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Square and Cube Difference Labeling of Cycle Cactus, Special Tree and a New Key Graphs

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Abstract. Let G be a (p, q) graph. *G* is said to be a square difference labeling if there exists a injection f : V(G) → {0,1,2,…,n-1} such that the edge set of G has assigned a weight defined by the absolute square difference of its end-vertices, the resulting weights are distinct. A graph which admits square difference labeling is called square difference graph. Shiama has obtained square difference labeling for some graphs like path, cycle, star ($K_{1,n-1}$), fan, crown ($C_n \odot K_1$). Let G be a (p, q) graph. G is said to be a cube difference labeling if there exists a injection f: V(G) → {0, 1, 2, …, n-1} such that the edge set of G has assigned a weight defined by the absolute cube difference of its end-vertices, the resulting weights are distinct. A graph which admits cube difference labeling is called cube difference graph. We have proved the square and cube difference labeling for graphs like cycle cactus graph $C_k^{(3)}$ and the tree <$K_{1,n}$: 2> and a newly defined key graph in this paper.

Keywords: Square difference labeling, Cube difference labeling, cycle cactus, tree <$K_{1,n}$: 2> and key graph

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

A function f is a square difference labeling of a graph G of size n if f is an injection from V(G) to the set {0, 1, 2, … , n-1} such that , when each edge uv of G has assigned the weight $\left| [f(u)]^2 - [f(v)]^2 \right|$, the resulting weights are distinct.

A function f is a cube difference labeling of a graph G of size n if f is an injection from V(G) to the set {0, 1, 2, … , n-1} such that , when each edge uv of G has assigned the weight $\left| [f(u)]^3 - [f(v)]^3 \right|$, the resulting weights are distinct. The notion of square difference and cube difference labeling were introduced by Shiama [3-6]. Graph labeling can also be applied in areas such as communication network, mobile telecommunications, and medical field. A dynamic survey on graph labeling is regularly updated by Gallian [2]. Khan and Pal have investyigated different types of labelling on cactus graphs [10-16]. The notation and terminology used in this paper are taken from [1].

Definition 1.1. Let G = (V(G), E(G)) be a graph .G is said to be square difference labeling if there exist an injection f : V(G) → {0, 1, …., n-1} such that the induced function f * : E(G) → N given by f *(uv) = $\left| [f(u)]^2 - [f(v)]^2 \right|$ is injective.

Definition 1.2. A graph which satisfies the square difference labeling is called the square difference graph.

Definition 1.3. Let G = (V(G), E(G)) be a graph .G is said to be cube difference labeling if there exist an injection f : V(G) → {0, 1, …., n-1} such that the induced function f * : E(G) → N given by f *(uv) = $\left| [f(u)]^3 - [f(v)]^3 \right|$ is injective.

Definition 1.4. A graph which satisfies the cube difference labeling is called the cube difference graph.

Definition 1.5. A cactus is a connected graph in which any two simple cycles have at most one vertex in common $C_k^{(n)}$ (n copies of cycles C_k).

Definition 1.6. A <$K_{1,n}$: 2> is a tree of diameter 4 obtained from the n bistar $B_{n,n}$ by sub dividing the middle edge with a new vertex.

Definition 1.7. A key graph is a graph obtained from K_2 by appending one vertex of C_5 to one end point and Hoffman tree $P_n \Theta K_1$ to the other end point of K_2

2. Main result

Theorem 2.1. The cycle cactus graph $C_k^{(3)}$ admits cube difference labeling $k \geq 3$.

Proof: Let $C_k^{(3)}$ be a cycle cactus graph, $k \geq 3$. Where k is the number of vertices in cycle C_k of cycle cactus graph $C_k^{(3)}$. Denote the vertices of the cycle C_k in the cycle cactus graph $C_k^{(3)}$ as $u_1, u_2, \ldots, u_n$ in the clockwise direction. Denote the vertices of first copy of cycle C_k as $u_{n+1}, \ldots, u_{n+m}$ in the clockwise direction. Similarly denote the vertices of second copy of cycle C_k as $u_{n+m+1}, \ldots, u_{n+m+p}$ in the clockwise direction. Note that $|V(G)| = 3n-2$ and $|E(G)| = 3n$.

