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A New and Simple Method of Solving Fully Fuzzy Linear System

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Abstract. The objective of this paper to find the positive solution of the fully fuzzy linear system of the form $\widetilde{A} \otimes \tilde{x} = \tilde{b}$, where $\widetilde{A}$ is a n x n fuzzy matrix consisting of positive fuzzy numbers, the unknown vector $\tilde{x}$ is a vector consisting of n positive fuzzy numbers and the constant $\tilde{b}$ is a vector consisting of n positive fuzzy numbers, using Cramer's rule along with Dodgson's Condensation.

Keywords: Fully fuzzy linear system, Triangular numbers, Cramer's rule, Dodgson's Condensation

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1. Introduction

System of simultaneous linear equations plays a vital role in mathematics, Operations Research, Statistics, Physics, Engineering and Social Sciences etc. In many applications at least some of the system's parameters and measurements are represented by fuzzy numbers rather than crisp numbers. Therefore it is imperative to develop mathematical models and numerical procedures to solve such a fuzzy linear system. The general model of a fuzzy linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy vector. In the fully fuzzy linear system all the parameters are considered to be fuzzy numbers. Dehghan et al. [1] have solved n x n fully fuzzy linear system with triangular fuzzy numbers using Cramer's rule, Gauss Elimination, LU decomposition method and linear programming approach. Nasseri et al. [2] proposed LU decomposition method for fully fuzzy linear system with triangular fuzzy numbers. The Concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh [4, 5]. In this paper we considered square fully fuzzy linear system of the form $\tilde{A} \otimes \tilde{x} = \tilde{b}$ with triangular fuzzy numbers which are non-negative. The main aim of this paper is to solve large scale of fully fuzzy linear systems using Dodgson's condensation method. Instead of using calculators, computers and other electronic modes we can solve the large scale linear systems manually by this method.

The Paper organized as follows. In section 2, we give some basic concepts of fuzzy sets theory and then define a fully fuzzy linear systems of equations. In section 3, we give the procedures for Dodgson's Condensation method and also Cramer's rule by employing Dodgson's Condensation method. The method of solving the n x n fully fuzzy linear system based on Cramer's rule along with Dodgson's Condensation method is discussed in section 4. Some numerical examples for the fully fuzzy linear system are designed in section 5. Section 6 contains Conclusion.

2. Preliminaries

Definition 2.1. A fuzzy subset $\widetilde{A}$ of R is defined by its membership function $\mu_{\widetilde{A}}$: R → [0,1], which assigns a real number $\mu_{\widetilde{A}}$ in the interval [0, 1], to each element x∈R, Where the value of $\mu_{\widetilde{A}}$ at x shows the grade of membership of x in $\widetilde{A}$.

Definition 2.2. A fuzzy number $\tilde{A}$ = (m, α, β) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}} = \begin{cases} 0, & x < m \\ \dfrac{x-m}{\alpha-m}, & m \leq x \leq \alpha \\ \dfrac{\beta-x}{\beta-\alpha}, & \alpha < x \leq \beta \\ 0, & x > \beta \end{cases}$$

Definition 2.3. A triangular fuzzy number $\tilde{A}$ is called positive (negative), denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$), if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0$, ∀ x ≤ 0 (∀ x ≥ 0).

Definition 2.4. Two triangular fuzzy numbers $\widetilde{A}$ = (m, α, β) and $\widetilde{B}$ = (n,γ, δ) are said to be equal if and only if m = n, α = γ and β = δ.

Definition 2.5. A Triangular fuzzy number $\tilde{A}$ = (m, α, β) is said to be zero triangular fuzzy number if and only if m = 0, α = 0, β = 0.

