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Properties of Pre-γ-Open Sets and Mappings

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Abstract. Hariwan Z. Ibrahim introduced the concepts of pre-γ-open sets and pre- γ-open maps in a topological space. In this paper, some characterizations of these notions are presented. Also, some topological operations such as: pre-γ-boundary, pre-γ-exterior and pre-γ-limit, etc. are introduced. Further we introduce and study some new classes of mappings called pre*- γ-open, pre*- γ-closed and super pre- γ-open by pre-γ-open sets. Also, the relationships between these mappings are discussed. Several properties of these types of mappings are presented.

Keywords: pre- γ-open sets, pre- γ-boundary, pre- γ-exterior, pre- γ-limit, locally pre- γ-closed, pre*- γ-open, super pre- γ-open mappings; pre- γ-compact; pre- γ-connected spaces

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1. Introduction

Ogata [2] introduced the notion of pre- γ-open sets which are weaker than open sets. The concept of pre- γ-open sets and pre- γ-open maps in topological spaces are introduced by Ibrahim [3, 4] and also fuzzy generalized γ-closed sets are introduced by De [1]. In this paper, some characterizations of these notions are presented. Also, some topological operations such as: pre- γ-boundary, pre- γ-exterior and pre- γ-limit, etc, are introduced. Further, we introduce and study some new classes of mappings called pre*- γ-open, pre*- γ-closed and super pre- γ-open by pre-γ-open sets. Also, the relationships between these mappings are discussed. Several properties of these types of mappings are presented.

2. Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. An operation γ [2] on a topology τ is a mapping from τ into power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [2] if for each $x \in A$, there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open sets in X. Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-

open sets are called γ-closed. The τ_γ -interior [5] of A is denoted by τ_γ-$int(A)$ and defined to be the union of all γ-open sets of X contained in A. A subset A of a space X is said to be pre-γ-open [3] if $A \subseteq \tau_\gamma$-$int(cl(A))$.

Definition 2.1. [4] A subset A of X is called pre-γ-closed if and only if its complement is pre- γ-open.

Moreover, pre- $\gamma O(X)$ denotes the collection of all pre- γ-open sets of (X,τ) and pre- $\gamma C(X)$ denotes the collection of all pre-γ-closed sets of (X, τ).

Definition 2.2. [4] Let A be a subset of a topological space (X,τ). The intersection of all pre- γ-closed sets containing A is called the pre- γ-closure of A and is denoted by pre-$\gamma Cl(A)$.

Definition 2.3. [4] A subset N of a space (X,τ) is called a pre- γ-neighborhood (briefly, pre- γ-nbd) of a point $p \in X$ if there exists a pre- γ-open set W such that $p \in W \subseteq N$.

The class of all pre-γ-nbds of $p \in X$ is called the pre-γ-neighborhood system of p and denoted by pre-γ- N_p .

Definition 2.4. [4] A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is called:
(i) pre- γ-continuous if $f^{-1}(V) \in$ pre- $\gamma O(X)$ for every open set V of Y,
(ii) pre- γ-irresolute if $f^{-1}(V) \in$ pre-$\gamma O(X)$ for every pre-γ-open set V of Y.

Definition 2.5. A space (X, τ) is called:
(i) pre- γ-T_1[3] if for every two distinct points x, y of X , there exist two pre- γ-open sets U, V such that $x \in U, y \notin U$ and $x \notin V, y \in V$,
(ii) pre- γ-T_2 [3] if for every two distinct points x, y of X , there exist two disjoint pre-γ-open sets U, V such that $x \in U, y \in V$,
(iii) pre- γ-compact if for every pre-γ-open cover of X has a finite subcover,
(iv) pre- γ-connected if it can not be expressed as the union of two disjoint non-empty pre- γ- open sets of X,
(v) pre- γ- Lindelöff if every pre- γ-open cover of X has a countable subcover.

3. Some topological operations

Definition 3.1. Let (X,τ) be a space and $\subseteq X$. Then the pre-γ-boundary of A (briefly, pre-$\gamma b(A)$) is given by pre- $\gamma b(A)$ = pre-$\gamma cl(A) \cap$ pre- $\gamma cl(X \backslash A)$.

Theorem 3.1. If A is a subset of a space (X,τ), then the following statements are hold:
(1) pre- $\gamma b(A)$ = pre-$\gamma b(X \backslash A)$.
(2) pre-$\gamma b(A)$ = pre-$\gamma cl(A) \backslash$pre- $\gamma int(A)$.
(3) pre- $\gamma b(A) \cap$ pre- $\gamma int(A) = \emptyset$.
(4) pre- $\gamma b(A) \cup$ pre- $\gamma int(A)$ = pre-$\gamma cl(A)$.

Proof: (1) Obvious from Definition 3.1.
(2) Since,
pre-γb(A) = pre- $\gamma cl(A) \cap$ pre- $\gamma cl(X \backslash A)$ = pre- $\gamma cl(A) \cap (X \backslash$pre-$\gamma int(A))$
=(pre-$\gamma cl(A) \cap X) \backslash$ [pre-$\gamma cl(A) \cap$ pre-$\gamma int(A)$]=pre-$\gamma cl(A) \backslash$ pre- $\gamma int(A)$.

(3) Also, by using (2),

$$\begin{aligned} \text{pre-}\gamma b(A) \cap \text{pre-}\gamma int(A) &= (\text{ pre-}\gamma cl(A)\backslash\text{pre-}\gamma int(A)) \cap \text{pre-}\gamma int(A) \\ &= (\text{ pre-}\gamma cl(A) \cap \text{pre-}\gamma int(A))\backslash \text{ pre-}\gamma int(A) \\ &= \text{pre-}\gamma int(A) \setminus \text{pre-}\gamma int(A) = \emptyset. \end{aligned}$$

(4) By using (3),

pre-$\gamma b(A) \cup$ pre-$\gamma int(A)$ = (pre-$\gamma cl(A)\backslash$ pre-$\gamma int(A)) \cup$ pre-$\gamma int(A)$ = pre-$\gamma cl(A)$.

Theorem 3.2. If A is a subset of a space X, then the following statements are hold:

(1) A is a pre-γ -open set if and only if $A \cap$ pre-$\gamma b(A) = \phi$.

(2) A is a pre-γ-closed set if and only if pre-$\gamma b(A) \subset A$.

(3) A is a pre-γ-clopen set if and only if pre-$\gamma b(A) = \phi$.

Proof: (1) Let A be a pre-γ-open set. Then A =pre-$\gamma int(A)$, hence $A \cap$ pre-$\gamma b(A) =$ pre-$\gamma int(A) \cap$ pre-$\gamma b(A) = \emptyset$.