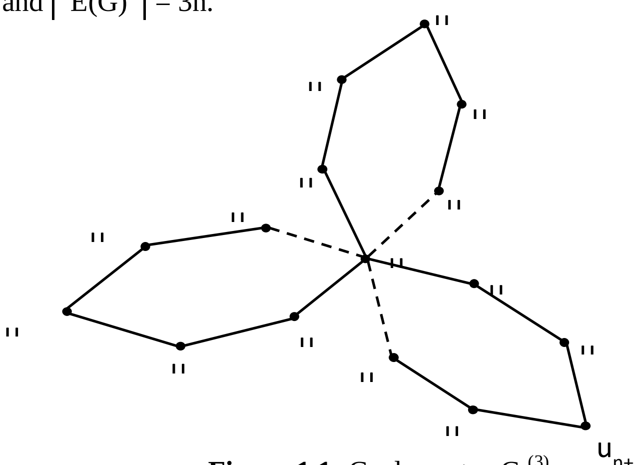

Figure 1.1: Cycle cactus $C_k^{(3)}$

The vertex labeling for the graph $C_k^{(3)}$ is defined as follows.

$f(u) = 0,\ f(u_i) = i,\ 1 \le i \le n$ (1.1)

Now the edge labels are obtained as follows.

f is called cube difference labeling if $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ for every $uv \in E(G)$ are all distinct where $u, v \ge 0$.

Let the edge sets be

$$E_1 = \{uu_i \ / \ 0 \le i \le n\} \text{ and}$$
$$E_2 = \{u_iu_{i+1} \ / \ 1 \le i \le n\} \quad (1.2)$$

Let the edge labels be

$$f^*(uu_i) = i^3, \ 1 \le i \le n \text{ and}$$
$$f^*(u_iu_{i+1}) = 3i^2 + 3i + 1, \ 1 \le i \le n \quad (1.3)$$

Hence the edges are distinct. Hence the cycle cactus graph $C_k^{(3)}$ admits a cube difference labeling.

An illustration of the above theorem is shown in Figure 1.2

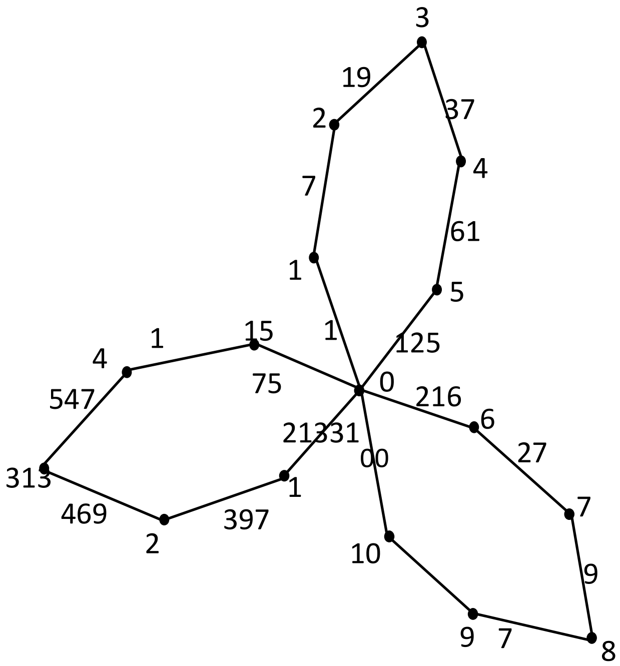

Figure 1.2: Cube difference labeling of cycle cactus $C_6^{(3)}$

Theorem 2.2. The $<K_{1,n}: 2>$ admits cube difference labeling.

Proof: Let $<K_{1,n}: 2>$ be a tree. Denote the vertices which are adjacent to u_1 as $u_2, \ldots, u_n$ in the anticlockwise direction. Denote the vertices which are adjacent to the vertex u_{n+1} as $u_{n+2}, u_{n+3}, \ldots, u_{n+m}$ in the clockwise direction. Let u_1, u_0, u_{n+1} be the path. Note that $|V(G)| = 2n + 3$ and $|E(G)| = 2n + 2$.

