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Some Cordial Labeling of Duplicate Graph of Ladder Graph

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Abstract. In this paper, we present that the duplicate graph of the ladder graph $L_m, m \geq 2$ is cordial, total cordial and prime cordial.

Keywords: Graph labeling, ladder graph, duplicate graph, cordial labeling, total cordial labeling, prime cordial labeling.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The concept of graph labeling was introduced by Rosa in 1967[2]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In the intervening years various labeling of graphs have been investigated in over 1100 papers. Cahit has introduced cordial labeling [3]. Labeled graphs serve as useful models in a broad range of applications such as circuit design, communication network addressing, X-ray crystallography, radar, astronomy, data base management and coding theory. M. Sundaram, R. Ponraj and S. Somasundaram have introduced prime cordial labeling of graph [5]. They have also proved the bistars and crowns are prime cordial. K.Thirusangu, P.P.Ulaganathan and B. Selvam, have proved that the duplicate graph of a path graph P_m is Cordial [4]. In this paper, we present that the duplicate graph of the ladder graph $L_m, m \geq 2$ is cordial, total cordial and prime cordial.

2. Preliminaries

In this section, we give the basic notions relevant to this paper.

Definition 1. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling).

Definition 2. The ladder graph L_m is a planar undirected graph with 2m vertices and 3m – 2 edges. It is obtained as the cartesian product of two path graphs, one of which has only one edge: $L_{m,1} = P_m \, X \, P_1$, where m is the number of rungs in the ladder.

Definition 3. Let $G(V, E)$ be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V' \; and \; V \cap V' = \emptyset$ and $f : V \to V'$ is bijective $(for \; v \in V, we \; write \; f(v) = v')$ and the edge set E_1 of DG is defined as : The edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.

Example 1. The following figures show the Ladder graph L_3 and its duplicate graph.

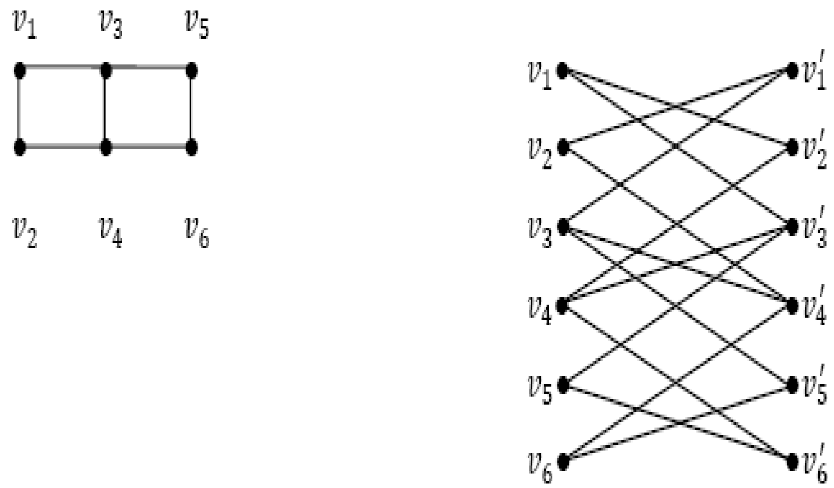

Figure 1:

Clearly the duplicate graph of the ladder graph L_m contains 4m vertices and 6m – 4 edges.

Definition 4. A function $f : V \to \{0 , 1\}$ is said to be a cordial labeling if each edge uv has the label $|f(u) - f(v)|$ such that (i) the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most "one" and (ii) the number of edges labeled '0' and the number of edges labeled '1' differ by at most "one". A graph which admits cordial labeling is called cordial.

Definition 5. A function $f : V \to \{0, 1\}$ such that each edge uv which is assigned the label $|f(u) - f(v)|$ is said to be a total cordial labeling if the number of vertices and edges labeled with '0' differ by at most one from the number of vertices and edges labeled with '1'. A graph which admits total cordial labeling is called total cordial.