Conversely, let $A \cap$pre-$\gamma b(A) = \emptyset$. Then by Theorem 3.1.,

$A \cap$ (pre-$\gamma cl(A)\backslash$pre-$\gamma int(A)) = (A \cap$ pre-$\gamma cl(A)) \setminus (A \cap$ pre-$\gamma int(A)) =$ $A\backslash$pre-γ $int(A) = \emptyset$. So, A =pre-$\gamma int(A)$ and hence A is pre-γ-open.

(2) Let A be a pre-γ-closed set. Then A = pre-$\gamma cl(A)$, but pre-$\gamma b(A)$ = pre-$\gamma cl(A)\backslash$ pre-$\gamma int(A) = A \setminus$ pre-$\gamma int(A)$, then pre-$\gamma b(A) \subset A$. Conversely, let pre-$\gamma b(A) \subset A$. Then by Theorem 3.1., pre-$\gamma cl(A)$ = pre-$\gamma b(A) \cup$ pre-$\gamma int(A) \subset A \cup$ pre-$\gamma int(A) = A$, thus pre-$\gamma cl(A) \subset A$ and $A \subset$ pre-$\gamma cl(A)$. Therefore, $A =$ pre-$\gamma cl(A)$.

(3) Let A be a pre-γ-clopen set. Then A = pre-$\gamma int(A)$ and A = pre-$\gamma cl(A)$, hence by Theorem 3.1., pre-$\gamma b(A)$ = pre-$\gamma cl(A)\backslash$pre-$\gamma int(A) = A \setminus A = \emptyset$. Conversely, suppose that pre-$\gamma b(A) = \emptyset$. Then pre-$\gamma b(A)$ = pre-$\gamma cl(A)\backslash$ pre-$\gamma int(A) = \emptyset$ and hence, A is pre-γ-clopen.

Definition 3.2. Let (X,τ) be a space and $A \subset X$. Then the set $X \setminus$(pre-$\gamma cl(A))$ is called the pre-γ-exterior of A and is denoted by pre-$\gamma ext(A)$. Each point $p \in X$ is called a pre-γ-exterior point of A, if it is a pre-γ-interior point of $X\backslash A$.

Theorem 3.3. If A and B are two subsets of a space (X, τ) , then the following statements are hold:

(1) pre-$\gamma ext(A)$ = pre-$\gamma int(X\backslash A)$.

(2) pre-$\gamma ext(A) \cap$ pre-$\gamma b(A) = \emptyset$.

(3) pre-$\gamma ext(A) \cup$ pre-$\gamma b(A)$ = pre-$\gamma cl(X\backslash A)$.

(4) {pre-$\gamma int(A)$, pre-$\gamma b(A)$ and pre-$\gamma ext(A)$} form a partition of X .

(5) If $A \subset B$, then pre-$\gamma ext(B) \subset$ pre-$\gamma ext(A)$.

(6) pre-$\gamma ext(A \cup B) \subset$ pre-$\gamma ext(A) \cup$ pre-$\gamma ext(B)$.

(7) pre-$\gamma ext(A \cap B) \supset$ pre-$\gamma ext(A) \cap$ pre-$\gamma ext(B)$.

(8) pre-$\gamma ext(\emptyset) = X$ and pre-$\gamma ext(X) = \emptyset$.

Proof: (1) Obvious from Definition 3.2.

(2) By Theorem 3.1., pre-$\gamma ext(A) \cap$ pre-$\gamma b(A)$ =pre-$\gamma int(X\backslash A) \cap$ pre-$\gamma b(X \setminus A) = \emptyset$.

(3) Also, by Theorem 3.1.,

pre-$\gamma ext(A) \cup$ pre-$\gamma b(A)$ = pre-$\gamma int(X\backslash A) \cup$ pre-$\gamma b(X\backslash A)$ = pre-$\gamma cl(X\backslash A)$.

(4) and (5) Obvious.

(6) pre-$\gamma ext(A \cup B) = X \backslash$pre- $\gamma cl(A \cup B) \subset X \backslash($ pre- $\gamma cl(A) \cup$ pre- $\gamma cl(B))$
$= (X \backslash ($ pre-$\gamma cl(A))) \cap (X \backslash ($ pre-$\gamma cl(B)))$
$=$ pre- $\gamma ext(A) \cap$ pre- $\gamma ext(B) \subset$pre- $\gamma ext(A) \cup$ pre- $\gamma ext(B)$.

(7) Further,
pre-$\gamma ext(A \cap B) = X \backslash$ pre-$\gamma cl(A \cap B) \supset X \backslash($ pre-$\gamma cl(A) \cap$ pre-$\gamma cl(B))$
$= (X \backslash ($ pre-$\gamma cl(A))) \cup (X \backslash ($ pre- $\gamma cl(B)))$
$=$ pre- $\gamma ext(A) \cup$ pre-$\gamma ext(B) \supset$ pre-$\gamma ext(A) \cap$ pre-$\gamma ext(B)$.

(8) Obvious.

Remark 3.1. The inclusion relation in part (5), (6) of the above theorem cannot be replaced by equality as is shown by the following example.

Example 3.1. Let $X = \{a, b, c, d\}$ with topologies $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} \text{int}(cl(A)) & if \ \ A \neq \{a\} \\ cl(A) & if \ \ A = \{a\} \end{cases}$$

If $A = \{a, d\}$ and $B = \{b, d\}$, then pre-$\gamma ext(A) = \{b, c\}$, pre- $\gamma ext(B) = \{a, c\}$. But pre- $\gamma ext(A \cup B) = \{c\}$, Therefore, pre-$\gamma ext(A) \cup$ pre-γext(B) $\not\subset$ pre-$\gamma ext(A \cup B)$. Also, pre-$\gamma ext(A \cap B) = \{a, b, c\}$, hence, pre-$\gamma ext(A \cap B) \not\subset$ pre-$\gamma ext(A) \cap$ pre-$\gamma ext(B)$.

Definition 3.3. If A is a subset of a space (X,τ) , then a point $p \in X$ is called a pre- γ-limit point of a set $A \subset X$ if every pre-γ-open set $G \subset X$ containing p contains a point of A other than p.

The set of all pre-γ-limit points of A is called a pre- γ -derived set of A and is denoted by pre-$\gamma d(A)$.

Proposition 3.1. Let (X, τ) be a topological space and $A \subseteq X$. Then, the following statements are hold:

(i) A is pre- γ-closed if and only if pre- $\gamma d(A) \subseteq A$

(ii) A is pre- γ -open if and only if it is pre- γ-nbd for every point $p \in A$

(iii) pre- $\gamma cl(A)$ = $A \cup$ pre- $\gamma d(A)$.

Proof: (i) Let A be a pre- γ-closed set and $p \in A$. Then $p \in X \backslash A$ which is open, hence there exists a pre-γ-open $(X \backslash A)$ such that $(X \backslash A) \cap A = \emptyset$, so $p \notin$ pre- $\gamma d(A)$, therefore, pre- $\gamma d(A) \subset A$.

