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Odd Graceful Labeling of Cycle with Parallel P_k Chords

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Abstract. A graph labeling is an assignment of integers to the vertices (or) edges or both subject to certain conditions. Graceful labeling was introduced by Rosa [6] while the concept of odd graceful labeling was introduced by Gnanajothi [2]. A Graph G (V, E), |V(G)| = p, |E(G)| = q is said to be odd graceful if there is an injection from V(G) to {0, 1, 2, ... 2q-1} such that when each edge xy is assigned the label |f(x) - f(y)| the resulting edge labels are distinct and are in the set {1, 3, 5, ... 2q-1}. In this paper we prove theodd gracefulness of every even cycle C_n, n ≥ 6 with parallel P_k chords for k = 3, 5 and also prove the odd gracefulness of every odd cycle C_nn≥ 7 with parallel $P_k$chords for k = 2,4 after the removal of 2 edges from the cycle C_n.

Keywords: Graceful labeling, odd graceful labeling, Dragon

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Many mathematicians have introduced different types of graph labeling techniques in the past five decades. This paper deals with one of the types of graph labeling namely odd graceful labeling. For a dynamic survey of various graph labeling problem Gallian [1] is referred.

The graph considered here is a finite undirected graph G = (V, E), where |V(G)| = p, |E(G)| = q without loops (or) multiple edges.β - Valuations defined by Rosa [6] is an injection from the vertices of G to {0, 1, 2, ... q} such that when each edge xy is assigned the label |f(x) - f(y)| the resulting edge labels are distinct and are in the set {1, 2, 3, ... q}. This was later renamed graceful by Golomb[3].

Several classes of graphs have been shown to be odd graceful and Gnanajothi[2] proved that every path P_n with n vertices, (n-1) edges is odd graceful for any n and every cycle C_n with n vertices and n edges is odd graceful iff n is even. She also proved that every graph with an odd cycle is not odd graceful.

A chord of a cycle is an edge joining two otherwise non-adjacent vertices of a cycle. Sethuraman and Elumalai[7] defined a cycle with parallel P_k chords as a graph obtained from a cycle C_n, $n \geq 6$ with consecutive vertices $v_0, v_1, v_2, \ldots v_{n-1}, v_n$ by adding disjoint paths (chords) of order k namely P_k, $k \geq 3$ between each pair of non-adjacent vertices $(v_1\, v_{n-1})$, $(v_2\, v_{n-2})$, ... $(v_\alpha v_\beta)$ where $\alpha = \left[\frac{n}{2}\right] - 1$, $\beta = \left[\frac{n}{2}\right] + 1$ if n is even and $\beta = \left[\frac{n}{2}\right] + 2$ if n is odd. In this paper we prove the odd gracefulness of cycle with parallel $P_k$chords.Gnanajothi[2] proved the following results on cycle related graphs.

Theorem 1.1. The disjoint union of copies of C_4, one point union of copies of C_4 are odd graceful graphs.

Theorem 1.2. P_n is odd graceful for any n and C_n is odd graceful iff n is even.

Lekha [5] proved the following results on cycle related graphs.

Theorem 1.3. Joint sum of two copies of C_n of even order admits odd graceful labeling.

Theorem 1.4. Joining two copies of C_n of even order by a path admits odd graceful labeling.

Theorem 1.5. Two copies of even cycles C_n sharing a common edge is odd graceful

Ibrahim Moussa [4] proved the following theorem

Theorem 1.6. $C_m \cup P_n$ is odd graceful if m = 4, 6, 8, 10 and $C_m \cup P_n$ is odd graceful iff m is even.

2. Gracefulness of cycle with parallel chords

Definition 2.1. A graph is called a cycle with parallel chords if G is obtained from the cycle $C_n : v_0\, v_1 \ldots v_{n-1}\, v_0$ $(n \geq 6)$ by adding chords $(v_1\, v_{n-1})$, $(v_2\, v_{n-2})$, ... $(v_\alpha v_\beta)$ where $\alpha = \left[\frac{n}{2}\right] - 1$, $\beta = \left[\frac{n}{2}\right] + 1$ if n is even and $\beta = \left[\frac{n}{2}\right] + 2$ if n is odd

Definition 2.2. A graph is called a cycle with parallel P_k chords if G is obtained from the cycle $C_n : v_0\, v_1\, v_2 \ldots v_{n-1}\, v_0$ $(n \geq 6)$ by adding disjoint paths P_ks of order k (chords) between the pair of vertices $(v_1\, v_{n-1})$, $(v_2\, v_{n-2})$, ... $(v_\alpha v_\beta)$ of cycle C_n where $\alpha = \left[\frac{n}{2}\right] - 1$, $\beta = \left[\frac{n}{2}\right] + 1$ if n is even, and $\beta = \left[\frac{n}{2}\right] + 2$ if n is odd.