Definition 2.6. Let $\tilde{A} = (\tilde{a}_{ij})$ and $\tilde{B} = (\tilde{b}_{ij})$ be two m x n and n x p fuzzy matrices.
We define $\tilde{A} \otimes \tilde{B} = \tilde{C} = (\tilde{c}_{ij})$ Which is the m x p matrix where

$$\tilde{C}_{ij} = \sum_{k=1,2....n}^{\oplus} \tilde{a}_{ik} \otimes \tilde{b}_{kj}$$

2.7. Arithmetic operations on triangular fuzzy numbers

Let $\tilde{A}_1 = (m, \alpha, \beta)$ and $\tilde{A}_2 = (n, \gamma, \delta)$ be two triangular fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m, \alpha, \beta) \oplus (n, \gamma, \delta)$ = (m+n,α +γ, β+δ)

(ii) $\tilde{A}_1 \geq 0$ and $\tilde{A}_2 \geq$ o then

$$\tilde{A}_1 \otimes \tilde{A}_2 = (m, \alpha, \beta) \otimes (n, \gamma, \delta)$$
$$= (mn, m\gamma + n\alpha, m\delta + n\beta)$$

(iii) $\tilde{A}_1 \ominus \tilde{A}_2 = (m, \alpha, \beta) \ominus (n, \gamma, \delta) = (m - \delta, \alpha - \gamma, \beta - n)$

Definition 2.8. A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ will be positive and denoted by $\tilde{A} > 0$, if each element of $\tilde{A}$ be positive. We may represent n x n fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})_{nxn}$, such that $\tilde{a}_{ij} = (a_{ij}, m_{ij}, n_{ij})$,with the new notation $\tilde{A} = (A, M, N)$, where $A = (a_{ij})$, $M = (m_{ij})$, $N = (n_{ij})$ are three n x n crisp matrices.

Definition 2.9. Consider the n x n fuzzy linear system of equations

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \dots \dots \dots \dots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$$

$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \dots \dots \dots \dots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$$
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$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \dots \dots \dots \dots \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$$

The matrix form of the above equations is $\tilde{A} \otimes \tilde{x} = \tilde{b}$,
where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, $1 \le i, j \le n$ is a n x n fuzzy matrix and $\tilde{x}_j \tilde{b}_j \in F(R)$. This system is called a fully fuzzy linear system.

3. Dodgson's Condensation method [3]
3.1. Dodgson's condensation of determinants consists of the following steps

1. Employ the elementary row and column operations to rearrange. If necessary, the given nth order determinant such that there are no zeros in its interior. The interior of a determinant is the minor formed after the first and last rows and columns of the determinant have been deleted.

2. Evaluate every 2nd order determinant formed by four adjacent elements. The values of the determinants form the (n-1)st order determinant.

3. Condense the (n-1)st order determinant in the same manner, dividing each entry by the corresponding element in the interior of the nth order determinant.

4. Repeat the condensation process until a single number is obtained. This number is the value of the nth order determinant.

3.2. The following are the steps for solving linear system Ax = b based on Well known Cramer's rule along with Dodgson's condensation.

1. Form the n x 2n matrix

$R_1 = [A \quad b \quad A']$

where A' is the array of numbers left when the last column of A is deleted.

2. Use Dodgson's condensation to condense R_1 to R_2, R_2 to R_3, and so on until the following row matrix is obtained.

$R_n = [A \quad A_1 \quad A_2 \quad A_3 \quad A_4 \ldots \ldots A_n]$

The values $A, A_1, A_2, A_3, A_4, \ldots \ldots A_n$ are the elements of A_n.

If n is even, then the solution is $x_1 = -\frac{A_1}{A}$, $x_2 = -\frac{A_2}{A}, \ldots \ldots, x_n = -\frac{A_n}{A}$

If n is odd, then the solution is $x_1 = \frac{A_1}{A}$, $x_2 = -\frac{A_2}{A}, \ldots \ldots, x_n = \frac{A_n}{A}$

4. Solving the fully fuzzy linear system based on Cramer's rule along with Dodgson's condensation

The n x n fully fuzzy linear system converted into three different linear systems and each linear system solved by this method.
Consider the fully fuzzy linear systems $\tilde{A} \otimes \tilde{x} = \tilde{b}$,

where $\widetilde{A} = (A, M, N)$, $\tilde{x} = (x, y, z)$, $\tilde{b} = (b, g, h) \geq 0$ and A is a full rank crisp matrix.