Conversely, suppose that pre- $\gamma d(A) \subset A$ and $p \notin A$. Then $p \notin$ pre- $\gamma d(A)$, hence there exists a pre- γ-open set U containing p such that $U \cap A = \emptyset$ and hence $X \backslash A = \bigcup_{p \in A} \{G, G$ is pre- γ-open$\}$, therefore A is pre- γ -closed.

(ii) Let A be a pre-γ-open set. Then A is a pre-γ -neighborhood for each $p \in A$. Conversely, let A be a pre- γ-neighborhood for each $p \in G$. Then there exists a pre- γ-

open set U_p containing p such that $p \in U_p \subseteq A$, so $A=\bigcup_{p\in G} U_p$, therefore, A is a pre- γ-open.

(iii) Since, pre-$\gamma d(A)$ $\subset$pre-$\gamma cl(A)$ and $A \subset$ pre- $\gamma cl(A)$, pre-$\gamma d(A) \cup A \subset$ pre-$\gamma cl(A)$. Conversely, suppose that $p \notin$ pre- γd(A) $\cup$ A. Then $p \notin$ pre- $\gamma d(A)$, $p \notin A$ and hence there exists a pre- γ-open set U containing p such that $U \cap A \neq \emptyset$. Thus $p \notin$ pre- γ cl(A) which implies that pre- $\gamma cl(A) \subset$ pre- $\gamma d(A) \cup A$. Therefore, pre- $\gamma cl(A)$ =pre- $\gamma d(A) \cup A$.

Theorem 3.4. If A and B are two subsets of a space X , then the following statements are hold:

(1) If $A \subset B$, then pre- $\gamma d(A) \subset$ pre- $\gamma d(B)$.

(2) A is a pre- γ-closed set if and only if it contains each of its pre- γ-limit points.

(3) pre- $\gamma cl(A) = A \cup$ pre-$\gamma d(A)$.

Proof: (1) Obvious.

(2) Let A be a pre- γ -closed set and $p \notin A$. Then $p \in X\backslash A$ which is pre- γ -open, hence there exists a pre- γ-open $(X\backslash A)$ such that $(X\backslash A) \cap A = \emptyset$, so $p \notin$ pre- γd(A), therefore, pre-$\gamma d(A) \subset A$.

Conversely, suppose that pre- $\gamma d(A) \subset A$ and $p \notin A$. Then p $\notin$ pre- $\gamma d(A)$, hence there exists a pre- γ-open set G containing p such that $G \cap A = \emptyset$ and hence

$$X\backslash A = \bigcup_{p\in A} \{G, G \text{ is pre-}\gamma\text{-open}\}$$, therefore A is pre-γ-closed.

(3) Since, pre-$\gamma d(A) \subset$ pre-$\gamma cl(A)$ and $A \subset$ pre-$\gamma cl(A)$, pre- $\gamma d(A) \cup A \subset$ pre-$\gamma cl(A)$. Conversely, suppose that $p \notin$ pre-$\gamma d(A) \cup A$. Then $p \notin$ pre-$\gamma d(A)$, $p \notin A$ and hence there exists a pre- γ-open set G containing p such that $G \cap A = \emptyset$. Thus $p \notin$ pre-$\gamma cl(A)$ which implies that pre-$\gamma cl(A) \subset$ pre- $\gamma d(A) \cup A$. Therefore, pre- $\gamma cl(A)$ =pre-$\gamma d(A) \cup A$.

Theorem 3.5. A subset G of a space X is pre-γ-open if and only if it is pre-γ-nbd, for every point $p \in G$.

Proof: Let G be a pre-γ-open set. Then G is a pre-γ-nbd for each $p \in G$. Conversely, let G be a pre-γ-nbd for each $p \in G$. Then there exists a pre-γ-open set W_p containing p such that $p \in W_p \subseteq G$, so $G = \bigcup_{p\in G} W_p$, therefore, G is a pre- γ-open.

Theorem 3.6. In a space (X, τ) . If pre- γ-N_p be the pre- γ-nbd. Systems of a point $p \in X$, then the following statements are hold:

(1) pre- γ-N_p is not empty and p belongs to each member of pre- γ-N_p,

(2) Each superset of members of N_p belongs pre- γ-N_p,

(3) Each member $N \in$ pre- γ-N_p is a superset of a member $W \in$ pre- γ-N_p, where W is pre-γ-nbd of each point $p \in W$.

Proof: Obvious.

Definition 3.4. A subset A of a topological space (X, τ) is said to be locally pre-γ-closed if $A = U \cap F$ for each $U \in \tau$ and $F \in$ pre-$\gamma C(X)$.

Theorem 3.7. Let H be a subset of a space X. Then H is locally pre-γ-closed if and only if $H = U \cap$ pre-$\gamma cl(H)$.

Proof: Since H is a locally pre-γ-closed set, $H = U \cap F$, for each $U \in \tau$ and $F \in$ pre-$\gamma C(X)$, hence $H \subseteq$ pre-$\gamma cl(H) \subseteq$ pre-$\gamma cl(F) = F$, thus $H \subseteq U \cap$ pre-$\gamma cl(H) \subseteq U \cap$ pre-$\gamma cl(F) = H$. Therefore $H = U \cap$ pre-$\gamma cl(H)$. Conversely, since pre-$\gamma cl(H)$ is pre-γ-closed and $H = U \cap$ pre-$\gamma cl(H)$, then H is locally pre-γ-closed.

Theorem 3.8. Let A be a locally pre-γ-closed subset of a space (X, τ). Then the following statements are hold:
(1) pre-$\gamma cl(A)\backslash A$ is a pre-γ-closed set.
(2) $(A \cup (X\backslash$pre-$\gamma cl(A)))$ is a pre-γ-open.
(3) A $\in$ pre-γ-$int(A \cup (X\backslash$ pre-$\gamma cl(A)))$.

Proof: (1) If A is a locally pre-γ-closed set, then there exists an open set U such that $A = U \cap$ pre-$\gamma cl(A)$. Hence,

pre-$\gamma cl(A)\backslash A$ = pre-$\gamma cl(A)\backslash(U \cap$ pre-$\gamma cl(A))$
= pre-$\gamma cl(A) \cap [X\backslash(U \cap$ pre-$\gamma cl(A))]$
= pre-$\gamma cl(A) \cap [(X\backslash U) \cup (X\backslash$ pre-$\gamma cl(A))]$
= pre-$\gamma cl(A) \cap (X\backslash U)$

which is pre-γ-closed.

(2) From (1), pre-$\gamma cl(A)\backslash A$ is pre-γ-closed, then $X\backslash[($ pre-γ-$cl(A)\backslash A)]$ is a pre-γ-open set and $X\backslash[($ pre-$\gamma cl(A)\backslash A)] = X\backslash$ pre-$\gamma cl(A) \cup (X \cap A) = A \cup (X\backslash$ pre-$\gamma cl(A))$, hence $A \cup (X\backslash$ pre-$\gamma cl(A))$ is pre-γ-open.