Definition 2.3. A Dragon $D_n(m)$ is a graph obtained by joining the end point of path P_m to cycle C_n.

Sethuraman and Elumalai [7] proved the following theorems.

Theorem 2.4. Every cycle $C_n n \geq 6$ with parallel chords is graceful.

Theorem 2.5. Every cycle $C_n n \geq 6$ with parallel P_k chords is graceful for k =3, k = 2r, $2 \leq r \leq 5$

3. Odd gracefulness of cycle with parallel chords

In this paper we have proved that every even cycle $C_n n \geq 6$ with parallel P_k chords is odd graceful for k = 3, 5 and every odd cycle C_n, $n \geq 7$ after the removal of 2 edges to get a Dragon with parallel P_k chords is odd graceful for k = 2, 4.

Theorem 3.1. Every even cycle $C_n n \geq 6$ with parallel P_3 chords is odd graceful.

Proof: Let G be the even cycle C_n with parallel P_3 chords. Then G has $N = [nk-\alpha(k-2)]/2$ vertices where $\alpha = 3$ if n is odd and $\alpha = 2$ if n is even and $M = [n(k+1) - \rho(k-1)]/2$ edges where $\rho = 3$ if n is odd and $\rho = 2$ if n is even, k is the order of the path P_k. Here $|V(G)| = [3n-2]/2 = N$, $|E(G)| = 2n-2 = M$. $v_0\ v_1\ v_2\ ...\ v_{N-1}$be a Hamiltonian path in G. Define the labeling $f : V(G) \rightarrow \{0, 1, 2, ... 2M-1\}$ as follows

$f(v_i) = i, i = 0, 2$

$f(v_i) = i + 2$, i - even $4 \leq i \leq N - 3$ if $n \equiv 0 \pmod 4$

$4 \leq i \leq N - 2$ if $n \equiv 2 \pmod 4$

$f(v_i) = 2M - (2i-1)$, iodd, $1 \leq i \leq 3$

$f(v_{2i+j}) = f(v_j) - 2i$, $1 \leq i \leq 2$

j = 3, 9, 15, ... N - 8 if $n \equiv 0 \pmod 4$

j = 3, 9, 15, ... N - 5 if $n \equiv 2 \pmod 4$

$f(v_{2+t}) = f(v_t) - 6$, t = 7, 13, 19, ... N - 4 if $n \equiv 0 \pmod 4$

t = 7, 13, 19, ... N - 7 if $n \equiv 2 \pmod 4$

$f(v_{N-1}) = f(v_{N-2}) + 1$ if $n \equiv 0 \pmod 4$

$= f(v_{N-2}) - 3$ if $n \equiv 2 \pmod 4$

clearly f is a 1-1 mapping of V(G) to {0, 1, 2, ... 2M-1}. Also the edge labels are distinct and are in the set {1, 3, 5, ... 2M-1}. From the above vertex labeling it is seen that the above graph is odd graceful. An example of the labeling defined in the above theorem is given in Figure 1.

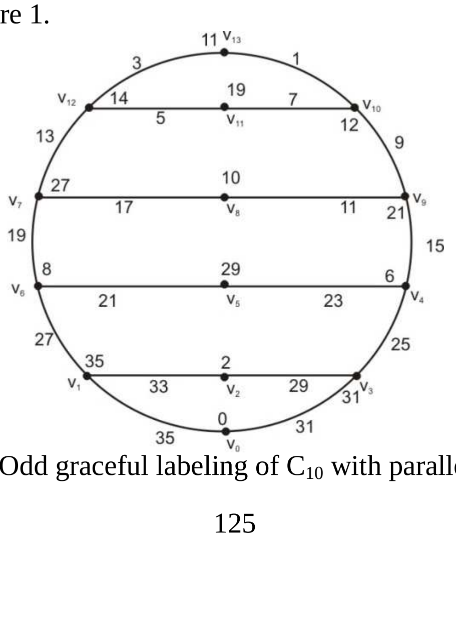

Figure 1: Odd graceful labeling of C_{10} with parallel P_3 chords

Theorem 3.2. Every even cycle C_n, $n \geq 6$ with parallel P_5 chords is odd graceful.
Proof: Let G be the even cycle with parallel P_5 chords.
Then G has $|V(G)| = [5n-6]/2 = N$, $|E(G)| = 3n-4 = M$.
Define the labeling $f: V(G) \rightarrow \{0, 1, 2, ... 2M-1\}$ as follows

