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Strongly Connectedness in Fuzzy Closure Space

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Abstract. A fuzzy Čech closure space (X, k) is a non-empty fuzzy set X with fuzzy Čech closure operator k: $I^X \rightarrow I^X$ where I^X is a power set of fuzzy subsets of X, which satisfies k (∅) =∅, $\lambda_1 \leq \lambda_2 \Rightarrow k(\lambda_1) \leq k(\lambda_2)$, $k(\lambda_1 \cup \lambda_2) = k(\lambda_1) \cup k(\lambda_2)$ for all $\lambda_1, \lambda_2 \in I^X$. The pair (X, k) is called fuzzy Čech closure space. A fuzzy topological space X is said to be fuzzy strongly connected if it has no non zero fuzzy closed sets λ and δ such that $\lambda+\delta\leq 1$. If X is not fuzzy strongly connected then it is called fuzzy weakly disconnected. Many properties which hold in fuzzy topological space hold in fuzzy Čech closure space as well. A Čech closure space (X, u) is said to be strongly connected if and only if it cannot be expressed as a disjoint union of countably many but more than one closed subsets of X. In strongly connected Čech closure space (X, u), E_i's are nonempty disjoint closed subsets of X then $X \neq E_1 \cup E_2 \cup \dots$.

In this paper we introduce **strongly connectedness in fuzzy Čech closure space.**

Keywords: Fuzzy Čech closure space, connectedness in fuzzy Čech closure space, strongly connectedness in Čech closure space and strongly connectedness in fuzzy Čech closure space.

AMS Mathematics Subject Classifications (2010): 54A40

1. Introduction

In 1965 Zadeh [10] in his classical paper generalized characteristic functions to fuzzy sets. Chang [2] in 1968 introduced the topological structure of fuzzy sets. Pu and Liu [7] defined the concept of fuzzy connectedness using fuzzy closed set. Lowen [5] also defined an extension of a connectedness in a restricted family of fuzzy topologies. Fuzzy Čech closure operator and fuzzy Čech closure space were first studied by A.S. Mashhour and M.H. Ghanim [6].

In 1965 Levine [4] defined the concept of strongly connectedness in topological space. U. V. Fatteh and D.S. Bassan [3] in 1985 introduced the strongly connectedness in fuzzy topological space. In this paper we introduce **strongly connectedness in fuzzy Čech closure space and study some of its properties**.

2. Preliminaries

Definition 2.1.[1] Let X is a non-empty fuzzy set. A function k: $I^X \rightarrow I^X$ is called fuzzy Čech closure operator on X if it satisfies the following conditions

1. k (∅) = ∅.
2. $\lambda \leq k(\lambda)$, for all $\lambda \in I^X$.
3. $k(\lambda_1 \cup \lambda_2) = k(\lambda_1) \cup k(\lambda_2)$ for all $\lambda_1, \lambda_2 \in I^X$.
   The pair (X, k) is called fuzzy Čech closure space.

Definition 2.2. [8] Let X is a nonempty fuzzy set .A function k: $I^X \rightarrow I^X$ is called fuzzy Čech closure operator on X. A fuzzy Čech closure space (X, k) is said to be connected if and only if any F-continuous map f from X to the fuzzy discrete space {0, 1} is constant.

Definition 2.3. [3] A topological space X is strongly connected if and only if it is not a disjoint union of countably many but more than one closed sets of X. If X is strongly connected, and E_i's are nonempty disjoint closed subsets of X, then $X \neq E_1 \cup E_2 \cup$.............

Definition 2.4. [9] A Čech closure space (X, u) is said to be strongly connected if and only if it cannot be expressed as a disjoint union of countably many but more than one closed sets of X. In connected Čech closure space (X, u), let E_1 and E_2 are two nonempty disjoint closed subsets of X then $X \neq E_1 \cup E_2$.

In strongly connected Čech closure space (X, u), E_i's are nonempty disjoint closed subsets of X then $X \neq E_1 \cup E_2 \cup$............

3. Strongly connectedness in fuzzy Čech closure space

Definition 3.1. A fuzzy Čech closure space (X, k) is said to be strongly connected if and only if it cannot be expressed as a disjoint union of countably many but more than one fuzzy closed sets. In strongly connected fuzzy Čech closure space (X, k), let E_1 and E_2 are two nonempty disjoint fuzzy closed subsets of X then $X > E_1 \cup E_2$.

In strongly connected fuzzy Čech closure space (X, u), Ei's are nonempty disjoint closed subsets of X then
$X > E_1 \cup E_2 \cup$............

Example 3.1. Let X= {a, b, c} be a fuzzy set. Define fuzzy Čech closure operator

k: $I^X \rightarrow I^X$ such that

$$k(A) = \begin{cases} 0_x; & A = 0_x \\ 1_{\{b,c\}}; & \text{if } 0 \in A \leq 1_{\{b,c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \in A \leq 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \in A \leq 1_{\{b\}} \\ 1_x; & \text{otherwise} \end{cases}$$

Then (X, k) is called fuzzy Čech closure space.