(3) It is clear that, $A \subseteq (A \cup (X\backslash$ pre-$\gamma cl(A)))$ = pre-$\gamma int(A \cup (X\backslash$ pre-$\gamma cl(A)))$.

4. Pre-γ -open and pre-γ-closed mappings

Definition 4.1. [3] A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be
(i) pre-γ-open if the image of each open set of (X, τ) is pre-γ-open in (Y, σ),
(ii) pre-γ-closed if the image of each closed set of (X, τ) is pre-γ-closed in (Y, σ).

Definition 4.2. For a space (X, τ) and $A \subseteq X$:
(i) pre-γ- $b(A)$ = pre-$\gamma cl(A)\backslash$ pre-$\gamma int(A)$
(ii) pre-$\gamma Bd(A) = A\backslash$ pre-$\gamma int(A)$.

The set of pre-γ-boundary (respectively pre-γ-border) of A is denoted by pre-γ-$b(A)$ (respectively pre-γ-$Bd(A)$).

Theorem 4.1. For a bijective mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:
(i) f^{-1} is pre-γ-continuous,
(ii) f is pre-γ-open,
(iii) f is pre-γ-closed.

Proof: Obvious.

Theorem 4.2. For a mapping $f:(X,\tau)\rightarrow(Y,\sigma)$, the following statements are equivalent:
(i) f is pre- γ-open,
(ii) For each $x\in X$ and each neighborhood U of x, there exists $V\in$ pre-$\gamma O(Y)$ containing $f(x)$ such that $V\subseteq f(U)$,
(iii) $f(int(A))\subseteq$ pre-$\gamma int(f(A))$, for each $A\subseteq X$,
(iv) $int(f^{-1}(B))\subseteq f^{-1}($ pre-$\gamma int(B))$, for each $B\subseteq Y$,
(v) $f^{-1}($ pre-$\gamma Bd(B))\subseteq Bd(f^{-1}(B))$, for each $B\subseteq Y$,
(vi) $f^{-1}($ pre-$\gamma cl(B))\subseteq cl(f^{-1}(B))$, for each $B\subseteq Y$.
Proof: (i) → (ii) Let U be neighborhood of x in X. Then there exists an open set G such that $x\in G\subseteq U$ and hence $f(x)\in f(G)\subseteq f(U)$. Since f is pre-γ-open, then $f(G)$ is pre-γ-open in Y. Put $f(G)=V$, then $f(x)\in V\subseteq f(U)$.
(ii) → (i) Let U be an open set containing x in X . Then U is neighborhood of each $x\in U$. By hypothesis, there exists $V\in$ pre- $\gamma O(Y)$ such that $(x)\in V\subseteq f(U)$.
Hence, $f(U)$ is pre-γ-neighborhood of each $f(x)\in f(U)$. By Proposition 3.1., $f(U)$ is pre-γ-open in Y. Therefore, f is pre-γ-open mapping.
(i) → (iii) Since $int(A)\subseteq A\subseteq X$ which is open and f is pre-γ-open, then $f(int(A))$ is pre-γ-open in Y. Hence, $f(int(A))\subseteq$ pre-$\gamma int(f(A))$, then $f(int(A))\subseteq$ pre-$\gamma int(f(A))\subseteq f(A)$.
(iii) → (iv) By replacing $f^{-1}(B)$ instead of A in (iii), we have $f(int(f^{-1}(B))\subseteq$ pre-γ $int(f(f^{-1}(B))$ and then $int(f^{-1}(B))\subseteq f^{-1}($ pre- γ- $int(f(f^{-1}(B))))\subseteq f^{-1}($ pre-$\gamma int(B))$.
(iv) → (i) Let $A\in\tau$. Then $f(A)\subseteq Y$ and by hypothesis, $int(f^{-1}(f(A)))\subseteq f^{-1}($ pre-$\gamma\, int(f(A)))$. This implies that, $int(A)\subseteq f^{-1}($ pre-$\gamma int(f(A)))$. Thus $f\big(int(A)\big)\subseteq$ pre-$\gamma int(f(A))$. Therefore, f is pre-γ-open.
(iv) → (v) Let $B\subseteq Y$. Then by hypothesis, $f^{-1}(B)\backslash f^{-1}($ pre-$\gamma int(B))\subseteq f^{-1}(B)\backslash int(f^{-1}(B))$ and hence, $f^{-1}(B\backslash pre\text{-}\gamma int(B))\subseteq f^{-1}(B)\backslash int(f^{-1}(B))$. Therefore, $f^{-1}($ pre-$\gamma Bd(B))\subseteq Bd(f^{-1}(B))$.
(v) → (iv) Let $B\subseteq Y$. Then by Definition 2.3., we have $f^{-1}(B\backslash$pre-$\gamma int(B))\subseteq f^{-1}(B)\backslash int(f^{-1}(B))$ and hence $f^{-1}(B)\backslash f^{-1}($pre-$\gamma int(B))\subseteq f^{-1}(B)\backslash int(f^{-1}(B))$. Therefore, $int(f^{-1}(B))\subseteq f^{-1}($ pre- $\gamma int(B))$.
(i) → (vi) Let $B\subseteq Y$ and $x\in f^{-1}(pre\text{-}\gamma cl(B))$. Then $f(x)\in$ pre- $\gamma cl(B)$. Assume that U is an open set containing x. Since f is pre-γ-open, then $f(U)$ is pre-γ-open in Y. Hence, $B\cap f(U)\neq\emptyset$. Thus $U\cap f^{-1}((B)\neq\emptyset$. Therefore, $x\in cl(f^{-1}((B))$. So, $f^{-1}(($ pre- $\gamma cl(B))\subseteq cl(f^{-1}((B))$.
(vi) → (i) Let $B\subseteq Y$. Then $Y\backslash B\subseteq Y$. By hypothesis, $f^{-1}($ pre-$\gamma cl(Y\backslash B))\subseteq cl(f^{-1}(Y\backslash B))$ and hence $X\backslash f^{-1}($ pre-$\gamma int(B))\subseteq X\backslash int(f^{-1}(B))$ that implies $int(f^{-1}(B))\subseteq f^{-1}($ pre- $\gamma int(B))$. Then by (iv), f is pre-γ-open.

Theorem 4.3. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a pre-γ-open mapping. If $W\subseteq Y$ and $F\subseteq X$ is a closed set containing $f^{-1}(W)$, then there exists a pre- γ-closed set H of Y containing W such that $f^{-1}(H)\subseteq F$.

Proof: Let $H = Y\backslash f(X\backslash F)$ and F be a closed set of X containing $f^{-1}(W)$. But f is pre-γ-open mapping, then $f(X\backslash F)$ is pre-γ-open set of Y . Therefore, H is pre-γ-closed and $f^{-1}(H) = X\backslash f^{-1}[f(X\backslash F)] \subseteq X\backslash(X\backslash F) = F$.

