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The Local Metric Dimension of Cyclic Split Graph

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Abstract. Let $G(V,E)$be a graph with vertex set V and edge set E. Let $W \subseteq V$ then W is said to be a local metric basis of G, if for any two adjacent vertices $u, v \in V/W$, there exists a $w \in W$ such that $d(u,w) \neq d(v,w)$. The minimum cardinality of local metric basis is called the local metric dimension (lmd) of graph G. In this paper we investigate the local metric basis and local metric dimension of Cyclic Split Graph $C_n K_r^k$.

Keywords: Cyclic Split Graph, local metric basis, local metric dimension

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The Metric dimension arises in many diverse areas, including telecommunication [3] connected joints in graph and chemistry [8] the robot navigation [18] and geographical routing protocols [19] etc.

The metric dimension problem is an application to network discovery and verification in the area of the telecommunication. Due to its fast dynamics, distributed growth process, it is hard to obtain an accurate map of the global network. A common way to obtain such map is to make certain local measurement at a small subsets of the nodes, and then to combine them in order to discover the actual graph. Each of these measurements is potentially quite costly. It is thus a natural objective to minimize the number of measurements, which still edge is cover the whole graph. That is to determine the metric dimension of the graph.

A basic problem in chemistry is to provide mathematical representation for a set of chemical compounds in a way that gives distinct representation to distinct compound. The structure of a chemical compound can be represented as a labeled graph where the vertex and edge labels specify the atoms and bond types respectively. Thus a graph-theoretical interpretation of this problem is to provide representations for the vertices of a graph in such a way that distinct representations. This observation can be used in drug discovery when it is to be determined whether the features of a compound are responsible for its pharmacological activity.

Robotics is the field of knowledge and techniques that permits the construction of robots. Computation of a collision – free path for a movable object among the obstacles is an important problem in the field of robotics. Navigation of a robot can be studied in a graph structured framework. The navigation agent can be assumed to be a point robot, which moves from node to node of graph space. For this robot there is neither the concept of direction nor that of visibility. But it is assumed that it can sense the distances to a set of landmarks. Evidently if the robot knows its distances to a sufficiently large set of landmarks, its position in the graph is uniquely determined. Consider a robot which is navigating in a space modeled by a graph and which wants to know its current location. It can send a signal to find out how far it is from each among a set of fixed landmarks. This suggested a problem of computing the minimum number of landmarks and their portions such that the robot can uniquely determine its location is equivalent to the metric dimension problem..

A more common problem in graph theory concerns distinguishing every two neighbours in a graph G by means of some coloring rather than distinguishing all the vertices of a connected graph G has been studied with the aid of distances in G. This suggests the topic of using distances to distinguish the two vertices in each pair of neighbours only and thus Okamoto et.al [21] introduced the local metric dimension problem.

Let $G(V, E)$be a graph with vertex set V and edge set E. Let $W \subseteq V$ then W is said to be a local metric basis of G, is for any two adjacent vertices $u, v \in V/W$, there exists a $w \in W$ such that $d(u, w) \neq d(v, w)$. The minimum cardinality of local metric basis is called the local metric dimension (lmd) of graph G.

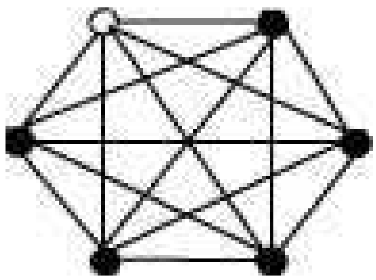

Figure 1: Complete graph K_6 with local metric dimension 5

We define a wheel W_n as a graph obtained from the cycle C_n by adding a new vertex and edges joining it to all the vertices of the cycle, where $n \geq 3$. A Cyclic Split Graph $C_n K_r^k$ [4] has a complete graph K_r with vertices $v_1, v_2, \dots v_r$ and kr wheels $W_{i,j}$ attached at the each vertex v_i in K_r, such that $W_{i,j} = v_i + C_{n,i,j}$, $1 \leq i \leq r$ and $1 \leq j \leq k$. The deletion of the spokes of the wheel results in the disjoint union of the complete graph K_r and kr independent cycles $C_{n,i,j}$, $1 \leq i \leq r$ and $1 \leq j \leq k$, where each cycle has n vertices which are labeled as $a_{n,i,j}$.

2. Literature survey

The metric dimension problem was introduced by Slater [24] where the metric generators were called locating set and by Harary & Metler [10], where metric generators received the name of resolving sets. After these papers, the metric dimension of several interesting classes of graphs have been investigated: Grassmann graph [1], Johnson & Kneser graph [2], Cartesian product of graphs [7], Cayley digraphs [9], Convex polytops [11], generalized Peterson graphs [12,13], Cayley graphs [14], Silicate networks [20], Circulant graphs [22], Cyclic Split graphs [4], etc…It also has been shown that some infinite graphs have infinite metric dimension.

Okamoto et.al.[21] characterize all nontrivial connected graphs of order n having local metric dimension $1, n-2$ or $n-1$ and establish sharp bounds for the local metric dimension of a graph. The local metric dimension of G is the cardinality of its local metric basis. Each resolving set of G is vertex distinguishing and each local resolving set is neighbour-distinguishing. Thus every resolving set is also a local resolving set of G, so if G is a nontrivial connected graph of order n, then $1 \leq \mathrm{lmd}(\mathrm{G}) \leq \beta(G) \leq n-1$.

