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Optimization of EPQ Inventory Models of two Level Trade Credit with Payment Policies Under Cash Discount

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Abstract. The main purpose of this paper is to investigate the retailer's optimal cycle time and optimal payment time under the supplier's cash discount and trade credit policy within the economic production quantity (EPQ) framework. In this paper, we assume that the retailer will provide a full trade credit to his/her good credit customers and request his/her bad credit customers pay for the items as soon as receiving them. Under this assumption, mathematical models have been derived for determining the retailer's optimal inventory cycle time so that the annual total profit is maximized.

Keyword: Inventory; EPQ; Two levels of trade credit; Cash discount

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

The traditional economic order quantity (EOQ) model assumes that the retailer must pay for the purchased items as soon as items are received. This is not always true in the actual business world. In fact, the supplier usually permits the retailer a delay of a fixed time period to settle the total amount owed to him. During such period, the retailer can sell the goods, accumulate revenue and earn interest. Over years, a number of researches have been published which dealt with the inventory model under trade credit. Goyal [1] suggested a mathematical model for obtaining the economic order quantity under permissible delay in payments. Aggarwal and Jaggi [2] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Jamal et al. [3] extended this issue with allowable shortage. Chung and Huang [4] extended Goyal's model by considering the units are replenished at a finite rate. Teng [5] amended Goyal's model by considering the difference between unit price and unit cost, and found that the economic replenishment interval and order quantity decrease under the permissible delay in payments in certain cases. Chung and Huang [6] extended Goyal's model by considering allowable shortage and presented a theorem to determine the optimal order quantity.

However, in most business transactions, the supplier not only allows a certain fixed period for settling the account but may also offer a cash discount to encourage the retailer to pay for his purchases quickly. The retailer can obtain the cash discount when the payment is paid within cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. For example, the supplier agrees to a 2% discount off the retailer's purchasing price if payment is made within 10 days. Otherwise, full payment is required within 30 days after the delivery. Huang and Chung [8] extended Goyal's [1] model with cash discount and determined the optimal cycle time and the optimal payment policy in the EOQ model under cash discount and trade credit so that the annual total relevant cost is minimized. Huang [9] extended Huang and Chung's [8] model by considering the difference between unit price and unit cost. Huang [10] extended Huang and Chung's [8] model by considering the replenishment rate is finite. Ouyang et al. [11] established an EOQ model with limited storage capacity, in which the supplier provides cash discount and permissible delay in payments for the retailer. Ho et al. [12] formulated an integrated supplier–buyer inventory model with the assumptions that the market demand is sensitive to the retail price and the supplier offers two payment options: trade credit and early-payments with discount price to the buyer.

All of the above models assumed that the supplier would offer the retailer a permissible delay of payments. That is one level of trade credit. Huang [13] pointed out that the retailer may also adopt the trade credit policy to stimulate his/her customer demand in most business transactions. Huang [14] defined this situation as two levels of trade credit policy, and incorporated both Chung and Huang [4] and Huang [13] to investigate the optimal retailer's replenishment decisions with two levels of trade credit policy in the EPQ framework. Teng and Chang [15] overcame a shortcoming in Huang's [14] model and proposed the generalized formulation to the problem. Huang and Hsu [16] extended Huang's [13] model by considering the retailer just offers the partial trade credit to his/her customer. Teng [17] established an inventory lot-sizing model for a retailer who receives a full trade credit from its supplier, and offers either a partial trade credit to its bad credit customers or a full trade credit to its good credit customers.

Therefore, in this study, for advancing practical use in a real world, we propose an inventory model with defective products under cash discount and two-level trade credit policy under the replenishment rate is finite. We model the retailer's inventory system as a profit maximization problem to determine the retailer's optimal inventory cycle time and optimal payment time under cash discount and two-level trade credit within the EPQ framework.

2. Notation and assumption

The following notation is used throughout this paper.

D	annual demand rate
P	annual replenishment rate, $P > D$
Q	order size
C	purchasing cost per unit
K	fixed ordering cost per order
s	selling price per unit
h	stock holding cost per unit per year

F	freight cost per delivery
d	unit screening cost
C_s	unit disposal cost of scrap items
x	annual screening rate
n	number of shipments from supplier to retailer
p	percentage of defective items in Q
q	percentage of scrap items in defective items
v	the unit price of imperfect items, $v < c$
I_d	interest earned per \$ per year
I_c	interest charged per \$ per year
M_1	retailer's fixed period of cash discount
M_2	retailer's fixed period of permissible delay in settling accounts , with $M_2 > M_1$
δ	cash discount rate, $0 < \delta < 1$
α	the fraction of the customers who pay for the items immediately upon receiving them, $0 \leq \alpha \leq 1$.
$1 - \alpha$	the portion of the customers who receive a permissible delay of payment
T	inventory cycle length of each cycle
T^*	Optimal replenishment cycle time
Q^*	Optimal order quantity

In addition, the following assumptions are made in this model.