Remark 4.1. The converse of above theorem is not true in general. Suppose that $X = Y = \{a, b, c, d\}$ with topologies τ= $\{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}\}$ and σ=$\{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Define an operation γ on σ by

$$\gamma(A) = \begin{cases} \text{int}(cl(A)) & if \ A \neq \{a\} \\ cl(A) & if \ A = \{a\}. \end{cases}$$

Hence the identity map $f : (X, \tau) \to (Y, \sigma)$ is satisfying the condition but it is not pre-γ-open. Since $\{d\} \subseteq Y$ and $\{d\} \subseteq X$ is a closed set containing $f^{-1}(\{d\}) = \{d\}$, hence there exists $\{d\} \in$ pre-$\gamma C(Y)$ containing $\{d\}$ such that $f^{-1}(\{d\}) \subseteq \{d\}$ but, $\{a\} \in \tau$ and $f(\{a\}) = \{a\} \notin$ pre-$\gamma O(Y)$.

Theorem 4.4. Let $f : (X, \tau) \to (Y, \sigma)$ be a pre-γ-closed mapping. Then the following statements are hold:
(i) If f is a surjective and $f^{-1}(B)$, $f^{-1}(C)$ have disjoint neighborhoods of X, then B and C are disjoint of Y,
(ii) pre-$\gamma int($ pre-$\gamma cl(f(A))) \subseteq f(cl(A))$, for each $A \subseteq X$.
Proof: (i) Let M , N be two disjoint neighborhoods of $f^{-1}(B)$, $f^{-1}(C)$. Then there exist two pre-γ-open sets U , V such that $f^{-1}(B) \subseteq U \subseteq M$, $f^{-1}(C) \subseteq V \subseteq N$. But, f is a surjective map, then $ff^{-1}(B) = B \subseteq f(U) \subseteq f(M)$, $ff^{-1}(C) = C \subseteq f(V) \subseteq f(N)$. Since M, N are disjoint, then also $f(M \cap N) = \emptyset$ and hence $B \cap C \subseteq f(U \cap V) \subseteq f(M \cap N) = \emptyset$. Therefore, B and C are disjoint of Y.
(ii) Since $A \subseteq cl(A) \subseteq X$ and f is a pre-γ -closed mapping, then $f(cl(A))$ is pre-γ-closed in Y. Hence, $f(A) \subseteq$pre-$\gamma cl(f(A)) \subseteq f(cl(A))$. So pre-γ $int($ pre -$\gamma cl(f(A))) \subseteq f(cl(A))$.

Theorem 4.5. For a mapping $f : (X, \tau) \to (Y, \sigma)$, then the following are equivalent:
(i) f is pre-γ -closed,
(ii) pre-$\gamma cl(f(A)) \subseteq f(cl(A))$ for each $A \subseteq X$,
(iii) If f is surjective, then for each subset B of Y and each open set U in X containing $f^{-1}(B)$, there exists a pre-γ-open set V of Y containing B such that $f^{-1}(V) \subseteq U$.
Proof: (i) → (ii) Let $cl(A) \subseteq X$ be a closed set. Since f is pre-γ-closed, then $f(cl(A)) \in$ pre-$\gamma C(Y)$. Hence, pre-$\gamma cl(f(A)) \subseteq f(cl(A))$.
(ii) → (i) Let $A \subseteq X$ be a closed set. By hypothesis, pre-$\gamma cl(f(A)) \subseteq f(cl(A)) = f(A)$. Hence, $f(A) \in$ pre-$\gamma C(Y)$. Therefore, f is pre-γ -closed.
(i) → (iii) Suppose that $V = Y\backslash f(X\backslash U)$ and U is an open set of X containing$f^{-1}((B)$. Then by hypothesis, V is pre-γ-open in Y. But, $f^{-1}(B) \subseteq U$, then $B \subseteq f(U)$ and $f(X\backslash U) \subseteq Y\backslash B$, that is, $B \subseteq V$ and$f^{-1}((V) \subseteq U$.
(iii) → (i) Let $F \subseteq X$ be a closed set and y be any point of $Y\backslash f(F)$. Then $f^{-1}(y) \in X\backslash F$ which is open in X. Hence by hypothesis, there exists a pre-γ-open set V

containing y such that $f^{-1}(V) \subseteq X\setminus F$. But f is surjective, then $y \in V \subseteq Y\setminus f(F)$ and $Y\setminus f(F)$ is the union of pre- γ -open sets and hence, $f(F)$ is pre- γ-closed. Therefore, f is pre- γ -closed.

Remark 4.2. The restriction of pre-γ-open mapping is not pre-γ-open. Consider the Remark 4.1., the function $f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a) = f(c) = c$, $f(b) = b$ and $f(d) = a$ is pre-γ-open. But $A = \{a,d\} \subseteq X$ and, $f_A : (A,\tau_A) \to (Y,\sigma)$ is not pre-γ-open. Since $\{d\} \in \tau_A$ but $f(\{d\}) = \{d\} \notin$ pre-$\gamma O(Y)$.

Remark 4.3. The composition of two pre-γ -open mappings may not be pre-γ -open. Let $X = Y = Z = \{a,b,c,d\}$ with topologies $\tau_X = \{X,\emptyset,\{a,b\},\{c,d\}\}$, τ_Y is an indiscrete topology and $\tau_Z = \{Z,\emptyset,\{a\},\{c\},\{a,c\}\}$. Let $f : (X,\tau_X) \to (Y,\tau_Y)$, $g : (Y,\tau_Y) \to (Z,\tau_Z)$ be the identity mappings and define an operation γ on τ_Y and τ_Z defined by $\gamma(A) = A$. Clearly f and g are pre-γ-open but $(g \circ f)$ is not pre-γ-open. Since $\{c,d\} \subseteq X$ is an open set of X, but $(g \circ f)(\{c,d\}) = \{c,d\} \notin$ pre-$\gamma O(Z)$. In the following, we give some further properties of the composition of two pre-γ-open (respespectly pre-γ-closed) mappings.

Theorem 4.6. Let $f : (X,\tau_X) \to (Y,\tau_Y)$ and $g : (Y,\tau_Y) \to (Z,\tau_Z)$ be two mappings. Then the following statements are hold:
(i) If f is an open and g is a pre-γ -open mappings, then $g \circ f$ is pre-γ -open,
(ii) If $g \circ f$ is a pre-γ -open and f is a surjective continuous map, then g is pre-γ-open,
(iii) If $g \circ f$ is an open and g is an injective pre-γ-continuous map, then f is pre-γ-open.
Proof: (i) Let $U \in \tau_X$. Then by hypothesis, $f(U) \in \tau_Y$. But g is a pre-γ-open map, then $g(f(U)) \in$ pre-$\gamma O(Z)$. Hence, $g \circ f$ is pre-γ -open.
(ii) Let $U \in \tau_Y$ and f be a continuous map. Then $f^{-1}(U) \in \tau_X$. But $g \circ f$ is a pre-γ -open map, then $(g \circ f)(f^{-1}(U)) \in$ pre-$\gamma O(Z)$. Hence by surjective of f, $g(U) \in$ pre-$\gamma O(Z)$. Hence, g is pre-γ-open.
(iii) Let $U \in \tau_X$ and $g \circ f$ be an open map. Then $(g \circ f)(U) = g(f(U)) \in \tau_Z$. Since g is an injective pre-γ-continuous map, hence $f(U) \in$ pre-$\gamma O(Y)$. Therefore, f is pre-γ-open.

