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# Edge Magic Total Labeling of the Cycle $C_n$ with $P_3$ Chords

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***Abstract.*** In this paper, we have proved edge-magic total labeling of cycles with $P_3$ chords.

***Keywords:*** Graph, graph labeling, edge magic total, cycle with $P_3$ chords

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## 1. Introduction

All graphs are finite, simple and undirected. The graph *G* has vertex-set *V* (*G*) and edge-set E (G). Unless otherwise noted, V (G) = v and E (G) = e.

A *labeling* of a graph is any map that carries some set of graph elements to numbers (usually to the positive or non-negative integers). Magic labeling was introduced by Sedlacek in 1963 [3]. Antimagic labeling of some path and cycle related graphs are shown by S. K. Vaidya and N. B. Vyas in 2012 and 2013 [8], [9]. The seminal paper on edge-magic labeling was published in 1970 by Kotzig and Rosa [4] who called these labeling magic valuations. Magic labeling are one-to-one maps onto the appropriate set of consecutive integers starting from 1, with some kind of "constant-sum" property.

An *edge-magic total labeling* on G is one-to- one map f from V (G) U E(G) onto the integers 1, 2, . . . , |V (G) U E(G)| with the property that, given any edge (x, y),

$$f(x) + f(x, y) + f(y) = k$$

for some constant k. Wallis and others [5] introduced Edge magic total labeling that generalize the idea of a magic square and can be referred for magic labeling. For a summary on various labeling see the dynamic survey of graph labeling by Gallian [7]. A chord of a cycle is an edge joining two adjacent vertices of a cycle.

Let G be a magic graph with magic labeling f. Then the following equation is true for the magic graph G.

$$e(G)k = [n(G)+e(G)][n(G)+e(G)+1]/2 + \Sigma\, (d(u_i)-1)\, f(u_i)$$

That is,

$$ek = [n+e][n+e+1]/2 + \Sigma\, (d(u_i)-1)\, f(u_i) \text{ for } u_i \in V(G) \qquad (1)$$

where n is the no. of vertices, e is the no. of edges and k denotes the magic sum.

A *cycle with $P_3$ chords* is a graph obtained from a cycle $C_n$ ($n \geq 5$, $n \neq 6$) by adding path $P_3$ joining two non-consecutive vertices of the cycle.

In this section we proved the edge-magic total labeling of the cycle $C_n$ with $P_3$ chords in two theorems.

**Theorem 1.** A cycle $C_n$ with $P_3$ chord has an edge-magic total labeling.

**Proof:** Let $C_n$ be a cycle on n vertices. We denote the vertices of $C_n$ as $v_1, v_2, v_3, \ldots, v_n$ in the clockwise direction and denote the edges of $C_n$ with $P_3$ chords as $e_1, e_2, e_3, \ldots, e_{n+2}$ such that $e_i = v_i v_{i+1}$ for $1 \leq i \leq n-1$, $e_n = v_n v_1$

**Case 1:** $C_n$, $n \geq 5$ (n is odd)

A vertex which divide the chord is names as $v_{n+1}$ and the edges of the chords are named as $e_{n+1} = v_{n+1} v_n$ and $e_{n+2} = v_2 v_{n+1}$

The labeling for the vertices of $C_n$ with $P_3$ chords are given as follows.

Define

$$f(v_i) = (2n+i+5)/2, \qquad 1 \leq i \leq n, \quad i \text{ odd}$$
$$f(v_i) = (3n+5+i)/2, \qquad 2 \leq i \leq n-1, \ i \text{ even}$$
$$f(v_{n+1}) = 2n+3$$

Let $s_1$ denotes the sum of vertex labels of degree 2 and $s_2$ denotes the sum of vertex labels of degree 1.

$$s_1 = 12n+24/2$$
$$s_2 = (3n^2+3n-6)/2$$
$$s = s_1 + s_2$$
$$= 3n^2+15n+18/2$$

Now the magic sum k is computed by recalling equation (1).

$$ek = [n+e][n+e+1]/2 + \sum (d(u_i)-1)f(u_i) \quad \text{for } u_i \in V(G)$$
$$(n+2)k = (2n+3)(2n+4)/2 + (3n^2+15n+18)/2$$
$$k = (7n+15)/2$$

Using the vertex labels $f(v_i)$ and the magic sum k, the edge labels $f(e_i)$, $f(e_{n+1})$ and $f(e_{n+2})$ can be obtained from the definition of edge magic labeling as

$$f(e_i) = k - f(v_i) - f(v_{i+1}) \qquad \text{for } 1 \leq i \leq n-1,$$
$$f(e_n) = k - f(v_n) - f(v_1)$$
$$f(e_{n+1}) = k - f(v_{n+1}) - f(v_n) \text{ and}$$
$$f(e_{n+2}) = k - f(v_2) - f(v_{n+1})$$

Hence the cycle $C_n$ with $P_3$ chords has an edge magic total labeling with magic sum $k = (7n+15)/2$ and all the vertex and edge labels where distinct.