1. There is a single supplier and single retailer for a single product in this model.
2. The demand rate is known, constant and continuous and the replenishment rate is known and constant.
3. The lead time is zero.
4. Shortages are not allowed.
5. Each order is subjected to a 100% inspection process at a rate of x units per unit time, the screening rate x is sufficiently large such that $t = \frac{Q}{x} \leq M_1$
6. To speed up cash in flow and reduce the risk of cash flow shortage, the supplier offers a discount $\delta(0 < \delta < 1)$, off the retailer's unit purchasing price, if the retailer settles the account at time M_1, otherwise, the full price of the purchase is charged.
7. During the credit period ((ie) M_1 or M_2), the retailer sells the items and uses the sales revenue to earn interest at a rate of I_d. At the end of this period, the retailer pays off all purchasing cost to the supplier and starts paying for the interest charges for the items in stock with rate I_c.
8. $s \geq c$, $I_c \geq I_d$ and $c(1 - \delta)I_c \geq sI_d$.
9. Each production lot Q has defective rate of p. Those pQ defective items in each cycle comprise $(1 – q)pQ$ imperfect (or reworkable items) and q pQ scrap (or unworkable) items. The scrap items must be removed from inventory at the end of the screening process at a disposal cost C_s per unit. Re-workable items are sold in a single batch at a discount price v per unit at the end of the cycle.

3. Model formulation

In this section, we formulate an inventory model under cash discount and two levels of trade credit policy. The total relevant cost consists of the following elements.

1. **Ordering cost per cycle:** The ordering cost per cycle is $\frac{K}{T}$.

2. **Holding cost per cycle:** With the stock holding cost per unit per year h, the holding cost per cycle is

$$\frac{hT(P - D)\frac{DT}{P}}{2} = \frac{DT^2h}{P}\left(1-\frac{D}{P}\right) = \frac{DT^2h\rho}{2} \text{ where } \rho = 1-\frac{D}{P}.$$

3. **Purchasing cost per cycle:**

Case 1 : Payment is paid at M_1. The cost of purchasing units is c(1 - δ)Q.
Case 2 : Payment is paid at M_2. The cost of purchasing units is cQ.

4. **Screening cost:** According to assumption (5), the screening cost is $\frac{dQ}{T}$.

5. **Freight cost:** The freight cost per unit time is $\frac{F}{T}$.

6. **Disposal cost per cycle:** The disposal cost per cycle is $\frac{C_s qpQ}{T}$.

7. According to the assumption (7), as well as the values of N and M_1 or M_2. There are two cases that occur in interest charged and interest earned per cycle.

Case 1: Payment is paid at M_1.

Case 1.1: If $N \le M_1$, there are four sub-cases in terms of interest charged and interest earned per cycle.

Case 1.1A: $M_1 \le \frac{PM_1}{D} \le T$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period $[0, M_1]$ from the portion of immediate payment and earn interest on average realized credit sales revenue for the time period $[N, M_1]$ from the portion of delayed payment. At M_1, the accounts are settled, and the retailer must pay for all items sold after M_1 for the proportion of immediate payment and all items sold after $M_1 - N$ for the portion of delayed payment with rate I_c.

Consequently, the interest charged is

$$c(1-\delta)I_c\left[\alpha\left(\frac{DT^2\rho}{2}-\frac{(P - D)M_1^2}{2}\right)+(1-\alpha)\left(\frac{DT^2\rho}{2}-\frac{(P - D)(M_1-N)^2}{2}\right)\right]$$

and the interest earned is $sI_d\left[\alpha\frac{DM^2}{2}+(1-\alpha)\frac{D(M_1-N)^2}{2}\right]$.

Case 1.1B: $M_1 \le T \le \frac{PM_1}{D}$

Same discussion as above case 1.1.A.

The interest charged is $c(1-\delta)I_c\left[\dfrac{\alpha D(T-M_1)^2}{2}+\dfrac{(1-\alpha)D(T+N-M_1)^2}{2}\right]$, and the interest earned is $sI_d\left[\alpha\dfrac{DM_1^2}{2}+(1-\alpha)\dfrac{D(M_1-N)^2}{2}\right]$.

Case 1.1C: $T \le M_1 \le T + N$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, T] and full sales revenue for the time period [T, M_1] from the portion of immediate payment and earn interest on average realized credit sales revenue for the time period [N, M_1] from the portion of delayed payment. At M_1, the accounts are settled, and the retailer must pay for all items sold after $M_1 - N$ for the portion of delayed payment with rate I_c. Consequently, the interest charged is $c(1-\delta)I_c\dfrac{(1-\alpha)D(T+N-M_1)^2}{2}$ and the interest earned is $\dfrac{sDI_d}{2}\left[\alpha T^2+2\alpha T(M_1-T)+(1-\alpha)(M_1-N)^2\right]$.

Case 1.1D: $T + N \le M_1$

In this case, the permissible payment time expires on or after the credit sales are completely realized. Consequently, there is no interest payable, and the interest earned is $\dfrac{sI_dDT}{2}\left[T+2\alpha(M_1-T)+2(1-\varepsilon)(M_1-T-N)\right]$.

Case 1.2: If $M_1 \le N$ there are three sub-cases in terms of interest charged and interest earned per cycle.