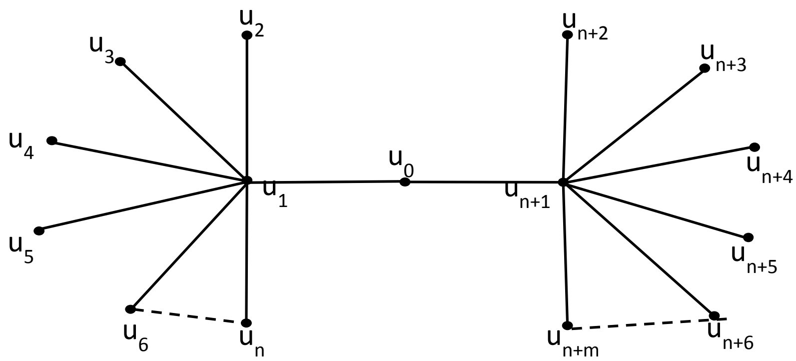

Figure 2.1. Tree K<$_{1,n}$: 2>

The vertex labeling for the tree <$K_{1,n}$: 2> is defined as follows.

$$f(u) = 0,$$
$$f(u_i) = i, \quad 1 \leq i \leq n \qquad (2.1)$$

From the above definition in (2.1) it is obvious that the vertex labels are distinct.
Now the edge labels are obtained as follows.

f is called cube difference labeling if f $*(uv) = \left| [f(u)]^3 - [f(v)]^3 \right|$ for every uv ∈ E(G) are all distinct where u, v ≥ 0.

Let the edge sets be

$$E_1 = \{uu_i \ / \ 1 \leq i \leq n\}$$
$$E_2 = \{u_iu_j \ / \ 1 \leq i \leq n \ , \ 2 \leq j \leq n+m\} \qquad (2.2)$$

Let the edge labels be

$$f\ *(uu_i) = i^3 , \quad 1 \leq i \leq n$$
$$f\ *(u_iu_j) = i^3 - j^3 , \quad 1 \leq i \leq n \ , \ 2 \leq j \leq n+m \qquad (2.3)$$

Hence the edges are distinct. Hence the tree <$K_{1,n}$: 2> admits cube difference labeling.
An illustration of the above theorem as follows.

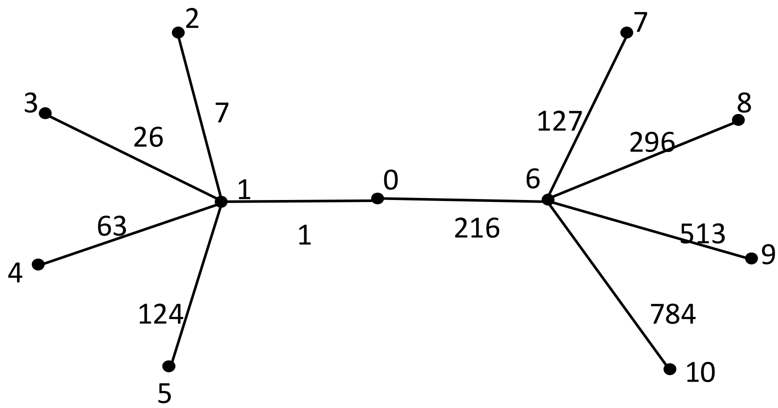

Figure 2.2: Cube difference labeling of <$K_{1,4}$: 2>

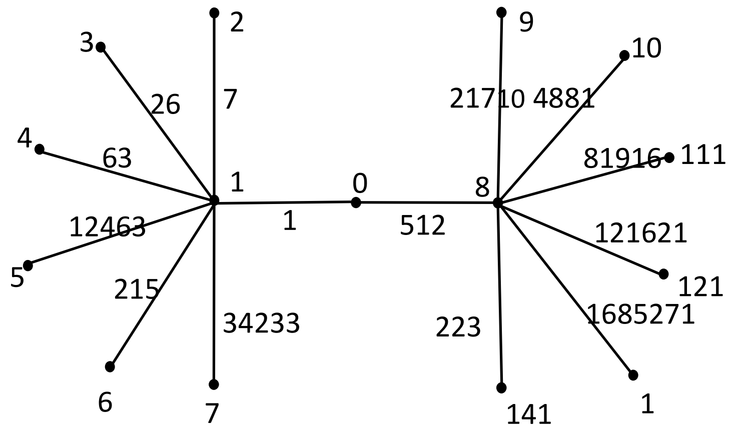

Figure 2.3: Cube difference labeling of <$K_{1,6}$: 2>

Theorem 2.3. The key graph $C_5 \odot P_n$ admits a square difference labeling.
Proof: Let the graph G be a key graph $C_5 \odot P_n$. The vertex set of G is { $w_1, w_2, \ldots\ldots\ldots w_4$, v_0, $v_1, v_2, \ldots\ldots\ldots v_n$, $u_1, u_2, \ldots\ldots\ldots u_n$} where w_i , $1 \le i \le 4$, u_i, v_i $0 \le i \le n$. Clearly G has 2n+5 vertices and 2n+5 edges.