Definition 6. A prime cordial labeling of a graph G with vertex set V is a bijection $f : V \to \{1, 2, 3 \ldots\ldots |V|\}$ such that each edge uv is assigned the label as

$$f(uv) = \begin{cases} 1 \; if \; g.c.d \, \{ f(u), f(v)\} = 1 \\ 0 \; if \; g.c.d \, \{ f(u), f(v)\} > 1 \end{cases}$$

and the number of edges labeled with ‘0’ and the number of edges labeled with ‘1’ differ at most by one.

Algorithm 1.1:

Procedure (Construction of duplicate graph of the ladder graph$L_{m,}$)

Case (i) For k = 2, 4, 6, …(2m – 2)

$$v_k v'_{k+2} \leftarrow e_{\frac{3k}{2}} \; ; \; v_{k+2} v'_k \leftarrow e'_{\frac{3k}{2}}$$

Case (ii) For k = 1, 2, 3…m

$$v_{2k-1} v'_{2k} \leftarrow e_{3k-2} \; ; \; v_{2k} v'_{2k-1} \leftarrow e'_{3k-2}$$

Case (iii) For k = 1, 2, 3, …(m – 1)

$$v_{2k-1} v'_{2k+1} \leftarrow e_{3k-1}; \; v_{2k+1} v'_{2k-1} \leftarrow e'_{3k-1}$$

Illustration: The ladder graphs L_5 , L_6 and their duplicate graphs are shown below:

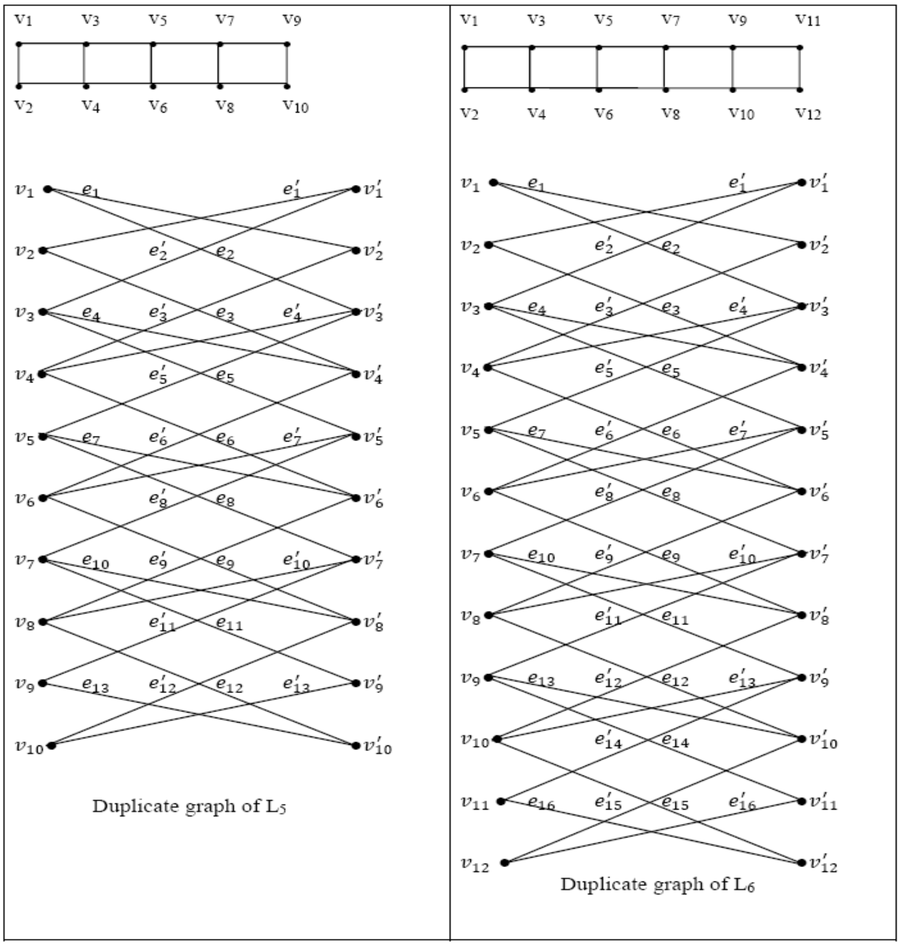

Figure 2:

Algorithm 1.2: (Cordial labeling)

$V \leftarrow \{v_1, v_2, v_3 \dots v_{2m}, v_1', v_2', v_3' \dots v_{2m}'\}$

$E \leftarrow \{e_1, e_2, e_3 \dots e_{3m-2}, e_1', e_2', e_3' \dots e_{3m-2}'\}$

Assignment of label to vertices

Case (i): When 'm' is odd and $m \geq 3$.

$fix\ v_1' \leftarrow 0$

For $1 \leq k \leq \left(\frac{m-1}{2}\right)$

$v_{4k}, v_{4k-1} \leftarrow 0;\ v_{4k-2}, v_{4k-3} \leftarrow 1;$

$v_{4k}',\ v_{4k-2}' \leftarrow 0;\ v_{4k-1}' \leftarrow 1;$

For $1 < k \leq \left(\frac{m-1}{2}\right)$

$v_{4k-3}' \leftarrow 1;$

For $k = \left(\frac{m+1}{2}\right)$

$v_{4k-2}, v_{4k-3}, v_{4k-3}' \leftarrow 1;\ v_{4k-2}' \leftarrow 0;$

Case (ii): When m is even and $m \geq 4$

For $1 \leq k \leq \left(\frac{m}{2}\right)$

$v_{4k}, v_{4k}', v_{4k-1}, v_{4k-2}' \leftarrow 0;\ v_{4k-2}, v_{4k-3}, v_{4k-1}',\ v_{4k-3}' \leftarrow 1;$

Theorem 1.1. The duplicate graph of the ladder graph L_m , m ≥ 2 is cordial.

Proof: First let us prove the theorem for $m \geq 3$

In order to label the vertices, define a function $f : V \rightarrow \{0, 1\}$ as given in algorithm 1.2. Then the vertices are labeled as follows:

Case (i): When m is odd

The vertex v_1' is fixed with label 0.

For $1 \leq k \leq \left(\frac{m-1}{2}\right)$, the $(2\mathrm{m} - 2)$ vertices $v_{4k}, v_{4k-1}, v_{4k}'\ and\ v_{4k-2}'$ receive label '0' and the $\frac{3}{2}(m-1)$ vertices $v_{4k-2}, v_{4k-3}\ and\ v_{4k-1}'$ receive label '1'. For $1 < k \leq \left(\frac{m-1}{2}\right)$, the $\left(\frac{m-3}{2}\right)$ vertices v_{4k-3}' receive label 1.

For $k = \left(\frac{m+1}{2}\right)$, the "three" vertices $v_{4k-2}, v_{4k-3}\ and\ v_{4k-3}'$ receive label '1', and the vertex v_{4k-2}' receive label '0'.

Thus the entire 4m vertices are labeled in such a way that the number of vertices labeled with '0' and '1' are same as 2m.

The induced function $f^* : E \rightarrow \{0, 1\}$ is defined such that

$f^*(v_i v_j) = |f(v_i) - f(v_j)|,\ v_i, v_j \in V$.

Using the induced function, we see that the (3m–2) edges namely, $e_1, e_3, e_5, e_7 \dots e_{3m-6}, e_{3m-4}, e_{3m-2}, e_4', e_6', e_8' \dots e_{3m-5}', e_{3m-3}'\ and\ e_1'$ receive label '1'. The 3m–2) edges namely, $e_2, e_4, e_6, e_8 \dots e_{3m-5}, e_{3m-3}, e_3', e_5', e_7' \dots e_{3m-4}', e_{3m-2}'\ and\ e_2'$ receive label '0'.