Case 1.2A: $M_1 \le \dfrac{PM_1}{D} \le T$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, M_1] from the proportion of immediate payment. At M_1, the accounts are settled and the retailer must pay all items sold after M_1, for the proportion of immediate payment and the entire amount of the delayed payment with rate I_c. Consequently, the interest charged is $c(1-\delta)I_c\left[\alpha\left(\dfrac{DT^2\rho}{2}-\dfrac{(P-D)M_1^2}{2}\right)+(1-\alpha)\left(\dfrac{DT^2\rho}{2}+DT(N-M_1)\right)\right]$ and the interest earned is $\alpha sI_d\dfrac{DM_1^2}{2}$.

Case 1.2B: $M_1 \le T \le \dfrac{PM_1}{D}$

Same discussion as above case 1.2A.

The interest charged is $c(1-\delta)I_c\left[\frac{\alpha D(T - M_1)^2}{2} + (1-\alpha)\left(\frac{DT^2\rho}{2} + DT(N - M_1)\right)\right]$ and the interest earned is $\alpha sI_d \frac{DM_1^2}{2}$.

Case 1.2C: $T \le M_1 \le \frac{PM_1}{D}$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, T] and full sales, revenue for the time period [T, M_1] from the portion of immediate payment. At M_1, the accounts are settled and the retailer must pay for the entire amount of the delayed payment with rate I_c. Consequently, the interest charged is $c(1-\delta)I_c(1-\alpha)\left(\frac{DT^2\rho}{2} + DT(N - M_1)\right)$ and the interest earned is $\alpha sI_d\left(\frac{DT^2}{2} + DT(M_1 - T)\right)$.

Case 2: Payment is paid at M_2.

Case 2.1A: $M_2 \le \frac{PM_2}{D} \le T$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, M_2] from the portion of immediate payment, an earn interest on average realized credit sales revenue for the time period [N, M_2] from the proportion of delayed payment. At M_2, the accounts are settled, and the retailer must pay for all items sold after M_2 – N for the portion of delayed payment with rate I_c. Consequently, the interest charged is $cI_c\left[\alpha\left(\frac{DT^2\rho}{2} - \frac{(P - D)M_2^2}{2}\right) + (1-\alpha)\left(\frac{DT^2\rho}{2} - \frac{(P - D)(M_2 - N)^2}{2}\right)\right]$ and the interest earned is $sI_d\left[\alpha\frac{DM^2}{2} + (1-\alpha)\frac{D(M_2 - N)^2}{2}\right]$.

Case 2.1B: $M_2 \le T \le \frac{PM_2}{D}$

Same discussion as above case 2.1.A.

The interest charged is $cI_c\left[\frac{\alpha D(T - M_2)^2}{2} + \frac{(1-\alpha)D(T + N - M_2)^2}{2}\right]$, and the interest earned is $sI_d\left[\alpha\frac{DM_2^2}{2} + (1-\alpha)\frac{D(M_2 - N)^2}{2}\right]$.

Case 2.1C: $T \le M_2 \le T + N$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, T] and full sales revenue for the time period [T, M_2] from the portion of immediate payment and earn interest on average realized credit sales revenue for the time period [N, M_2] from the portion of delayed payment. At M_2 the accounts are settled, and the retailer must pay all items sold after M_2 – N for the portion of delayed payment with rate I_c. Consequently, the interest charged is $cI_c\dfrac{(1-\alpha)D(T+N-M_2)^2}{2}$ and the interest earned is $\dfrac{sDI_d}{2}\left[\alpha T^2 + 2\alpha T(M_2-T) + (1-\alpha)(M_2-N)^2\right]$.

Case 2.1D: T + N ≤ M_2

In this case, the permissible payment time expires on or after the credit sales are completely realized. Consequently, there is no interest payable and the interest earned is $\dfrac{sDI_dT}{2}\left[T + 2\alpha(M_2-T) + 2(1-\alpha)(M_2-T-N)\right]$.

Case 2.2: If $M_2 \le N$, there are three sub-cases in terms of interest charged and interest earned per cycle.

Case 2.2A: $M_2 \le \dfrac{PM_2}{D} \le T$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, M_2] from the proportion of immediate payment. At M_2, the accounts are settled, and the retailer must pay all items sold after M_2 for the proportion of immediate payment and the entire amount of the delayed payment with rate I_c. Consequently, the interest charged is $cI_c\left[\alpha\left(\dfrac{DT^2\rho}{2}-\dfrac{(P-D)M_2^2}{2}\right)+(1-\alpha)\left(\dfrac{DT^2\rho}{2}-DT(N-M_2)\right)\right]$ and the interest earned is $\alpha sI_d\dfrac{DM_2^2}{2}$.

Case 2.2B: $M_2 \le T \le \dfrac{PM_2}{D}$

Same discussion as above case 2.1.A.

The interest charged is $cI_c\left[\dfrac{\alpha D(T-M_2)^2}{2}+(1-\alpha)\left(\dfrac{DT^2\rho + DT(N-M_2)}{2}\right)\right]$ and the interest earned is $\alpha sI_d\dfrac{DM_2^2}{2}$.