FOS(X) = {{a}, {b}, {c}, {a, b}, {a, c}, 0_X, 1_X }.

Here E_1={b, c} is only a fuzzy closed subset of X, so we cannot express X as a union of count ably many but more than one fuzzy closed subsets of X. Hence fuzzy Čech closure space (X, k) is strongly connected.

Example 3.2. Let X= {a, b, c} be a fuzzy set. Define fuzzy Čech closure operator

k: $I^X \rightarrow I^X$ such that

$$k(A) = \begin{cases} 0_x\,; & A = 0_x \\ 1_{\{a,b\}}; & \text{if } 0 \in A \le 1_{\{a\}} \\ 1_{\{b,c\}}; & \text{if } 0 \in A \le 1_{\{b\}} \\ 1_{\{c,a\}}; & \text{if } 0 \in A \le 1_{\{c\}} \\ 1_x; & \text{otherwise} \end{cases}$$

Then (X, k) is called fuzzy Čech closure space.
FOS(X) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, 0_x, 1_X}.
Here there are no fuzzy closed subsets of X. Hence fuzzy Čech closure space (X, k) is strongly connected.

Definition 3.2. A fuzzy Čech closure space (X, k) is said to be weakly disconnected if and only if it can be expressed as a disjoint union of count ably many but more than one fuzzy closed sets.

Example 3.3. Let X= {a, b} be a fuzzy set. Define fuzzy Čech closure operator k: $I^X \rightarrow I^X$ such that

$$k(A) = \begin{cases} 0_x\,; & A = 0_x \\ 1_{\{a\}}; & \text{if } 0 \in A \le 1_{\{a\}} \\ 1_{\{b\}}; & \text{if } 0 \in A \le 1_{\{b\}} \\ 1_x; & \text{otherwise} \end{cases}$$

Then (X, k) is called fuzzy Čech closure space.
FOS(X) = {1_x, 0_x}.
FCS(X) = {{a}, {b}, 1_x, 0_x}.
Here E_1= {a}, E_2= {b} are fuzzy closed subsets of X. So we can express X= $E_1 \cup E_2$.
Hence fuzzy Čech closure space (X, k) is not strongly connected. It is also called weakly disconnected fuzzy Čech closure space.

Theorem 3.1. An F-continuous image of a strongly connected fuzzy Čech closure space is strongly connected fuzzy Čech closure space.
Proof: Let (X, k) is a fuzzy Čech closure spaces. Define an F-continuous function f: X→ f(X). Since (X, k) is a strongly connected fuzzy Čech closure space. If f(X) is not strongly connected fuzzy Čech closure space then by definition it can be expressed as a disjoint union of countably many but more than one fuzzy closed subsets of f(X). Since f is F-continuous and the inverse image of fuzzy closed set is still fuzzy closed, so X can be expressed as a disjoint union of count ably many but more than one fuzzy closed

subsets of X. Therefore fuzzy Čech closure space (X, k) is not strongly connected, which is a contradiction. Hence f(X) is a strongly connected fuzzy Čech closure space.

Theorem 3.2. A strongly connected fuzzy Čech closure space is a connected fuzzy Čech closure space. But converse is not true.

Example 3.4. Consider a **strongly connected fuzzy Čech closure space.**
Let X= {a, b, c} be a fuzzy set. Define fuzzy Čech closure operator k: $I^X \to I^X$ such that

$$k(A) = \begin{cases} 0_x ; & A = 0_x \\ 1_{\{a,b\}}; & \text{if } 0 \in A \leq 1_{\{a\}} \\ 1_{\{b,c\}}; & \text{if } 0 \in A \leq 1_{\{b\}} \\ 1_{\{c,a\}}; & \text{if } 0 \in A \leq 1_{\{c\}} \\ 1_x; & \text{otherwise} \end{cases}$$

Then (X, k) is called fuzzy Čech closure space.
FOS(X) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}, 0_x, 1_x}.
Here there are no fuzzy closed subsets of X. Hence fuzzy Čech closure space (X, k) is strongly connected.
Now we define a F-continuous function f: X→ {0, 1} such that
$f^{-1}\{1\}$= {a}={b}= {c}= {a, b}= {b, c}= {a, c}=1_x , $f^{-1}\{0\}=0_x$. Here f is a constant function.