Theorem 4.7. Let $f : (X,\tau) \to (Y,\sigma)$ be a bijective pre-γ-open mapping. Then the following statements are hold:
(i) If X is a T_i-space, then Y is pre-γ- T_i where i = 1, 2.
(ii) If Y is a pre-γ-compact (respectively pre-γ-Lindelöff .) space, then X is compact (respectively Lindelöff).

Proof: (i) We prove that for the case of a T_1-space. Let y_1, y_2 be two distinct points of Y. Then there exist x_1, $x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is a T_1 -space, then there exist two open sets U, V of X such that $x_1 \in U, x_2 \notin U$ and $x_2 \in V, x_1 \notin V$. But, f is a pre-γ-open map, then $f(U)$, $f(V)$ are pre-γ-open sets of Y with $y_1 \in f(U)$, $y_2 \notin f(U)$ and $y_2 \in f(V)$, $y_1 \notin f(V)$. Therefore, Y is pre-γ -T_1.
(ii) We prove that the theorem for pre-γ-compact. Let $\{U_i : i \in I\}$ be a family of open cover of X and f be a surjective pre-γ-open mapping. Then $\{f(U_i) : i \in I\}$ is a pre-γ -open cover of Y. But, Y is pre-γ-compact space, hence there exists a finite subset I_o of

I such that $Y = \cup \{f\ (U_i) : i \in I_o\}$. Then by injective of f, $\{U_i : i \in I_o\}$ is a finite subfamily of X . Therefore, X is compact.

Theorem 4.8. If $f : (X,\tau) \to (Y,\sigma)$ is a surjective pre- γ-open mapping and Y is pre- γ-connected space, then X is connected.
Proof: Suppose that X is a disconnected space. Then there exist two non-empty disjoint open sets U, V of X such that $X = U \cup V$. But f is a surjective pre-γ-open map, then $f(U)$ and $f(V)$ are non-empty disjoint pre-γ-open sets of Y with $Y = f(U) \cup f(V)$ which is a contradiction with the fact that Y is pre-γ-connected.

5. Super pre- γ-open and super pre- γ-closed mappings
In the following, we introduce and study the concept of super pre- γ-open and super pre-γ-closed mappings. Also, some of their properties are investigated.

Definition 5.1. A mapping $f : (X,\tau) \to (Y,\sigma)$ is called:
(i) super pre- γ -open if $f(U)$ is open in Y for each $U \in$ pre- $\gamma O(X,\tau)$.
(ii) super pre- γ-closed if $f(U)$ is closed in Y for each $U \in$ pre-$\gamma C(X,\tau)$.

Example 5.1. Let $X = Y = \{a,b,c,d\}$ with topologies
$\tau = \{X,\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$ and
$\sigma = \{Y,\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{c,d\},\{b,c\},\{a,b,c\},\{a,c,d\},\{b,\ c,d\}\}$. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} \text{int}(cl(A)) & if\ \ A \neq \{a\} \\ cl(A) & if\ \ A = \{a\}. \end{cases}$$

Also the map $f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a) = b, f(b) = a, f(c) = c$ and $f(d) = d$ is super pre- γ-open.

Proposition 5.1. Every super pre- γ-open mapping is pre- γ -open.
Proof: Let $A \subseteq X$ be an open set and hence A is pre-γ-open. But, f is super pre-γ-open, then $f(A)$ is open in Y, hence $f(A)$ is pre-γ-open in Y. Therefore, f is pre- γ-open.

Remark 5.1. The converse of the above proposition is not true as shown in the following Example. Suppose that $X = Y = \{a,b,c,d\}$ with topologies
$\tau = \{X,\emptyset,\{a\},\{c\},\{a,c\}\}$ and $\sigma = \{Y,\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$.
Define an operation γ on τ by $\gamma(A) = A$ and an operation γ on σ by

$$\gamma(A) = \begin{cases} \text{int}(cl(A)) & if\ \ A \neq \{a\} \\ cl(A) & if\ \ A = \{a\}. \end{cases}$$

Also a mapping $f : (X,\tau) \to (Y,\sigma)$which defined by $f(a) = b, f(b) = a, f(c) = c$ and $f(d) = d$ is pre-γ-open but not super pre-γ-open. Since $\{a,c,d\} \in$ pre-$\gamma O(X)$ and $f(\{a,c,d\}) = \{b,c,d\} \notin \sigma$.

Theorem 5.1. If $f : (X,\tau) \to (Y,\sigma)$ is a mapping, then the following statements are equivalent:
(i) f is super pre- γ-open,

(ii) for each $x \in X$ and each pre- γ-neighborhood U of x , there exists a neighborhood V of $f(x)$ such that $V \subseteq f(U)$,
(iii) $f($ pre- $\gamma int(A)) \subseteq int(f(A))$, for each $A \subseteq X$,
(iv) pre- $\gamma int(f^{-1}(B)) \subseteq f^{-1}(int(B))$, for each $B \subseteq Y$,
(v) $f^{-1}(Bd(B)) \subseteq$ pre-$\gamma Bd(f^{-1}(B))$, for each $B \subseteq Y$,
(vi) $f^{-1}(cl(B)) \subseteq$pre-$\gamma cl(f^{-1}(B))$, for each $B \subseteq Y$,
(vii) If f is surjective, then for each subset B of Y and for any set $F \in$ pre-$\gamma C(X)$ containing $f^{-1}(B)$, there exists a closed subset H of Y containing B such that $f^{-1}(H) \subseteq F$.

Proof: (i)→ (ii): Let U be a pre- γ-neighborhood of x in X. Then there exists $W \in$ pre-$\gamma O(X)$ such that $x \in W \subseteq U$ and hence $f(x) \in f(W) \subseteq f(U)$. Hence by hypothesis, $f(W) \in \sigma$ and containing (x) . Put $f(W) = V$, then $f(x) \in V \subseteq f(U)$.

(ii)→ (i): Suppose that U is pre- γ -open set of X and containing $x \in X$. Then $f(x) \in f(U)$. Hence by hypothesis, there exists $V \in \sigma$ containing $f(x)$ such that $f(x) \in V \subseteq f(U)$. Hence, $f(U)$ is neighborhood for $f(x) \in f(U)$. Thus $f(U)$ is open in Y and hence f is super pre- γ-open.