**Case 2:** $C_{n=4m}$ for $m \geq 2$ (n is even)

A vertex which divide the chord is names as $v_{n+1}$ and the edges of the chords are named as $e_{n+1} = v_{n+1} v_1$ and $e_{n+2} = v_{n-2} v_{n+1}$.

The labeling for the vertices of $C_n$ with $P_3$ chords are given as follows.

Define

$$f(v_i) = (2n+i+5)/2, \qquad 1 \leq i \leq \frac{n-2}{2}, \ i \text{ odd}$$
$$f(v_i) = (3n+i+5)/2, \qquad \frac{n+2}{2} \leq i \leq n-3, \ i \text{ odd}$$

$f(v_i) = (2n+i+6/2)$ , $n/2 \le i \le n$, i even

$f(v_i)= ( 3n+i+6)/2$ , $2 \le i \le \frac{n-4}{2}$ , i even

$f(v_{n-1}) = 2n+3$ ,

$f(v_{n+1})= 2n+2$

Let $s_1$ denotes the sum of vertex labels of degree 2 and $s_2$ denotes the sum of vertex labels of degree 1.

$s_1=10n+20/2$

$s_2=(3n^2+4n-4)/2$

$s= s_1+ s_2$

$= 3n^2+14n+16/2$

Now the magic sum k is computed by recalling equation (1)

$(n+2)k= (2n+3)(2n+4)/2+(3n^2+14n+16)/2$

$k=(7n+14)/2$

Using the vertex labels $f(v_i)$ and the magic sum k, the edge labels $f(e_i)$, $f(e_n)$, $f(e_{n+1})$ and $f(e_{n+2})$ can be obtained from the definition of edge magic labeling as

$f(e_i )=k- f(v_i )-f(v_{i+1} )$ for $1 \le i \le n-1$,

$f(e_n)=k-f(v_n)-f(v_1)$

$f(e_{n+1}) = k- f(v_{n+1})-f(v_1)$ and

$f(e_{n+2})=k-f(v_{n-2})-f(v_{n+1})$

Hence the cycle $C_n$ with $P_3$ chords has a edge magic total labeling with magic sum $k = (7n+14)/2$ and all the vertex and edge labels where distinct.

**Case 3:** $C_{n=4m+2}$ for $m \ge 2$ (n is even)

A vertex which divide the chord is names as $v_{n+1}$ and the edges of the chords are named as $e_{n+1}=v_{n+1}v_2$ and $e_{n+2}=v_{n-3}v_{n+1}$.

The labeling for the vertices of $C_n$ with $P_3$ chords are given as follows.

Define

$f(v_i) = (2n+i+5)/2$, , $1 \le i \le \frac{n}{2}$, i odd

$f(v_i) = (3n+i+7)/2$, $\frac{n+4}{2} \le i \le n-1$, i odd

$f(v_i) = (3n+i+8/2)$ , $2 \le i \le \frac{n-2}{2}$, i even

$f(v_i)= (2n+i+6)/2$ , $\frac{n+2}{2} \le i \le n-4$, i even

$f(v_i) = (2n+i+8)/2$ , $n-2 \le i \le n$

$f(v_{n+1})= (3n+4)/2$,

Let $s_1$ denotes the sum of vertex labels of degree 2 and $s_2$ denotes the sum of vertex labels of degree 1.

$s_1 =10n+20/2$

$s_2 =(3n^2+4n-4)/2$

$s = s_1+ s_2$

$= 3n^2+14n+16/2$

Now the magic sum k is computed by recalling equation (1).

$(n+2)k= (2n+3)(2n+4)/2+(3n^2+14n+16)/2$

$k= (7n+16)/2$

Using the vertex labels $f(v_i)$ and the magic sum k, the edge labels $f(e_i)$,$f(e_n)$, $f(e_{n+1})$ and $f(e_{n+2})$ can be obtained from the definition of edge magic labeling as

$$f(e_i)=k- f(v_i)-f(v_{i+1}) \qquad \text{for } 1\le i\le n-1,$$
$$f(e_n)=k-f(v_n)-f(v_1) \text{ and}$$
$$f(e_{n+1}) = k- f(v_{n+1})-f(v_2) \text{ and}$$
$$f(e_{n+2})=k-f(v_{n-3})-f(v_{n+1})$$

Hence the cycle $C_n$ with $P_3$ chords has an edge magic labeling with magic sum $k = (7n+16)/2$ and all the vertex and edge labels where distinct.