$f(v_i) = i$, i even, $0 \leq i \leq N - 3$ if $n \equiv 0 \pmod 4$
$\qquad\qquad\qquad\quad 0 \leq i \leq N - 2$ if $n \equiv 2 \pmod 4$

$f(v_{2i+1}) = 2M - 3^i$, $0 \leq i \leq 2$

$f(v_{2i+j}) = f(v_j) - 2i$, $1 \leq i \leq 3$,
$\qquad j = 5, 15, ... N - 12$ if $n \equiv 0 \pmod 4$
$\qquad j = 5, 15, ... N - 7$ if $n \equiv 2 \pmod 4$

$f(v_{2i+t}) = f(v_t) - 4i$, $1 \leq i \leq 2$,
$\qquad t = 11, 21, ... N - 16$ if $n \equiv 0 \pmod 4$
$\qquad t = 11, 21, ... N - 11$ if $n \equiv 2 \pmod 4$

$f(v_{N-1}) = f(v_{N-3}) + 2$ if $n \equiv 0 \pmod 4$
$\qquad\quad = f(v_{N-2}) - 3$ if $n \equiv 2 \pmod 4$

$$\left.\begin{array}{l} f(v_{N-2}) = f(v_{N-1}) + 1 \\ f(v_{N-4}) = f(v_{N-1}) + 3 \end{array}\right\} \text{if } n \equiv 0 \pmod 4$$

Clearly f is a 1-1 mapping of $V(G) \rightarrow \{0, 1, 2, ... 2M-1\}$ And the edge labels are distinct and are in the set $\{1, 3, 5, ... 2M-1\}$. From the above vertex labeling it is seen that the above graph is odd graceful. An example of the labeling defined in the above theorem is given in Figure 2.

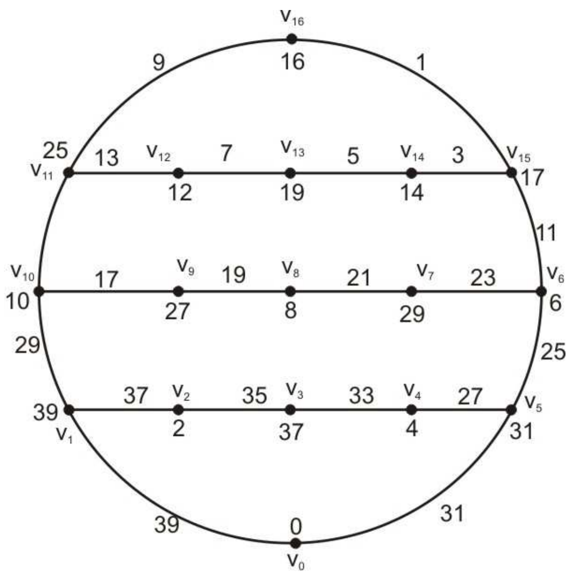

Figure 2: Odd graceful labeling of cycle C_8 with parallel P_5 chords

Theorem 3.3. Every odd cycle C_n, $n \geq 7$ with parallel chords is odd graceful after the removal of 2 edges from the cycle C_n.i.e Dragon $D_{n-3}(4)$ with parallel chords is odd graceful.

Proof: Let G be the cycle C_n, n - odd with parallel chords. After the removal of 2 edges from the cycle C_n it becomes a Dragon $D_{n-3}(4)$ with parallel chords.

Then $|V(G)| = n = N$

$|E(G)| = (3n-7)/2 = M$

Define the labeling f : V(G) → {0, 1, 2, ... 2M-1} as follows

$f(v_i) = i, \quad 0 \leq i \leq 4, \; i \text{ even}$
$f(v_1) = 2M-1$
$f(v_{2i+1}) = f(v_1) - 2i, \quad i = 1, 2, ... (n-3)/2$
$f(v_{2i+4}) = f(v_4) + 4i, \quad i = 1, 2, 3, ... (n-7)/2$
$f(v_{n-1}) = f(v_{n-2}) + 3$

The above mapping f is a 1-1 mapping of V(G) to {0,1,2,... 2M-1}. Also the edge labels are district and are in the set {1,3,5,....2M-1}. From the above vertex labeling it is clear that the above graph admits odd graceful labeling. An example of the labeling defined in the above theorem is given in Figure 3 where the dotted lines show the removal of edges.

