg}(c) = e_{sg}(d) = 2$. So G is self-centered with respect to the metric d_{sg}. Also, $e_{ss}(a) = e_{ss}(b) = e_{ss}(c) = e_{ss}(d) = 1$. That is, G is self-centered with respect to the metric d_{ss}.

$e_\delta(a) = 1 + \Delta_w - \min_{v \in V} w(a, v) \Rightarrow e_\delta(a) = e_\delta(b) = e_\delta(c) = e_\delta(d) = 2$. So G is self-centered with respect to the metric δ.

Remark 4.6. If G is a weighted cycle with n edges in which there exist at least 2 arcs having weakest weight w_o, Then $e_{ss}(a) = \frac{1}{w_0}, \forall\, a \in V$.

Moreover all edges of G are strong and hence

$$e_{sg}(a) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ \frac{n-1}{2}, & \text{if n is odd} \end{cases}$$

This is true for every vertex a in G.

Also $e_{\delta}(a) = 1 + \Delta_w - w_o, \forall a \in V$. So G is self-centered with respect to d_{sg}, d_{ss} and δ.

Theorem 4.7. Every connected weighted graph G is ss-self-centered as well as δ-self centered.
Proof: Let P be a strongest $u - v$ path in G having strength s and let s be the least strength of connectivity between any pair of vertices in G.
Let $w(x, y) = s$, where (x, y) is an edge in P. Now consider any vertex z in V.
Claim: $CONN_G(z, x) = s$ or $CONN_G(z, y) = s$.
Suppose not, that is there is a strongest z- x path P_1 with strength s_1 and a strongest $z - y$ path P_2 with strength s_2 such that $s_1, s_2 > s$.
Clearly both P_1 and P_2 do not contain the edge (x, y) and every edges in P_1 and P_2 has weight greater than s. Also $P_1 \cup P_2$ contains an $x - y$ path and it does not contain the edge (x, y). So this path has strength greater than s. Then there exist a $u - v$ path P_3, which has strength greater than s. This contradicts our assumption.
Therefore $CONN_G(z, x) = s$ or $CONN_G(z, y) = s$, $\forall z \in V$. Since s is least, $min\{CONN_G(z, x), CONN_G(z, y)\} \leq CONN_G(z, a)$, where a is any vertex in G.
This gives, $d_{ss}(z, x) \vee d_{ss}(z, y) \geq d_{ss}(z, a)$, where $\vee$ denotes the maximum.
That is, $e_{ss}(z) = d_{ss}(z, x) \vee d_{ss}(z, y) = \frac{1}{s}$, $\forall z \in V$. Therefore G is ss-self-centered. To prove that G is δ- self-centered, we have
$min\{CONN_G(z, x), CONN_G(z, y)\} \leq CONN_G(z, a), \forall a \in V$.
This gives, $1 + \Delta_w - min\{CONN_G(z, x), CONN_G(z, y)\} \geq 1 + \Delta_w - CONN_G(z, a)$.
That is $\delta(z, x) \vee \delta(z, y) \geq \delta(z, a)$, where a is any vertex in G. So, $e_{\delta}(z) = \delta(z, x) \vee \delta(z, y)$, where $\vee$ denotes the maximum. $e_{\delta}(z) = 1 + \Delta_w - s$, $\forall z \in V$. Therefore G is δ-self-centered.

5. Conclusion

Weighted graph theory has many applications in various fields like network analysis, information theory, database theory, operations research etc. Metric concepts play a key role in applications related with graphs. In this article, an attempt is made to define some metrics in weighted graphs. The properties of these metrics in partial trees are studied. Some relations between these metrics are studied. Also we proved that every connected weighted graph is ss-self-centered and δ-self-centered.

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