Hence fuzzy Čech closure space (X, k) is connected. This shows that a strongly connected fuzzy Čech closure space (X, k) is a connected fuzzy Čech closure space.
Now consider a **connected fuzzy Čech closure space:**
Let X= {a, b} be a fuzzy set. Define fuzzy Čech closure operator k: $I^X \to I^X$ such that

$$k(A) = \begin{cases} 0_x ; & A = 0_x \\ 1_{\{a\}}; & \text{if } 0 \in A \leq 1_{\{a\}} \\ 1_{\{b\}}; & \text{if } 0 \in A \leq 1_{\{b\}} \\ 1_{\{c,a\}}; & \text{if } 0 \in A \leq 1_{\{c\}} \\ 1_x; & \text{otherwise} \end{cases}$$

Then (X, k) is called fuzzy Čech closure space. We define an F-continuous function f: X→ {0, 1} such that $f^{-1}\{1\} = 1_x = 0_x$. Therefore (X, k) is a **fuzzy connected Čech closure space**.

FOS(X) = {1_x, 0_x}.
FCS(X) = {{a}, {b}, 1_x, 0_x}.
Here E_1= {a}, E_2= {b} are fuzzy closed subsets of X. So we can express X= $E_1 \cup E_2$.
Hence (X, k) is not strongly connected fuzzy Čech closure space. It is also called **weakly disconnected fuzzy Čech closure space.**

Theorem 3.3. A fuzzy Čech closure space (X, k) is said to be strongly connected fuzzy Čech closure space if and only if it has no non zero fuzzy open sets λ and δ such that $\lambda \neq 1$, $\delta \neq 1$ and $\lambda + \delta \geq 1$.

Proof: Necessary: Let fuzzy Čech closure space (X, k) is strongly connected. If X has no non zero fuzzy closed sets f and g such that f+g<1, so that it has not non zero fuzzy open setsλ=f' and δ=g' such that $\lambda \neq 1$, $\delta \neq 1$ and $\lambda + \delta \geq 1$.Hence it has no non zero fuzzy open sets λ and δ such that $\lambda \neq 1$, $\delta \neq 1$ and $\lambda + \delta \geq 1$.

Sufficient: Let X has no non zero fuzzy open sets λ and δ such that $\lambda \neq 1$, $\delta \neq 1$ and $\lambda + \delta \geq 1$. so that it has no non zero fuzzy closed sets λ'=f and δ'=g such that f≠1, g≠1 and f+ g≤ 1. Hence fuzzy Čech closure space (X, k) is strongly connected.

Theorem 3.4. If A is a subset of fuzzy connected Čech closure space (X, k) and A is a strongly connected fuzzy subset of X if and only if for any fuzzy open sets λ and δ in X, $\mu_A \leq \lambda + \delta$ implies either $\mu_A \leq \lambda$ or $\mu_A \leq \delta$.

Proof: Necessary: Let A is strongly connected fuzzy subset of connected fuzzy Čech closure space (X, k). Let fuzzy open sets λ and δ such that $\mu_A \leq \lambda + \delta$, if $\mu_A \nleq \lambda$ and $\mu_A \nleq \delta$, then λ/A≠1, δ/A≠1, and λ/A + δ/A ≥1.So A is not strongly connected fuzzy subset of X. It is a contradiction. Hence there exists fuzzy open sets λ and δ in X, $\mu_A \leq \lambda + \delta$ implies either $\mu_A \leq \lambda$ or $\mu_A \leq \delta$.

Sufficient: Let any fuzzy open sets λ and δ in X, $\mu_A \leq \lambda + \delta$ implies either $\mu_A \leq \lambda$ or $\mu_A \leq \delta$. If A is not a strongly fuzzy connected subset of X. Then there exists fuzzy closed sets f and g in X such that
(1) f/A≠0,
(2) g/A≠0, and
(3) f/A+ g/A≤1.

If we put λ=1-f and δ=1-g, then λ/A=1-f/A, δ/A=1-g/A. So (1), (2), and (3) imply that $\mu_A \leq \lambda + \delta$ but $\mu_A \nleq \lambda$ and $\mu_A \nleq \delta$. It is a contradiction. Hence A is a fuzzy strongly connected subset of X.

Theorem 3.5. If F is a subset of a fuzzy connected Čech closure space X such that ⍰$_F$ is fuzzy closed in X, then X is strongly connected fuzzy Čech closure space implies that F is a fuzzy strongly connected subset of X.

Proof: Suppose F is not fuzzy strongly connected subset of X. Then there exists fuzzy closed sets f and k in X such that (1) f/F≠0, (2) k/F≠0, and (3) f/F + k/F≤1.(3) implies that (f $\wedge \mu_F$) + (k $\wedge \mu_F$) ≤ 1, by (1) and (2) f $\wedge \mu_F \neq 0$, k $\wedge \mu_F \neq 0$. So X is not fuzzy strongly connected, which is a contradiction. Hence X is a fuzzy strongly connected subset of X.

Conclusion: In this paper the idea of fuzzy strongly connectedness was introduced and relationship between the fuzzy strongly connectedness and fuzzy Čech closure space were explained.

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