(A, M, N) $\otimes$ (x, y, z) = (b, g, h)

(Ax, Ay + Mx, Az + Nx) = (b, g, h) [by 2.7. (ii)]

Using 2.4 we have

Ax = b

Ay + Mx = g

Az +Nx = h

The above three linear systems are solved by using the above section 3.

5. Numerical Examples

5.1 Solve the following 2x2 fully fuzzy linear system

$(2,3,4) \otimes (x_1, y_1, z_1) \oplus (6,7,8) \otimes (x_2, y_2, z_2) = (26,65,78)$

$(1,3,5) \otimes (x_1, y_1, z_1) \oplus (2,4,6) \otimes (x_2, y_2, z_2) = (9,31,44)$

Solution:

$A = \begin{bmatrix} 2 & 6 \\ 1 & 2 \end{bmatrix}, M = \begin{bmatrix} 3 & 7 \\ 3 & 4 \end{bmatrix}, N = \begin{bmatrix} 4 & 8 \\ 5 & 6 \end{bmatrix}, b = \begin{bmatrix} 26 \\ 9 \end{bmatrix}, g = \begin{bmatrix} 65 \\ 31 \end{bmatrix}, h = \begin{bmatrix} 78 \\ 44 \end{bmatrix}$

Now, Ax = b

$A_1 = \begin{bmatrix} 2 & 6 & 26 & 2 \\ 1 & 2 & 9 & 1 \end{bmatrix}$; $A_2 = [-2 \quad 2 \quad 8]$; $x_1 = -\frac{2}{-2} = 1$; $x_2 = -\frac{8}{-2} = 4$

Now, Ay + Mx = g ⇒ Ay = g – Mx

g – Mx = $\begin{bmatrix} 34 \\ 12 \end{bmatrix}$

$A_1 = \begin{bmatrix} 2 & 6 & 34 & 2 \\ 1 & 2 & 12 & 1 \end{bmatrix}$; $A_2 = [-2 \quad 4 \quad 10]$; $x_1 = -\frac{4}{-2} = 2$; $x_2 = -\frac{10}{-2} = 5$

Now, Az + Nx = h ⇒ Ay = h – Nx

h – Nx = $\begin{bmatrix} 42 \\ 15 \end{bmatrix}$

$A_1 = \begin{bmatrix} 2 & 6 & 42 & 2 \\ 1 & 2 & 15 & 1 \end{bmatrix}$; $A_2 = [-2 \quad 6 \quad 12]$; $x_1 = -\frac{6}{-2} = 3$; $x_2 = -\frac{12}{-2} = 6$

$\tilde{x}_1 = (1,2,3)$, $\tilde{x}_2 = (4,5,6)$

5.2.3. Solve the following 3x3 fully fuzzy linear system

$(7,8,9) \otimes (x_1, y_1, z_1) \oplus (4,5,6) \otimes (x_2, y_2, z_2)) \oplus (2,7,9) \otimes (x_3, y_3, z_3) = (13,46,63)$

$(6,7,8) \otimes (x_1, y_1, z_1) \oplus (3,6,9) \otimes (x_2, y_2, z_2) \oplus (1,4,7) \otimes (x_3, y_3, z_3) = (10,37,54)$

$(2,5,8) \otimes (x_1, y_1, z_1) \oplus (2,4,6) \otimes (x_2, y_2, z_2) \oplus (1,5,9) \otimes (x_3, y_3, z_3) = (5,24,38)$

Solution:

$A = \begin{bmatrix} 7 & 4 & 2 \\ 6 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix}, M = \begin{bmatrix} 8 & 5 & 7 \\ 7 & 6 & 4 \\ 5 & 4 & 5 \end{bmatrix}, N = \begin{bmatrix} 9 & 6 & 9 \\ 8 & 9 & 7 \\ 8 & 6 & 9 \end{bmatrix}, b = \begin{bmatrix} 13 \\ 10 \\ 5 \end{bmatrix}, g = \begin{bmatrix} 46 \\ 37 \\ 24 \end{bmatrix}, h = \begin{bmatrix} 63 \\ 54 \\ 38 \end{bmatrix}$

Now, Ax = b

$A_1 = \begin{bmatrix} 7 & 4 & 2 & 13 & 7 & 4 \\ 6 & 3 & 1 & 10 & 6 & 3 \\ 2 & 2 & 1 & 5 & 2 & 2 \end{bmatrix}$

$A_2 = \begin{bmatrix} -3 & -2 & 7 & 8 & -3 \\ 6 & 1 & -5 & -10 & 6 \end{bmatrix}$

$A_3 = \begin{bmatrix} \frac{9}{3} & \frac{3}{1} & \frac{-30}{10} & \frac{18}{6} \end{bmatrix} = \begin{bmatrix} 3 & 3 & -3 & 3 \end{bmatrix}$

$x_1 = \frac{3}{3} = 1;\ x_2 = -\frac{(-3)}{3} = 1; x_3 = \frac{3}{3} = 1$

Now, Ay + Mx = g ⇒ Ay = g – Mx

g – Mx = $\begin{bmatrix} 26 \\ 20 \\ 10 \end{bmatrix}$

$A_1 = \begin{bmatrix} 7 & 4 & 2 & 26 & 7 & 4 \\ 6 & 3 & 1 & 20 & 6 & 3 \\ 2 & 2 & 1 & 10 & 2 & 2 \end{bmatrix}$

$A_2 = \begin{bmatrix} -3 & -2 & 14 & 16 & -3 \\ 6 & 1 & -10 & -20 & 6 \end{bmatrix}$

$A_3 = \begin{bmatrix} \frac{9}{3} & \frac{6}{1} & \frac{-120}{20} & \frac{36}{6} \end{bmatrix} = \begin{bmatrix} 3 & 6 & -6 & 6 \end{bmatrix}$

$y_1 = \frac{6}{3} = 2;\ y_2 = -\frac{(-6)}{3} = 2; y_3 = \frac{6}{3} = 2$

Now, Az + Nx = h ⇒ Ay = h – Nx

h – Nx = $\begin{bmatrix} 39 \\ 30 \\ 15 \end{bmatrix}$

$A_1 = \begin{bmatrix} 7 & 4 & 2 & 39 & 7 & 4 \\ 6 & 3 & 1 & 30 & 6 & 3 \\ 2 & 2 & 1 & 15 & 2 & 2 \end{bmatrix}$

$A_2 = \begin{bmatrix} -3 & -2 & 21 & 24 & -3 \\ 6 & 1 & -15 & -30 & 6 \end{bmatrix}$

$A_3 = \begin{bmatrix} \frac{9}{3} & \frac{9}{1} & \frac{-270}{30} & \frac{54}{6} \end{bmatrix} = \begin{bmatrix} 3 & 9 & -9 & 9 \end{bmatrix}$

$z_1 = \frac{9}{3} = 3;\ z_2 = -\frac{(-9)}{3} = 3; z_3 = \frac{9}{3} = 3$

$\tilde{x}_1 = (1,2,3),\ \tilde{x}_2 = (1,2,3), \tilde{x}_3 = (1,2,3)$

6. Conclusion

In this paper, the n x n fully fuzzy linear system converted into three different n x n crisp linear system then we solved each system by Cramer's rule along with Dodgson's Condensation method. In this method it is necessary that the given nth order determinant such that there are no zeros in its interior.

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