**Theorem 2:** A cycle $C_n$ with $2P_3$ chord has an edge-magic total labeling .

**Proof:** Let $C_n$ be a cycle on n vertices. We denote the vertices of $C_n$ as $v_1,v_2,v_3,\ldots,v_n$ in the clockwise direction and denote the edges of $C_n$ with $2P_3$ chords as $e_1,e_2,e_3,\ldots,e_{n+4}$ such that $e_i= v_iv_{i+1}$ for $1\le i\le n-1$, $e_n=v_nv_1$

**Case 1:** $C_{n,}$ $n\ge 5$ (n is odd)

The vertices which divide the chords is named as $v_{n+1}$ and $v_{n+2}$ and the edges of the chords are named as $e_{n+1}=v_{n+1}v_2$ , $e_{n+2}=v_{n+2}v_2$ and $e_{n+3}=v_4v_{n+2,}$ $e_{n+4}=v_nv_{n+1.}$

The labeling for the vertices of $C_n$ with $2P_3$ chords are given as follows.

Define

$$f(v_i) = (2n+i+9)/2, \qquad 1\le i\le n, \text{ i odd}$$
$$f(v_i) = (3n+i+9)/2 , \qquad 2\le i\le n-1, \text{ i even}$$
$$f(v_{n+1}) = 2n+5$$
$$f(v_{n+2})= 2n+6$$

Let $s_1$ denotes the sum of vertex labels of degree 3, $s_2$ denotes the sum of vertex labels of degree 2 and $s_3$ denotes the sum of vertex labels of degree 1 .

$$s_1 = (9n+33)/2$$
$$s_2 =12n+44/2$$
$$s_3 =(3n^2+8n-11)/2$$
$$s = s_1+ s_2+ s_3$$
$$= 3n^2+29n+66/2$$

Now the magic sum k is computed by recalling equation (1).

$$(n+4)k= (2n+6)(2n+7)/2+(3n^2+29n+66)/2$$
$$k=(7n+27)/2$$

Using the vertex labels $f(v_i)$ and the magic sum k, the edge labels $f(e_i)$, $f(e_n)$,$f(e_{n+1})$, $f(e_{n+2})$, $f(e_{n+3})$ and $f(e_{n+4})$can be obtained from the definition of edge magic labeling as

$$f(e_i)=k- f(v_i)-f(v_{i+1}) \qquad \text{for } 1\le i\le n-1,$$
$$f(e_n)=k-f(v_n)-f(v_1) \text{ and}$$
$$f(e_{n+1})=k-f(v_{n+1})-f(v_2) ,$$
$$f(e_{n+2})=k-f(v_{n+2})-f(v_2)$$
$$f(e_{n+3})=k-f(v_4)-f(vn_{+2}),$$
$$f(e_{n+4})=k-f(v_n)-f(vn_{+1}).$$

Hence the cycle $C_n$ with $2P_3$ chords has an edge magic total labeling with magic sum $k = (7n+27)/2$ and all the vertex and edge labels where distinct.

**Case 2:** $C_{n=4m}$ for $m\ge 2$ (n is even)

The vertices which divide the chords is names as $v_{n+1}$ and $v_{n+2}$ and the edges of the chords are named as $e_{n+1}=v_{n+1}v_1$ , $e_{n+2}=v_{n+2}v_1$ and $e_{n+3}=v_{n-2}v_{n+2,}$ $e_{n+4}=v_{n-1}v_{n+1}$.

The labeling for the vertices of $C_n$ with $2P_3$ chords are given as follows.

Define

$$f(v_i) = (2n+i+9)/2, \quad , \quad 1 \le i \le \frac{n-2}{2}, \text{ i odd}$$
$$f(v_i) = (3n+i+13)/2, \quad \frac{n+2}{2} \le i \le n-1, \text{ i odd}$$
$$f(v_i) = (2n+i+10/2), \quad n/2 \le i \le n, \text{ i even}$$
$$f(v_i) = (3n+i+14)/2, \quad 2 \le i \le \frac{n-4}{2}, \text{ i even}$$
$$f(v_{n+1}) = (3n+12)/2$$
$$f(v_{n+2}) = (3n+14)/2$$

Let $s_1$ denotes the sum of vertex labels of degree 3, $s_2$ denotes the sum of vertex labels of degree 2 and $s_3$ denotes the sum of vertex labels of degree 1.

$$s_1 = (3n+15)$$
$$s_2 = 14n+40/2$$
$$s_3 = (3n^2+8n-8)/2$$
$$s = s_1 + s_2 + s_3$$
$$= 3n^2+28n+62/2$$

Now the magic sum k is computed by recalling equation (1).