Case 2.2C: $T \le M_2 \le \dfrac{PM_2}{D}$

In this case, the retailer can earn interest on average realized credit sales revenue for the time period [0, T] and full sales revenue for the time period [T, M_2] from the portion of immediate payment. At M_2, the accounts are settled and the retailer must pay for the entire amount of the delayed payment with rate I_c. Consequently, the interest charged is

$cI_c(1-\alpha)\left(\frac{DT^2\rho}{2} + DT(N - M_2)\right)$ and the interest earned is $\alpha sI_d\left[\frac{DT^2}{2} + DT(M_2 - T)\right]$.

The annual total cost for that retailer is the total relevant cost (which includes ordering cost, holding cost, purchasing cost, screening cost, freight cost, disposal cost and interest charged) minus interest earned.

From the above arguments, we show that the annual total cost for that retailer is given by,

$$TC(T) = \begin{cases} TC_1(T), \text{ if the retailer settles the account at } M_1 \\ TC_2(T), \text{ if the retailer settles the account at } M_2 \end{cases}$$

Case 1: If the retailer settles the account at M_1.

Case 1.1: $M_1 \geq N$

In this case, the retailer's fixed period of cash discount M_1 is equal to or larger than the customer's fixed period of permissible delay in payments N. From the above discussion, the annual total cost for the retailer consists of the following four cases

(1) $M_1 \leq \frac{PM_1}{D} \leq T$ (2) $M_1 \leq T \leq \frac{PM_1}{D}$ (3) $T \leq M_1 \leq T + N$ and (4) $T + N \leq M_1$

$$\text{i.e. } TC_1(T) = \begin{cases} TC_{11}(T), \text{ if } M_1 \leq \frac{PM_1}{D} \leq T \\ TC_{12}(T), \text{ if } M_1 \leq T \leq \frac{PM_1}{D} \\ TC_{13}(T), \text{ if } T \leq M_1 \leq T + N \\ TC_{14}(T), \text{ if } T + N \leq M_1 \end{cases}$$

$$\text{where } TC_{11}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + c(1-\delta)Q + \frac{dQ}{T} + \frac{F}{T} + \frac{C_sqPQ}{T} + c(1-\delta)I_c\left[\alpha\left(\frac{DT^2\rho}{2} - \frac{(P - D)M_1^2}{2}\right) + (1-\alpha)\left(\frac{DT^2\rho}{2} - \frac{(P - D)(M_1 - N)^2}{2}\right)\right] - sI_d\left[\alpha\frac{DM_1^2}{2} + (1-\alpha)\frac{D(M_1 - N)^2}{2}\right].$$

$$= \frac{1}{T}\left\{K - \frac{c(1-\delta)I_c}{2}\left[\alpha(P-D)M_1^2 + (1-\alpha)(P-D)(M_1-N)^2\right] - \frac{sI_d}{2}\left[\alpha DM_1^2 + (1-\alpha)D(M_1-N)^2\right] + dD + F + C_sqpD\right\}$$

$$T\left[\frac{Dh\rho}{2}+\frac{c(1-\delta)I_c D\rho}{2}\right] + c(1-\delta)D \qquad \ldots (1)$$

$$TC_{12}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + c(1-\delta)Q + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} + c(1-\delta)I_c\left[\frac{\alpha D(T - M_1)^2}{2}+\frac{(1-\alpha)D(T + N - M_1)^2}{2}\right] - sI_d\left[\alpha\frac{DM_1^2}{2}+(1-\alpha)\frac{D(M_1-N)^2}{2}\right]$$

$$TC_{12}(T) = \frac{1}{T}\left\{K + \frac{c(1-\delta)I_c}{2}\left[\alpha D(M_1^2+(1-\alpha)D(M_1-N)^2\right] - \frac{sI_d}{2}\left[\alpha DM_1^2+(1-\alpha)D(M_1-N)^2\right] + dD + F + C_s qpD\right\} + T\left[\frac{Dh\rho}{2}+\frac{c(1-\delta)I_c D}{2}\right] + c(1-\delta)D - c(1-\delta)I_c D - \frac{sI_d}{2}\left[\alpha M_1+(1-\alpha)(M_1-N)\right] \qquad \ldots (2)$$

$$TC_{13}(T)=\frac{K}{T} + \frac{DT^2h\rho}{2} + c(1-\delta)Q + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} + c(1-\delta)I_c(1-\alpha)\frac{D(T + N-M_1)^2}{2} - \frac{sI_dD}{2}\left[\alpha T^2+2\alpha T(M_1-T) + (1-\alpha)(M_1-N)^2\right]$$

$$= \frac{1}{T}\left\{K + \frac{c(1-\delta)I_c}{2}(1-\alpha)D(M_1-N)^2 - \frac{sI_d}{2}(1-\alpha)D(M_1-N)^2 + dD + F + C_s qpD\right\} + T\left[\frac{Dh\rho}{2}+\frac{c(1-\delta)I_c(1-\alpha)D}{2}+\frac{\alpha sI_dD}{2}\right] + c(1-\delta)D - \alpha sDI_dM_1 - c(1-\delta)I_cD(1-\alpha)(M_1-N) \qquad \ldots (3)$$

$$TC_{14}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + c(1-\delta)Q + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} + - \frac{sI_dD}{2}\left[T + 2\alpha(M_1-T) + 2(1-\alpha)(M_1-T-N)\right]$$

$$= \frac{K}{T} + T\left(\frac{Dh\rho}{2} + \frac{sDI_d}{2}\right) + c(1-\delta)D + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPD}{T} - sDI_d M_1 + (1 - \alpha)sDI_d N \qquad \ldots (4)$$