(i) → (iii): Since pre- $\gamma int(A) \subseteq A \subseteq X$ is pre-γ-open set and f is super pre- γ-open, then $f($ pre-$\gamma int(A)) \subseteq f(A)$ is open in Y. Hence, $f($ pre-$\gamma int(A)) \subseteq int(f(A))$.

(iii) → (iv): By replacing $f^{-1}(B)$ instead of A of (iii), we have $f($pre- $\gamma int(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$ and hence, pre-$\gamma int(f^{-1}(B)) \subseteq f^{-1}(int(B))$.

(iv) → (v): Let $B \subseteq Y$. Then by hypothesis and Definition 2.3., we have $f^{-1}(B)\backslash f^{-1}(int(B)) \subseteq f^{-1}(B)\backslash$ pre- $\gamma int(f^{-1}(B))$ and hence, $f^{-1}(Bd(B)) \subseteq$ pre-$\gamma Bd(f^{-1}(B))$.

(v) → (iv): Let $B \subseteq Y$. Then by hypothesis and Definition 2.3., we have $f^{-1}(B\backslash int(B)) \subseteq f^{-1}(B)\backslash$pre-$\gamma int(f^{-1}(B))$ and hence $f^{-1}(B)\backslash f^{-1}(int(B)) \subseteq f^{-1}(B)\backslash pre\text{-}\gamma int(f^{-1}(B))$. Therefore, pre-$\gamma int(f^{-1}(B)) \subseteq f^{-1}(int(B))$.

(iv) → (vi): Let $B \subseteq Y$. Then $Y\backslash B \subseteq Y$, hence by hypothesis, we have pre-$\gamma int(f^{-1}(Y \backslash B)) \subseteq f^{-1}(int(Y \backslash B))$ and hence $X\backslash$ pre-$\gamma cl(f^{-1}(B)) \subseteq X\backslash f^{-1}(cl(B))$. Therefore, $f^{-1}(cl(B))$ □ pre-$\gamma cl(f^{-1}(B))$.

(vi) → (iv): Let $B \subseteq Y$. Then $Y \backslash B \subseteq Y$. So by hypothesis, we have $f^{-1}(cl(Y\backslash B)) \subseteq$ pre-$\gamma cl(f^{-1}(Y \backslash B))$ and hence $X\backslash f^{-1}(int(B)) \subseteq X\backslash$ pre-$\gamma int(f^{-1}(B))$. Therefore, pre-$\gamma int(f^{-1}(B) \subseteq f^{-1}(int(B))$.

(iv) → (i): Let $A \in$ pre- $\gamma O(X)$. Then $f(A) \subseteq Y$ and by hypothesis,
pre-$\gamma int(f^{-1}(f(A))) \subseteq f^{-1}(int(f(A)))$. This implies that,
pre-$\gamma int(A) \subseteq f^{-1}(int(f(A)))$. Thus $f($ pre-$\gamma int(A)) \subseteq int(f(A))$. Therefore by (iii), f is super pre-γ-open.

(i) → (vii): Let $H = Y \backslash f(X\backslash F)$ and F be a pre-γ-closed set of X containing $f^{-1}(B)$. Then $X\backslash F$ is a pre-γ-open set. But f is a super-pre-γ-open mapping, then $f(X\backslash F)$ is open in Y. Therefore, H is a closed set of Y and $f^{-1}(H) = X\backslash f^{-1}f(X\backslash F) \subseteq X\backslash(X\backslash F) = F$.

(vii) → (i): Let $U \in$ pre- $\gamma O(X)$ and put $B = Y \backslash f(U)$. Then $X\backslash U \in$ pre- $\gamma C(X)$ with $f^{-1}(B) \subseteq X\backslash U$. By hypothesis, there exists a closed set H of Y such that $B \subseteq H$ and $f^{-1}(H) \subseteq X\backslash U$. Hence, $f(U) \subseteq Y \backslash H$ and since $B \subseteq H$, then $Y \backslash H \subseteq Y \backslash B = f(U)$. This implies $f(U) = Y \backslash H$ which is open. Therefore, f is super pre-γ-open.

Theorem 5.2. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective super pre-γ-open mapping. Then the following statements are hold:
(i) If X is a pre-γ- T_i-space, then Y is T_i, where i = 1, 2 .
(ii) If Y is a compact (respectively Lindelöff.) space, then X is pre-γ-compact (respectively pre-γ-Lindelöff).
Proof: (i) We prove that for the case of a pre-γ-T_2 -space. Let y_1, y_2 be two distinct points of Y. Then there exist $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is a pre-γ-T_2-space, then there exist two disjoint pre-γ-open sets U, V of X such that $x_1 \in U$ and $x_2 \in V$. But, f is super pre-γ-open map, then $f(U)$, $f(V)$ are open sets of Y with $y_1 \in f(U)$, $y_2 \in f(V)$, and $f(U) \cap f(V) = \emptyset$. Therefore, Y is T_2 .
(ii) We prove that the theorem for pre-γ-Lindelöff. space. Let $\{U_i$: i ∈ I} be a family of pre- γ-open cover of X and f be a surjective super pre- γ-open mapping. Then $\{f(Ui) : i \in I\}$ is an open cover of Y. But, Y is a Lindelöff space, hence there exists a countable subset I_0 of I such that $= \cup \{f(Ui) : i \in I_0\}$. Then by injective of f, $\{U_i : i \in I_0\}$ is a countable subfamily of X . Therefore, X is pre-γ-Lindelöff

Theorem 5.3. If $f : (X, \tau) \to (Y, \sigma)$ is a surjective super pre-γ-open mapping and Y is a connected space, then X is pre-γ-connected.
Proof: Obvious.

6. Pre*-γ-open and pre*-γ-closed mappings

In this section, we introduce the concepts of pre*-γ-open and pre*-γ-closed mappings. Also, we study some of their basic properties and characterizations.

Definition 6.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be:
(i) pre*-γ-open if $f(V) \in$ pre-$\gamma O(Y)$ for each $V \in$ pre-$\gamma O(X)$,
(ii) pre*- γ-closed if $f(V) \in$ pre-$\gamma C(Y)$ for each $V \in$ pre- $\gamma C(X)$.

Theorem 6.1. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then the following Statements are equivalent:
(i) f is pre*-γ-closed,
(ii) f is pre *-γ-open,
(iii)f^{-1} is pre-γ-irresolute.
Proof: Obvious.

Proposition 6.1. (i) Every super pre-γ-open mapping is pre*-γ-open, (ii) Every pre*-γ-open mapping is pre-γ-open.
Proof: (i) Let $A \subseteq X$ be a pre-γ-open set and f be super pre-γ-open, then $f(A)$ is open in Y and hence $f(A)$ is pre-γ-open. Therefore, f is pre*-γ-open.
(ii) Let $A \subseteq X$ be an open set and hence A is pre-γ-open. But, f is pre*-γ-open, then $f(A)$ is pre- γ-open in Y. Therefore, f is pre- γ-open.