Salman et al. [23] gave integer linear programming formulations of the local metric dimension of two convex polytops S_n & U_n. Rodr´ıguez-Vel´azquezr et al. [17] shows that the computation of the local metric dimension of a graph with cutvertices is reduced to the computation of the local metric dimension of the primary subgraphs, they applied this computation to specific constructions including bouquets of graphs, rooted product graphs, corona product graphs, block graphs and chain graphs. In [16] they studied the problem of finding exact values for local metric dimension of corona product graph.

3. Main result

Theorem 3.1. The local metric dimension of a Cyclic Split graph $C_nK_r^k$ is kr for $n = 4$ and $(m+1)kr$ for $n \geq 5$ & $n = 4m + i,$ where $i \in \{1, \ldots, 4\}$ and $m = 1,2, \ldots$

Proof:

Case (i) $n = 4$:

Let $a_{1,i,j}$ be a member of a local metric basis, where $a_{1,i,j} \in W_{i,j}$, whose vertex of attachment with K_r be v_i . Then $d(a_{1,i,j}, v_i) = d(a_{1,i,j}, a_{2,i,j}) = d(a_{1,i,j}, a_{n,i,j}) = 1$ where $\{v_i, a_{2,i,j}\}$ and $\{v_i, a_{n,i,j}\}$ are pairs of vertices adjacent to each other. But $d(a_{1,i',j}, v_i) = 2$ and $d(a_{1,i',j}, a_{n,i,j}) = 3$ for any $i' \neq i$ & $i, i' \in \{1, \ldots, r\}$. Hence $\{a_{1,i,j}\}$ forms a local metric basis for $C_nK_r^k$, where $i = 1, \ldots, r$ & $j = 1, \ldots, k$. Thus local metric dimension of $C_nK_r^k$ is kr.

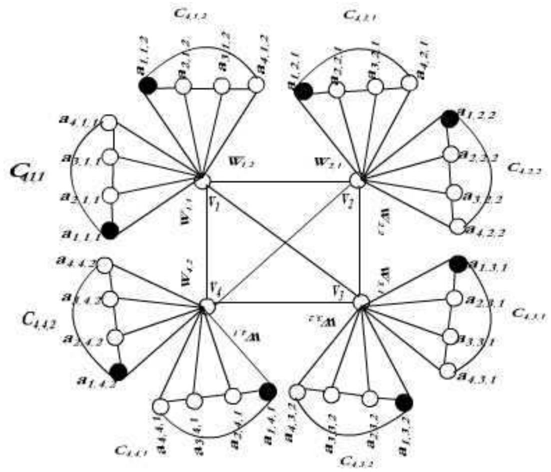

Figure 2: Cyclic Split Graph $\boldsymbol{C_4K_4^2}$ with local metric dimension 8

Case (ii) $n \geq 5$ & $n = 4m + i,$ where $i \in \{1, \ldots, 4\}$ and $m = 1,2, \ldots$

Let $a_{1,i,j}$ be a member of a local metric basis for $C_nK_r^k$. Then $d(a_{1,i,j}, v_i) = d(a_{1,i,j}, a_{2,i,j}) = d(a_{1,i,j}, a_{n,i,j}) = 1$. By the above argument another vertex $a_{1,i',j}, i' \neq i$ & $i, i' \in \{1, \dots, r\}$ resolves the pair of adjacent vertices.

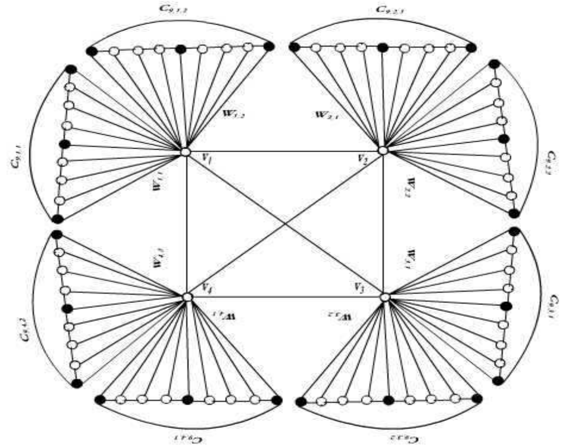

Figure 3: Cyclic Split Graph $\boldsymbol{C_9K_4^2}$ with local metric dimension 24

Consider $d(a_{1,i,j}, a_{k,i,j}) = d(a_{1,i,j}, a_{k+1,i,j}) = 2$, where $k \in \{3, \dots, n-2\}$ where $((n-2)-3)$ adjacent pairs of vertices are equidistant from $a_{1,i,j}$. Hence we select vertices $\{a_{5,i,j}, a_{9,i,j}, \dots, a_{4m+1,i,j}\}$ where any $d(a_{k,i,j}, a_{k-1,i,j}) = 1$ and $d(a_{k,i,j}, a_{k-2,i,j}) = 2$, where $a_{k,i,j}$ is a member of the local metric basis. Similarly $d(a_{k,i,j}, a_{k+1,i,j}) = 1$ whereas $d(a_{k,i,j}, a_{k+2,i,j}) = 2$. Thus $\{a_{1,i,j}, a_{5,i,j}, \dots, a_{4m+1,i,j}\}$ resolves the adjacent pair of vertices in $C_nK_r^k$. Thus we have $\left(\frac{4m+1-1}{4}\right) + 1$ vertices in each $W_{i,j}$ as a member of a local metric basis. Hence local metric dimension of $C_nK_r^k$ is $(m+1)kr$.

Corollary 3.2. The local metric dimension of $C_nK_r^k$, $n = 3$ is $2kr$.

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