Case 1.2: $M_1 \le N$

In this case, the retailer's fixed period of cash discount M_1 is equal to or less than the customer's fixed period of permissible delay in payments N. The annual total cost for the retailer consists of the following three cases

(1) $M_1 \le \frac{PM_1}{D} \le T$ (2) $M_1 \le T \le \frac{PM_1}{D}$ (3) $T \le M_1 \le \frac{PM_1}{D}$

$$\text{i.e. } TC_1(T) = \begin{cases} TC_{15}(T), \text{ if } M_1 \le \frac{PM_1}{D} \le T \\ TC_{16}(T), \text{ if } M_1 \le T \le \frac{PM_1}{D} \\ TC_{17}(T), \text{ if } T \le M_1 \le \frac{PM_1}{D} \end{cases}$$

$$\text{where } TC_{15}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + c(1-\alpha)Q + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} + c(1-\delta)I_c\left[\alpha\left(\frac{DT^2\rho}{2} - \frac{(P - D)M_1^2}{2}\right) + (1-\alpha)\left(\frac{DT^2\rho}{2} - DT(N - M_1)\right)\right] - \alpha sI_d\frac{DM_1^2}{2}.$$

$$= \frac{1}{T}\left\{K - \frac{\alpha c(1-\delta)I_c(P-D)M_1^2}{2} - \frac{\alpha sI_d DM_1^2}{2} + dD + F + C_s qPD\right\} T\left(\frac{Dh\rho}{2} + \frac{c(1-\delta)I_c D\rho}{2}\right) + c(1 - \delta)D + c(1-\delta)I_c(1-\alpha)D(N - M_1) \qquad \ldots (5)$$

$$TC_{16}(T) = \frac{1}{T}\left\{K + \frac{\alpha c(1-\delta)I_c DM_1^2}{2} - \frac{\alpha sI_d DM_1^2}{2} + dD + F + C_s qPD\right\} + T\left\{\frac{Dh\rho}{2} + \frac{c(1 - \delta)I_c D}{2}\left[\alpha + (1 - \alpha)\rho\right]\right\} + c(1 - \delta)D - c(1 - \delta)I_c D\left[\alpha M_1 - (1 - \alpha)(N - M_1)\right] \qquad \ldots (6)$$

$$TC_{17}(T) = \frac{K}{T} + T\left(\frac{DTh\rho}{2} + \frac{c(1-\delta)I_c(1-\alpha)D\rho}{2} + \frac{\alpha sI_d}{2}\right) + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} + c(1 - \delta)D - \alpha sDI_d M_1 + c(1 - \delta)I_c D(1 - \alpha)(N - M_1) \ldots (7)$$

Case 2: If the retailer settles the account at M_2.

Case 2.1: $M_2 \geq N$

In this case, the retailer's fixed period of permissible delay in payments M_2 is equal to or larger than the customer's fixed period of permissible delay in payments N.

From the above discussion, the annual total cost for the retailer consists of the following four cases

(1) $M_2 \leq \frac{PM_2}{D} \leq T$ (2) $M_2 \leq T \leq \frac{PM_2}{D}$ (3) $T \leq M_2 \leq T + N$ and (4) $T + N \leq M_2$

$$\text{i.e. } TC_2(T) = \begin{cases} TC_{21}(T), \text{ if } M_2 \leq \dfrac{PM_2}{D} \leq T \\ TC_{22}(T), \text{ if } M_2 \leq T \leq \dfrac{PM_2}{D} \\ TC_{23}(T), \text{ if } T \leq M_2 \leq T + N \\ TC_{24}(T), \text{ if } T + N \leq M_2 \end{cases}$$

$$\text{where } TC_{21}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + CQ + \frac{dQ}{T} + \frac{F}{T} + \frac{C_sqPQ}{T} + cI_c\left[\alpha\left(\frac{DT^2\rho}{2} - \frac{(P-D)M_2^2}{2}\right) + (1-\alpha)\left(\frac{DT^2\rho}{2} - \frac{(P-D)(M_2-N)^2}{2}\right)\right] - sI_d\left[\alpha\frac{DM^2}{2} + (1-\alpha)\frac{D(M_2-N)^2}{2}\right].$$

$$= \frac{1}{T}\left\{K - \frac{cI_c}{2}\left[\alpha(P-D)M_2^2 + (1-\alpha)(P-D)(M_2-N)^2\right] + dD + F + C_sqpD - \frac{sI_d}{2}\left[\alpha DM_2^2 + (1-\alpha)D(M_2-N)^2\right]\right\} T\left[\frac{Dh\rho}{2} + \frac{cI_cD\rho}{2}\right] + cD \quad \text{. . . (8)}$$