Remark 6.1. According to the above proposition, we have the following diagram

super pre-γ-open → pre*-γ-open → pre-γ-open

The converse of the above implication is not true in general.

Example 6.1. In Remark 5.1., f is pre-γ-open but not pre*-γ-open. Since $\{a,c,d\} \in$ pre-$\gamma O(X)$ and $f(\{a,c,d\}) = \{b,c,d\} \notin$pre-$\gamma O(Y)$.

Example 6.2. Suppose that $X = Y = \{a,b,c,d\}$ with topologies $\tau = \{X,\emptyset,\{a\},\{b\},\{c\},\{a, b\}, \{a,c\},\{b,c\},\{a,b,c\}\}$ and $\sigma = \{Y,\emptyset,\{d\}\}$. Define an operation γ on σ by $\gamma(A) = A$ and an operation γ on τ by

$$\gamma(A) = \begin{cases} \text{int}(cl(A)) & if \;\; A \neq \{a\} \\ cl(A) & if \;\; A = \{a\}. \end{cases}$$

Also a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ which defined by $f(a) = b, f(b) = f(c) = d$ and $f(d) = c$ is pre-γ-open but not super pre-γ-open. Since $\{a,b\} \in$ pre-$\gamma O(X)$ and $f(\{a,b\})$ = {b, d} ∉ σ.

Theorem 6.2. For a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$, the following statements are equivalent:

(i) f is pre*-γ-open,

(ii) For each x ∈ X and each pre-γ-neighborhood U of x, there exists $V \in$ pre-$\gamma O(Y)$ containing $f(x)$ such that $V \subseteq f(U)$,

(iii) $f($ pre-$\gamma int(A)) \subseteq$ pre- $\gamma int(f(A))$ for each $A \subseteq X$,

(iv) pre-$\gamma int(f^{-1}(B)) \subseteq f^{-1}($ pre-$\gamma int(B))$ for each $B \subseteq Y$,

(v) $f^{-1}($ pre-$\gamma Bd(B)) \subseteq$ pre-$\gamma\, Bd(f^{-1}(B))$ for each $B \subseteq Y$,

(vi) $f^{-1}($ pre-$\gamma cl(B)) \subseteq$ pre-$\gamma\, cl(f^{-1}(B))$ for each $B \subseteq Y$.

Proof: It is similar to that of Theorem 5.1.

Theorem 6.3. If $f : (X,\tau) \rightarrow (Y,\sigma)$ is a surjective pre*-γ-closed mapping and $f^{-1}(B),\ f^{-1}(C)$ have disjoint pre-γ-neighborhoods of X, then $B,\ C$ are disjoint of Y.

Proof: Obvious.

Theorem 6.4. For a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$, then the following statements are equivalent:

(i) f is pre*-γ-closed,

(ii) pre- $\gamma cl(f(A)) \subseteq f($ pre-$\gamma cl(A))$, for each $A \subseteq X$,

(iii) If f is surjective for each subset B of Y and for each pre-γ-open set U of X containing $f^{-1}(B)$, there exists a pre-γ-open set V of Y containing B such that $f^{-1}(V) \subseteq U$.

Proof: Obvious.

Theorem 6.5. Let $f : (X,\tau_X) \rightarrow (Y,\tau_Y)$ and $g : (Y,\tau_Y) \rightarrow (Z,\tau_Z)$ be two mappings. Then the following statements are hold:

(i) $g \circ f$ is a pre*-γ-open mapping, if $f,\ g$ are pre*-γ-open,

(ii) $g \circ f$ is a pre-γ-open mapping if f is pre-γ-open and g is pre*-γ-open,

(iii) If f is a surjective pre-γ-continuous mapping and $g \circ f$ is pre*-γ-open, then g is pre-γ-open.

Proof: (i) Let $U \in$ pre-$\gamma O(X)$ and f be a pre*-γ-open mapping. Then $f(U) \in$ pre-$\gamma O(Y)$. But, g is pre*-γ-open, then $g(f(U)) \in$ pre-$\gamma O(Z)$. Hence, $g \circ f$ is pre*-γ-open.
(ii) Let $U \in \tau_X$ and f be a pre-γ-open mapping. Then $f(U) \in$ pre-$\gamma O(Y)$. But, g is pre*-γ-open, then $g(f(U)) \in$ pre-$\gamma O(Z)$. Hence, $g \circ f$ is pre-γ-open.
(iii) Let $U \in \tau_Y$ and f be a pre*-γ-continuous mapping. Then $f^{-1}(U) \in$ pre-$\gamma O(X)$. But, $g \circ f$ is pre*-γ-open, then $(g \circ f)(f^{-1}(U)) \in$ pre-$\gamma O(Z)$. Also, by surjective of f, $g(U) \in$ pre- $\gamma O(Z)$. Hence, g is pre-γ-open.

Theorem 6.6. Let $f : (X, \tau_X) \to (Y, \tau_Y)$ and $g : (Y, \tau_Y) \to (Z, \tau_Z)$ be two mappings such that $g \circ f : X \to Z$ is pre-γ-irresolute. Then:
(i) f is pre-γ-irresolute, if g is an injective pre*-γ-open mapping.
(ii) g is pre-γ-irresolute, if f is a surjective pre*-γ-open mapping.
Proof: (i) Let $U \in$ pre-$\gamma O(Y)$. Then $g(U) \in$ pre-$\gamma O(Z)$. But, $g \circ f$ is pre-γ-irresolute, then $(g \circ f)^{-1}(g(U)) \in$ pre-$\gamma O(X)$. Since g is an injective map, then $f^{-1}(U) \in$ pre-$\gamma O(X)$. Hence, f is pre-γ-irresolute.
(ii) Let $V \in$ pre-$\gamma O(Z)$. Then $(g \circ f)^{-1}(V) \in$ pre-$\gamma O(X)$. But, f is a pre*-γ-open mapping, then $f[(g \circ f)^{-1}(V)] \in$ pre-$\gamma O(Y)$. Since f is a surjective map, then $g^{-1}(V) \in$ pre-$\gamma O(Y)$. Therefore, g is pre- γ-irresolute.

Theorem 6.7. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective pre-γ-open mapping. Then the following statements are hold:
(i) If X is a pre-γ- T_i-space, then Y is pre-γ- T_i , where i = 1, 2 .
(ii) If Y is a pre-γ-compact (respectively pre-γ-Lindelöff.) space, then X is pre-γ-compact (respectively pre-γ-Lindelöff.).
Proof: Obvious.

Theorem 6.8. If $f : (X, \tau) \to (Y, \sigma)$ is a surjective pre*-γ-open mapping and Y is a pre-γ-connected space, then X is pre-γ-connected.
Proof: Obvious.

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