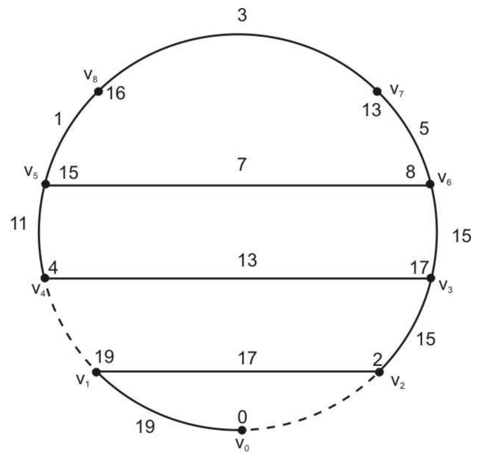

Figure 3: Odd graceful labeling of $D_6(4)$ with parallel chords

Theorem 3.4. Every odd cycle C_n, n ≥ 7 with parallel P_4 chords is odd graceful after the removal of 2 edges from the cycle $C_n$ie Dragon $D_{n-1}(6)$ with parallel P_4 chords is odd graceful.

Proof: Let G be the Dragon $D_{n-1}(6)$, n ≥ 7 and n-odd with parallel P_4 chords. The G has $|V(G)| = 2n-3 = N$ $|E(G)| = (5n-13)/2 = M$.

Define the labeling f: V(G) → {0, 1, 2, ... 2M-1} as follows :

$f(v_0) = 0$

for i - even, $\quad 2 \leq i \leq N - 3$

$f(v_i) = i \quad 2 \leq i \leq 8$
$= i + 2 \quad 10 \leq i \leq 16$
$= i + 4 \quad 18 \leq i \leq 24$

$= i + 6 \qquad 26 \leq i \leq 32$

.... and so on.

$f(v_1) = 2M-1$

$f(v_{2i+1}) = f(v_1) - 2i, \qquad 1 \leq i \leq 5$

$f(v_{2+P}) = f(v_p) - 4, \qquad P = 11, 19, 27, \ldots N-8 \quad \text{if} \quad n \equiv 3 \pmod 4$

$\qquad\qquad 11, 19, 27, \ldots N-12 \quad \text{if} \quad n \equiv 1 \pmod 4$

$f(v_{2i+Q}) = f(v_Q) - 2i, \qquad 1 \leq i \leq 3$

$Q = 13, 21, 29, \ldots N-6 \quad \text{if} \quad n \equiv 3 \pmod 4$

$= 13, 21, 29, \ldots N-10 \quad \text{if} \quad n \equiv 1 \pmod 4$

$f(v_{N-2}) = f(V_{N-3}) + 5 \quad \text{if} \quad n \equiv 1 \pmod 4$

$f(v_{N-1}) = f(v_{N-2}) + 3$

From the above vertex labeling it is seen that the labels of all the vertices are distinct and all the edge values are distinct and are in the set {1, 3, 5, ... 2M-1}. Hence the above graph is odd graceful. An example of the labeling defined in the above theorem is given in Figure 4where the dotted lines show the removal of edges.

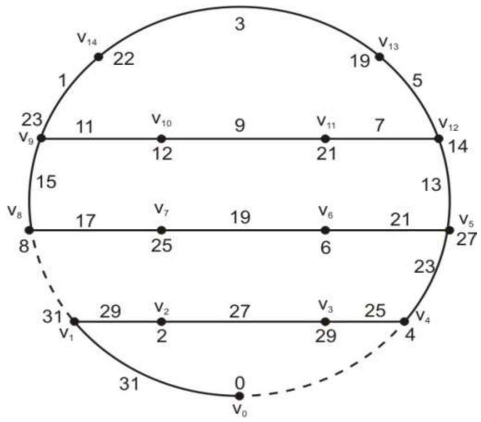

Figure 4: Odd graceful labeling of $D_8(6)$ with parallel P_4 chords

4. Conclusion

Graceful labeling and odd graceful labeling of a graph are entirely two different concepts and a graph may possess one (or) both of these (or) neither. In this paper we have proved that every even cycle C_n, $n \geq 6$ with parallel P_k chords is odd graceful for k = 3, 5. In continuation, our future work is on proving for any odd k. Also we have proved that every odd cycle C_n, $n \geq 7$ with parallel P_k chords is odd graceful for k = 2, 4 after the removal of 2 edges from the cycle C_n. Again in continuation, our future work is on proving for any even k.

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