$$TC_{22}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + cQ + \frac{dQ}{T} + \frac{F}{T} + \frac{C_sqPQ}{T} + cI_c\left[\frac{\alpha D(T-M_2)^2}{2} + \frac{(1-\alpha)D(T+N-M_2)^2}{2}\right] - sI_d\left[\alpha\frac{DM_2^2}{2} + (1-\alpha)\frac{D(M_2-N)^2}{2}\right]$$

$$= \frac{1}{T}\left\{K + \frac{cI_c}{2}\left[\alpha D(M_2^2 + (1-\alpha)D(M_2-N)^2\right] - \frac{sI_d}{2}\left[\alpha DM_2^2 + (1-\alpha)D(M_2-N)^2\right] + dD\right.$$

$$+ F + C_s qpD\Big\} + T\left[\frac{Dh\rho}{2} + \frac{cI_cD}{2}\right] + cD - cI_cD\left[\alpha M_2 + (1-\alpha)(M_2 - N)\right] \quad \ldots (9)$$

$$TC_{23}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + cQ + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} + \frac{cI_c(1-\alpha)D(T+N-M_2)^2}{2} - \frac{sI_dDT}{2}\left[\alpha T^2 + 2\alpha(M_2-T) + (1-\alpha)(M_2-N)^2\right]$$

$$TC_{23}(T) = \frac{1}{T}\left\{K + \frac{cI_c}{2}(1-\alpha)D(M_2-N)^2 - \frac{sI_d}{2}(1-\alpha)D(M_2-N)^2 + dD + F + C_s qpD\right\}$$

$$+ T\left[\frac{Dh\rho}{2} + \frac{cI_c(1-\alpha)D}{2} + \frac{\alpha sDI_d}{2}\right] + cD - \alpha sDI_dM_2 - cI_cD(1-\alpha)(M_2-N) \quad \ldots (10)$$

$$TC_{24}(T) = \frac{K}{T} + \frac{DT^2h\rho}{2} + cQ + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qPQ}{T} - \frac{sDI_dQ}{T}$$

$$- \frac{sI_dDT}{2}\left[T + 2\alpha(M_2-T) + 2(1-\alpha)(M_2-T-N)\right]$$

$$= \frac{K}{T} + T\left[\frac{Dh\rho}{2} + \frac{sDI_d}{2}\right] + \frac{dQ}{T} + \frac{F}{T} + \frac{C_s qpQ}{T} + cD - sDI_dM_2 + (1-\alpha)sDI_dN \quad (11)$$

Case 2.2: $M_2 \le N$

In this case, the retailer's fixed period of permissible delay in payments M_2 is equal to or less than the customer's fixed period of permissible delay in payments N. The annual total cost for the retailer consists of the following three cases

(1) $M_2 \le \frac{PM_2}{D} \le T$ (2) $M_2 \le T \le \frac{PM_2}{D}$ (3) $T \le M_2 \le \frac{PM_2}{D}$

$$\text{i.e. } TC_2(T) = \begin{cases} TC_{25}(T), \text{ if } M_2 \le \frac{PM_2}{D} \le T \\ TC_{26}(T), \text{ if } M_2 \le T \le \frac{PM_2}{D} \\ TC_{27}(T), \text{ if } T \le M_2 \le \frac{PM_2}{D} \end{cases}$$

where $TC_{25}(T) = \frac{1}{T}\left\{K - \frac{\alpha cI_c(P-D)M_2^2}{2} - \frac{\alpha sI_dDM_2^2}{2} + dD + F + C_s qpD\right\}$

$$+T\left[\frac{Dh\rho}{2}+\frac{cI_cD\rho}{2}\right] + cD + cI_c(1-\alpha)D(N - M_2) \qquad ...(12)$$

$TC_{26}(T)=$

$$\frac{1}{T}\left\{K+\frac{\alpha cI_cDM_2^2}{2}-\frac{\alpha sI_dDM_2^2}{2} + dD + F + C_sqpD\right\} + T\left[\frac{Dh\rho}{2}+\frac{cI_cD}{2}\left[\alpha+(1-\alpha)\rho\right]\right]$$
$$+ cD + cI_cD\left[\alpha M_2-(1-\alpha)(N - M_2)\right] \qquad ...(13)$$

$$TC_{27}(T) = \frac{K}{T} + T\left[\frac{Dh\rho}{2}+\frac{cI_c(1 - \alpha)D\rho}{2}+\frac{\alpha sDI_d}{2}\right] + \frac{dQ}{T}+\frac{F}{T}+\frac{C_sqpQ}{T}+cD$$
$$- \alpha sDI_dM_2 + cI_cD(1 - \alpha)(N - M_2) \qquad ...(14)$$

4. Model analysis and solution

Now, we shall determine the optimal replenishment cycle time that minimizes the annual total cost.

4.1. Decision rules for the optimal replenishment cycle time T^* when $M_1 \geq N$.

Let $TC_{li}'(T) = 0$ for i = 1, 2, 3, 4 we can obtain

$$TC_{11}^*(T) = \sqrt{\frac{2(K + dD + F + C_sqpD) - [\rho c(1 - \delta)I_cP + sI_dD][\alpha M_1^2+(1 - \alpha)(M_1-N)^2}{Dh\rho + Dc(1 - \delta)I_c\rho}} \qquad ...(15)$$

if $2(K + dD + F + C_sqpD) - [\rho c(1 - \delta)I_cP + sI_dD][\alpha M_1^2+(1 - \alpha)(M_1-N)^2 \geq 0$.