Thus the entire 6m – 4 edges are labeled in such a way that the number of edges labeled '1' and the number of edges labeled '0' are same as 3m – 2.

Case (ii): When m is even

The "2m" vertices $\left\{v_{4k}, v_{4k-1}, v'_{4k}, v'_{4k-2} \,/ 1 \leq k \leq \frac{m}{2}\right\}$ receive label '0' and the "2m" vertices $\left\{v_{4k-2}, v_{4k-3}, v'_{4k-1}\,,\; v'_{4k-3} \;/ 1 \leq k \leq \frac{m}{2}\right\}$ receive label '1'.

Thus the entire 4m vertices are labeled in such a way that the number of vertices receive label "0" and "1" are same as 2m.

Using the induced function as in case (i), the (3m – 2) edges $e_1, e_3, e_5 \ldots e_{3m-5}, e_{3m-3},\; e'_2, e'_4, e'_6 \ldots\; e'_{3m-4}, e'_{3m-2}$ receive label '1' and the (3m – 2) edges $e_2, e_4, e_6 \ldots e_{3m-4}, e_{3m-2,}\; e'_1, e'_3, e'_5 \ldots\; e'_{3m-5}, e'_{3m-3}$ receive label '0'.

Thus the entire 6m – 4 edges are labeled in such a way that the number of edges labeled '0' and the number of edges labeled '1' are same as 3m – 2.

Hence the duplicate graph of the ladder graphL_m, m ≥ 3 is cordial.

Case (iii): Now let us prove the theorem for m = 2.

In this case the four vertices v_1, v_2, v'_2 *and* v'_4 receive label '1' and the four vertices v_3, v_4, v'_1 *and* v'_3 receive label '0'.

Using the induced function as in case (i), the four edges e_2, e'_1, e'_2 *and* e'_3 receive label '1' and the four edges e_2, e_3, e_4 *and* e'_4 receive label '0'.

Thus the entire 6m – 4 edges are labeled in such a way that the number of edges labeled '0' and the number of edges labeled '1' are same as 3m – 2.

Hence the duplicate graph of the ladder graphL_m, m = 2 is cordial.

Illustration:

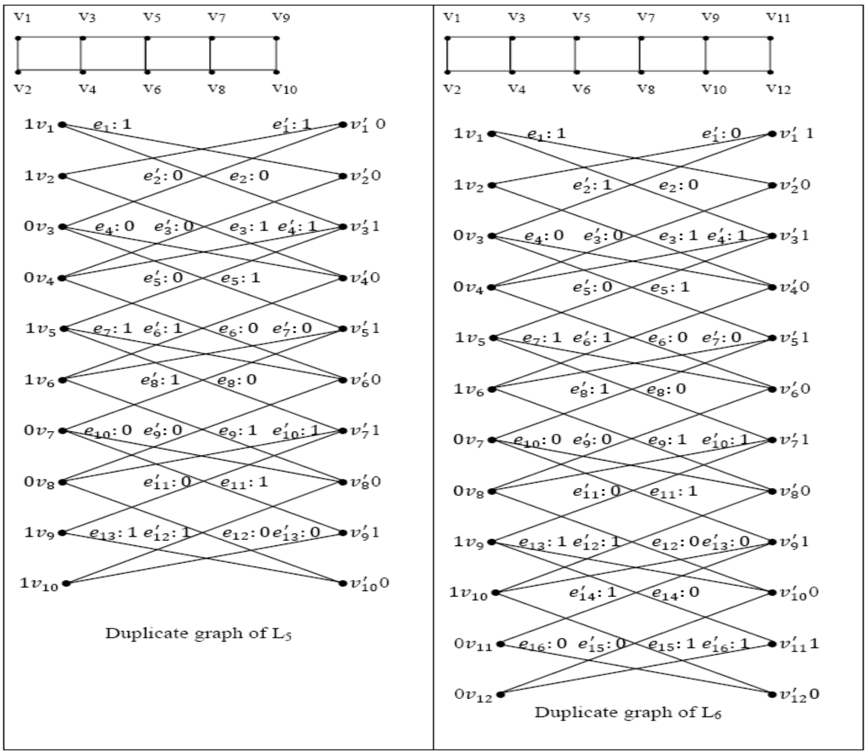

Figure 3:

Theorem 1.2. The duplicate graph of ladder graph $L_{m\,,}\ m\ \geq 2$ is total cordial.

Proof: In theorem 1.1, we have shown that the number of vertices labeled '1' and '0' are same as 2m, the number of edges labeled '1' and '0' are same as 3m – 2 . Thus we obtained that the number of vertices and edges labeled with '1' and the number of vertices and edges labeled with '0' are same as 5m – 2.

Hence the duplicate graph of the ladder graph $L_{m\,,}\ m\ \geq 2$ is total cordial.

Algorithm 1.3:

$V\ \leftarrow\ \{v_1, v_2, v_3 \ldots v_{2m}, v_1', v_2', v_3' \ldots v_{2m}'\}$

$E\ \leftarrow\ \{e_1, e_2, e_3 \ldots e_{3m-2}, e_1', e_2', e_3' \ldots e_{3m-2}'\}$

Assignment of labels to vertices

Case (i): When m is even

$For\ 1\ \leq k \leq \frac{m}{2}$

$v_{4k-3} \leftarrow 8k-7\ ;\ v_{4k-3}'\ \leftarrow 8k-6\ \ ; v_{4k-2}\ \leftarrow 8k-4\ ;\ v_{4k-2}'\ \leftarrow 8k-5;$

$v_{4k-1}\ \leftarrow 8k-2\ ;\ v_{4k-1}'\ \leftarrow 8k-3\ \ ; v_{4k}\ \leftarrow 8k-1\ \ ;\ v_{4k}'\ \leftarrow 8k;$

Case (ii): When m is odd

$For\ 1\ \leq k \leq \frac{m-1}{2}$

$v_{4k-3} \leftarrow 8k-7\ ;\ v_{4k-3}'\ \leftarrow 8k-6\ \ ; v_{4k-2}\ \leftarrow 8k-4\ ;\ v_{4k-2}'\ \leftarrow 8k-5;$

$v_{4k-1}\ \leftarrow 8k-2\ ;\ v_{4k-1}'\ \leftarrow 8k-3\ \ ; v_{4k}\ \leftarrow 8k-1\ \ ;\ v_{4k}'\ \leftarrow 8k;$

$For\ k =\ \frac{m+1}{2}$

$v_{4k-2}\ \leftarrow 8k-4\ ;\ v_{4k-2}'\ \leftarrow 8k-5\ ;\ v_{4k-3}\ \leftarrow\ 8k-7\ ;\ v_{4k-3}'\ \leftarrow 8k-6.$

Theorem 1.3. The duplicate graph of a ladder graph $L_m,\ m \geq 2,$ is prime cordial.

Proof: We prove the theorem for the duplicate graph of a ladder graph $L_m \geq 2$.

To label the vertices, define a function $f : V\ \rightarrow\ \{1,2,3 \ldots\ldots |V|\}$, as given in algorithm1.3, and then the vertices are labeled as follows:

Case (i): When m is even.