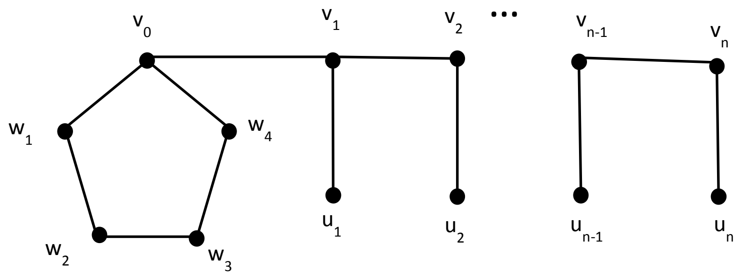

Figure 2.4: Key graph $C_5 \odot P_n$

Let $|V(G)| = 2n+5$ and $|E(G)| = 2n+5$.

The mapping f : V(G) → {0,1,2,…,n-1} is defined by f(u) = 0, $f(u_i) = i$, $1 \le i \le n$ and the induced function f * : E(G) → N is defined by

(i) $f^*(uu_i) = |[f(u)]^2 - [f(u_i)]^2| = i^2, \quad 1 \le i \le n.$

(ii) $f^*(u_iu_{i+1}) = |[f(u_i)]^2 - [f(u_{i+1})]^2| = 2i + 1, \quad 1 \le i \le n.$

(iii) $f^*(u_iu_{i+2}) = |[f(u_i)]^2 - [f(u_{i+2})]^2| = 4i + 4, \quad 1 \le i \le n.$

Here the edge sets are

(i) $E_1 = \{uu_i / 0 \le i \le n\}$

(ii) $E_2 = \{u_iu_{i+1} / 1 \le i \le n\}$

(iii) $E_3 = \{u_iu_{i+2} / 1 \le i \le n\}$

Here the edges are distinct. Hence the key graph $C_5 \odot P_n$ admits a square difference labeling.

Theorem 2.4. The key graph $k_n(C_5 \odot P_n)$admits cube difference labeling.
Proof: Denote the key graph $C_5 \odot P_n$ as G. The vertex set of G is { $w_{1,}$ $w_{2,...}$ w_4, v_0, $v_{1,}v_2$,………v_n, u_1,u_2,………u_n} where w_i , $1 \le i \le 4$, u_i, v_i $0 \le i \le n$. Clearly G has 2n+5 vertices and 2n+5 edges.

Define $f : V(G) \rightarrow \{0,1,2,\ldots,2n+4\}$ as follows,
$f(u_1) = 0$, $f(u_i) = i$, $2 \le i \le n$, $f(v_i) = i$, $1 \le i \le n$, $f(w_i) = i$, $1 \le i \le 5$,
and the induced function $f^* : E (G) \rightarrow N$ is defined by

(i) $f^*(uu_i) = |[f(u)]^3 - [f(u_i)]^3| = i^3$, $1 \le i \le 2n$.

(ii) $f^*(u_iu_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3| = 3i^2+3i+1$, $1 \le i \le 2n$.

(iii) $f^*(u_iu_{i+2}) = |[f(u_i)]^3 - [f(u_{i+2})]^3| = 6i^2+12i+8$, $1 \le i \le 2n$.

Here the edge sets is $E = \{ w_i w_{i+1}, 0 \le i \le 4\} \cup \{ v_{0,} w_5\} \cup \{ v_i v_{i+1,} 1 \le i \le n-1\} \cup$
$\{ u_i v_i , 1 \le i \le n-1\} \cup \{ v_0 v_1\}$

Here the edges are distinct. Hence the key graph $C_5 \odot P_n$ admits a cube difference labeling.

3. Conclusion

In this paper the cycle cactus graphs, tree $<K_{1,n}: 2>$ and key graph are investigated for the square and cube difference labeling. This labeling can be verified for some other graphs.

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