$$(n+4)k = (2n+6)(2n+7)/2+(3n^2+28n+62)/2$$
$$k = (7n+26)/2$$

Using the vertex labels $f(v_i)$ and the magic sum k, the edge labels $f(e_i)$, $f(e_n)$, $f(e_{n+1})$, $f(e_{n+2})$, $f(e_{n+3})$ and $f(e_{n+4})$ can be obtained from the definition of edge magic labeling as

$$f(e_i) = k - f(v_i) - f(v_{i+1}) \quad \text{for } 1 \le i \le n-1,$$
$$f(e_n) = k - f(v_n) - f(v_1)$$
$$f(e_{n+1}) = k - f(v_{n+1}) - f(v_1),$$
$$f(e_{n+2}) = k - f(v_{n+2}) - f(v_1) \text{ and}$$
$$f(e_{n+3}) = k - f(v_{n-2}) - f(v_{n+2}),$$
$$f(e_{n+4}) = k - f(v_{n-1}) - f(v_{n+1}).$$

Hence the cycle $C_n$ with $2P_3$ chords has an edge magic total labeling with magic sum $k = (7n+26)/2$ and all the vertex and edge labels where distinct.

**Case 3:** $C_{n=4m+2}$ for $m \ge 2$ (n is even)
The vertices which divide the chords is names as $v_{n+1}$ and $v_{n+2}$ and the edges of the chords are named as $e_{n+1}=v_{n+1}v_{n-3}$ , $e_{n+2}=v_{n+2}v_{n-3}$ and $e_{n+3}=v_3v_{n+2}$, $e_{n+4}=v_2v_{n+1}$.
The labeling for the vertices of $C_n$ with twin $P_3$ chords are given as follows.
Define

$$f(v_i) = (2n+i+9)/2, \quad , \quad 1 \le i \le \frac{n}{2}, \text{ i odd}$$
$$f(v_i) = (3n+i+13)/2, \quad \frac{n+4}{2} \le i \le n-1, \text{ i odd}$$
$$f(v_i) = (3n+i+14)/2, \quad 2 \le i \le \frac{n-2}{2}, \text{ i even}$$
$$f(v_i) = (2n+i+10/2), \quad \frac{n+2}{2} \le i \le n-4, \text{ i even}$$
$$f(v_i) = (2n+i+14)/2, \quad n-2 \le i \le n, \text{ i even}$$
$$f(v_{n+1}) = (3n+8)/2$$
$$f(v_{n+2}) = (3n+10)/2$$

Let $s_1$ denotes the sum of vertex labels of degree 3 , $s_2$ denotes the sum of vertex labels of degree 2 and $s_3$ denotes the sum of vertex labels of degree 1.

$$s_1 = (6n+15)$$

$$s_2 = 10n+56/2$$
$$s_3 = (3n^2+8n-16)/2$$
$$s = s_1 + s_2 + s_3$$
$$= 3n^2+30n+70/2$$

Now the magic sum k is computed by recalling equation (1).

$$(n+4)k= (2n+6)(2n+7)/2+(3n^2+30n+70)/2$$
$$k= (7n+28)/2$$

Using the vertex labels $f(v_i)$ and the magic sum k, the edge labels $f(e_i)$, $f(e_n)$, $f(e_{n+1})$, $f(e_{n+2})$, $f(e_{n+3})$ and $f(e_{n+4})$can be obtained from the definition of edge magic labeling as

$$f(e_i)=k- f(v_i)-f(v_{i+1}) \qquad \text{for } 1\leq i\leq n-1,$$
$$f(e_n)=k-f(v_n)-f(v_1) \text{ and}$$
$$f(e_{n+1})=k-f(v_{n+1})-f(v_{n-3}) ,$$
$$f(e_{n+2})=k-f(v_{n+2})-f(v_{n-3}) \text{ and}$$
$$f(e_{n+3})=k-f(v_3)-f(v_{n+2}),$$
$$f(e_{n+4})=k-f(v_2)-f(v_{n+1}).$$

Hence the cycle $C_n$ with $2P_3$ chords has an edge magic total labeling with magic sum $k = (7n+28)/2$ and all the vertex and edge labels where distinct.

**Illustration:**

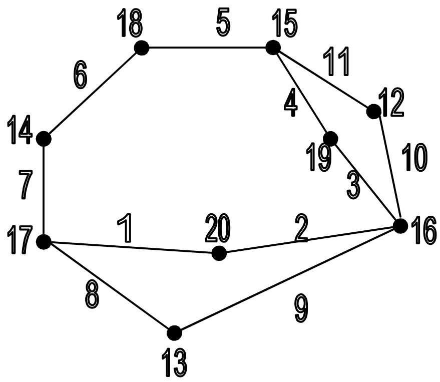


**Figure 1:** Edge magic total labeling of $C_7$ with the magic sum k=38

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