Equation (15) gives the optimal value of T for the case $M_1 \leq \frac{PM_1}{D} \leq T$, so that $M_1 \leq \frac{PM_1}{D} \leq T$ so that $M_1 \leq \frac{PM_1}{D} \leq T_{11}^*$.

$$TC_{12}^*(T) = \sqrt{\frac{2(K + dD + F + C_sqpD) - [c(1 - \delta)I_c - sI_d][\alpha DM_1^2+(1 - \alpha) D(M_1-N)^2}{Dh\rho + Dc(1 - \delta)I_c}} \qquad ...(16)$$

if $2(K + dD + F + C_sqpD) - [c(1 - \delta)I_c + sI_d][\alpha DM_1^2+ (1 - \alpha) D(M_1-N)^2 \geq 0$.

Equation (16) gives the optimal value of T for the case $M_1 \leq T \leq \frac{PM_1}{D}$, so that $T_{12}^* \leq \frac{PM_1}{D}$.

$$TC_{13}^*(T) = \sqrt{\frac{2(K + dD + F + C_sqpD) +(1 - \alpha)D(M_1- N)^2(c(1 - \delta)I_c - sI_d)}{Dh\rho + (1 - \alpha)cI_cD(1 - \delta) + \alpha sDI_d}} \qquad ...(17)$$

if $2(K + dD + F + C_sqpD) + (1 - \alpha)D(M_1-N)^2(c(1 - \delta)I_c - sI_d \geq 0$.

Similarly Eq.(17) gives the optimal value of T for the case when $T \le M_1 \le \frac{PM_1}{D} \le T + N$ so that $T_{13}^* \le M_1 \le \frac{PM_1}{D} \le T_{13}^* + N$.

$$\mathrm{TC}_{14}^*(\mathrm{T}) = \sqrt{\frac{2(\mathrm{K + dD + F + C_s qpD})}{\mathrm{Dh}\rho + \alpha \mathrm{sDI_d}}} \quad ...(18)$$

Similarly Eq.(18) gives the optimal value of T for the case when $T \le M_1 - N$ so that $T_{14}^* \le M_1 - N$.

4.2. Decision rules of the optimal replenishment cycle time T^* when $M_1 \le N$.

Let $TC_{li}'(T) = 0$ (i = 5, 6, 7). We can obtain that

$$\mathrm{TC}_{15}^*(\mathrm{T}) = \sqrt{\frac{2(\mathrm{K + dD + F + C_s qpD}) - \alpha c(1-\delta)\mathrm{I_c(P - D)M_1^2} - \alpha \mathrm{sI_d DM_1^2}}{\mathrm{Dh}\rho + \mathrm{Dc}(1-\delta)\mathrm{I_c}\rho}} \quad ...(19)$$

if $2(\mathrm{K + dD + F + C_s qpD}) - \alpha c(1-\delta)\mathrm{I_c(P-D)M_1^2} - \alpha \mathrm{sI_d DM_1^2} \ge 0$.

Eq.(19) gives the optimal value of T for the case when $T \ge \frac{PM_1}{D}$ so that $T_{15}^* \ge \frac{PM_1}{D}$.

$$\mathrm{TC}_{16}^*(\mathrm{T}) = \sqrt{\frac{2(\mathrm{K + dD + F + C_s qpD}) + \alpha c(1-\delta)\mathrm{I_c DM_1^2} - \alpha \mathrm{sI_d DM_1^2}}{\mathrm{Dh}\rho + \mathrm{Dc}(1-\delta)\mathrm{I_c}[\alpha + \rho(1-\alpha)]}} \quad ...(20)$$

if $2(\mathrm{K + dD + F + C_s qpD}) + \alpha c(1-\delta)\mathrm{I_c DM_1^2} - \alpha \mathrm{sI_d DM_1^2} \ge 0$.

Similarly Eq.(20) gives the optimal value of T for the case when $M_1 \le T \le \frac{PM_1}{D}$ so that $M_1 \le T_{16}^* \le \frac{PM_1}{D}$.

$$\mathrm{TC}_{17}^*(\mathrm{T}) = \sqrt{\frac{2(\mathrm{K + dD + F + C_s qpD})}{\mathrm{Dh}\rho + c(1-\delta)\mathrm{I_c}(1-\alpha)\mathrm{D}\rho + \alpha \mathrm{sDI_d}}} \quad ...(21)$$

Similarly Eq.(21) gives the optimal value of T for the case when $T \le M_1$ so that $T_{17}^* \le M_1$.

4.3. Decision rules for the optimal replenishment cycle time T* when $M_2 \ge N$.

Let $TC_{2i}'(T) = 0$ (i = 1,2,3,4). We can obtain that

$$\mathrm{TC}_{21}^*(\mathrm{T}) = \sqrt{\frac{2(\mathrm{K + dD + F + C_s qpD}) - [\rho c\mathrm{I_c P + sI_d D}][\alpha \mathrm{M_2^2} + (1-\alpha)(\mathrm{M_2 - N})^2]}{\mathrm{Dh}\rho + \mathrm{DcI_c}\rho}} \quad ...(22)$$

if $2(\mathrm{K + dD + F + C_s qpD}) - [\rho c\mathrm{I_c P + sI_d D}][\alpha \mathrm{M_2^2} + (1-\alpha)(\mathrm{M_2 - N})^2] \ge 0$.

Eq.(22) gives the optimal value of T for the case when $M_2 \le \frac{PM_2}{D} \le T$ so that $M_2 \le \frac{PM_2}{D} \le T_{21}{}^*$.

$$TC^*_{22}(T) = \sqrt{\frac{2(K + dD + F + C_s qpD) + [cI_c - sI_d]\,[\alpha DM_2^2 + (1 - \alpha)D(M_2 - N)^2]}{Dh\rho + DcI_c}} \quad \text{. . . (23)}$$

if $2(K + dD + F + C_s qpD) + [cI_c - sI_d]\,[\alpha DM_2^2 + (1 - \alpha)D(M_2 - N)^2] \ge 0$.

Similarly Eq.(23) gives the optimal value of T for the case when $M_2 \le T \le \frac{PM_2}{D}$ so that $M_2 \le T_{22}{}^* \le \frac{PM_2}{D}$.

$$TC^*_{23}(T) = \sqrt{\frac{2(K + dD + F + C_s qpD) + (1 - \alpha)D(M_2 - N)^2(cI_c - sI_d)}{Dh\rho + (1 - \alpha)cDI_c + \alpha sDI_d}} \quad \text{. . . (24)}$$

if $2(K + dD + F + C_s qpD) + (1 - \alpha)D(M_2 - N)^2(cI_c - sI_d) \ge 0$.

Similarly Eq.(24) gives the optimal value of T for the case when $T \le M_2 \le \frac{PM_2}{D} \le T + N$ so that $T_{23}{}^* \le M_2 \le \frac{PM_2}{D} \le T_{23}{}^* + N$.

$$TC^*_{24}(T) = \sqrt{\frac{2(K + dD + F + C_s qpD)}{Dh\rho + sDI_d}} \quad \text{. . . (25)}$$

Similarly Eq.(25) gives the optimal value of T for the case when $T \le M_2 - N$ so that $T_{24}{}^* \le M_2 - N$.

4.4. Decision rules for the optimal replenishment cycle time T* when $M_2 \le N$.

Let $TC_{2i}'(T) = 0$ (i = 5, 6, 7). We can obtain that

$$TC^*_{25}(T) = \sqrt{\frac{2(K + dD + F + C_s qpD) - \alpha cI_c(P - D)M_2^2 - \alpha sI_d DM_2^2}{Dh\rho + DcI_c\rho}} \quad \text{. . . (26)}$$

if $2(K + dD + F + C_s qpD) - \alpha cI_c(P - D)M_2^2 - \alpha sI_d DM_2^2 \ge 0$.

Eq.(26) gives the optimal value of T for the case when $T \ge \frac{PM_2}{D}$ so that $T_{25}{}^* \ge \frac{PM_2}{D}$.

$$TC^*_{26}(T) = \sqrt{\frac{2(K + dD + F + C_s qpD) + \alpha cI_c DM_2^2 - \alpha sI_d DM_2^2}{Dh\rho + DcI_c[\alpha + \rho(1 - \alpha)]^2}} \quad \text{. . . (27)}$$

if $2(K + dD + F + C_s qpD) + \alpha cI_c DM_2^2 - \alpha sI_d DM_2^2 \ge 0$.

Similarly Eq.(27) gives the optimal value of T for the case when $M_2 \le T \le \frac{PM_2}{D}$ so that $M_2 \le T_{26}{}^* \le \frac{PM_2}{D}$.

$$TC^*_{27}(T) = \sqrt{\frac{2(K + dD + F + C_s qpD)}{Dh\rho + cI_c(1-\alpha)D\rho + \alpha sDI_d}} \quad \text{. . (28)}$$

Similarly Eq.(28) gives the optimal value of T for the case when $T \le M_2$ so that $T_{27}{}^* \le M_2$.

5. Conclusion

In this paper, we develop a inventory model of the retailer to allow items with imperfect quality under cash discount and trade credit by considering the following situations simultaneously: (1) the retailer's unit selling price and the purchasing price per unit are not necessarily equal, (2) due to reduce default risks, the retailer only provide a full trade credit to his/her good credit customers, (3) to reduce the risk of cash flow shortage and bad debt, the supplier offer the credit terms mixing cash discount and trade credit to the retailer, (4) the replenishment rate is finite, (5) a random defective rate is assumed. Based on our analysis, it is found that the retailer may determine the optimal payment time by trading off the benefits of permissible delay against cash discount in view of several seasons such as the retailer's cash discount period, the retailer's permissible delay period, the customer's permissible delay period, the cash discount rate, and so on. A future study will further incorporate the proposed model into more realistic assumptions, such as the demand that depends on selling price, the permissible delay in payments that depends on order quantity.

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