$for\ 1\ \leq k \leq \frac{m}{2}$, the $\left(\frac{m}{2}\right)$ vertices v_{4k-3} receive label 8k – 7, the $\left(\frac{m}{2}\right)$ vertices v_{4k-2} receive label 8k – 4, the $\left(\frac{m}{2}\right)$ vertices v_{4k-1} receive label 8k – 2, the $\left(\frac{m}{2}\right)$ vertices v_{4k} receive label 8k – 1 , the $\left(\frac{m}{2}\right)$ vertices v_{4k-3}' receive label 8k – 6 , the $\left(\frac{m}{2}\right)$ vertices v_{4k-2}' receive label 8k – 5, the $\left(\frac{m}{2}\right)$ vertices v_{4k-1}' receive label 8k – 3 and the $\left(\frac{m}{2}\right)$ vertices v_{4k}' receive label 8k.

Thus the entire 4m vertices are labeled.

The induced function $f^* : E\ \rightarrow\ \{0\ ,1\}$ is defined such that

$$f^*(v_i v_j) = \begin{cases} 1\ \ if\ \ g.c.d\ [f(v_i), f(v_j)] = 1 \\ 0\ \ if\ \ g.c.d\ [f(v_i), f(v_j)]\ > 1 \end{cases}$$

Using the induced function, the (3m–2) edges namely e_1,e_2, e_6, e_7, e_8,e_{12}, e_{13}, e_{14},…. ..e_{3m-6}, e_{3m-5}, e_{3m-4},e_3', e_4', e_5', e_9', e_{10}', e_{11}',…. e_{3m-3}', e_{3m-2}' receive label '1' and the 3m – 2 edges namely e_3, e_4, e_5, e_9, e_{10}, e_{11}, ….e_{3m-3}, e_{3m-2}, e_1', e_2', e_6', e_7', e_8', ….e_{3m-4}', e_{3m-5}', e_{3m-6}' receive label '0'.

Hence the entire 6m – 4 edges are labeled in such a way that the number of edges labeled '0' and the number of edges labeled '1' are same as 3m – 2.

Case (ii): When m is odd

For $1 \le k \le \frac{m-1}{2}$, the $\left(\frac{m-1}{2}\right)$ vertices v_{4k-3} receive label 8k – 7, the $\left(\frac{m-1}{2}\right)$ vertices v_{4k-2} receive label 8k – 4, the $\left(\frac{m-1}{2}\right)$ vertices v_{4k-1} receive label 8k – 2, the $\left(\frac{m-1}{2}\right)$ vertices v_{4k} receive label 8k – 1 , the $\left(\frac{m-1}{2}\right)$ vertices v'_{4k-3} receive label 8k – 6 , the $\left(\frac{m-1}{2}\right)$ vertices v'_{4k-2} receive label 8k – 5, the $\left(\frac{m-1}{2}\right)$ vertices v'_{4k-1} receive label 8k – 3 and the $\left(\frac{m-1}{2}\right)$ vertices v'_{4k} receive label 8k.

For $k = \frac{m+1}{2}$, the four vertices v_{4k-2}, v_{4k-3}, v'_{4k-2} and v'_{4k-3} receive labels 8k – 4, 8k – 7, 8k – 5 and 8k – 6 respectively.

Thus the entire 4m vertices are labeled.

Using the induced function, as in case (i), the (3m–2) edges namely e_1 , e_2 , e_6 , e_7 , e_8 , e_{12} , e_{13} , e_{14} ,…... e_{3m-3}, e_{3m-2} , e'_3 , e'_4 , e'_5 , e'_9 , e'_{10}, e'_{11},…. e'_{3m-6}, e'_{3m-5} e'_{3m-4} receive label '1' and the (3m – 2) edges namely e_3, e_4, e_5, e_9, e_{10}, e_{11}, ….e_{3m-6}, e_{3m-5}, e_{3m-4}, e'_1, e'_2, e'_6, e'_7, e'_8, ….e'_{3m-3}, e'_{3m-2} receive label '0'.

Hence the entire 6m – 4 edges are labeled in such a way that the number of edges labeled '0' and the number of edges labeled '1' are same as 3m – 2.

Hence the duplicate graph of a ladder graph $L_m, m \ge 2$, is prime cordial.

Illustration:

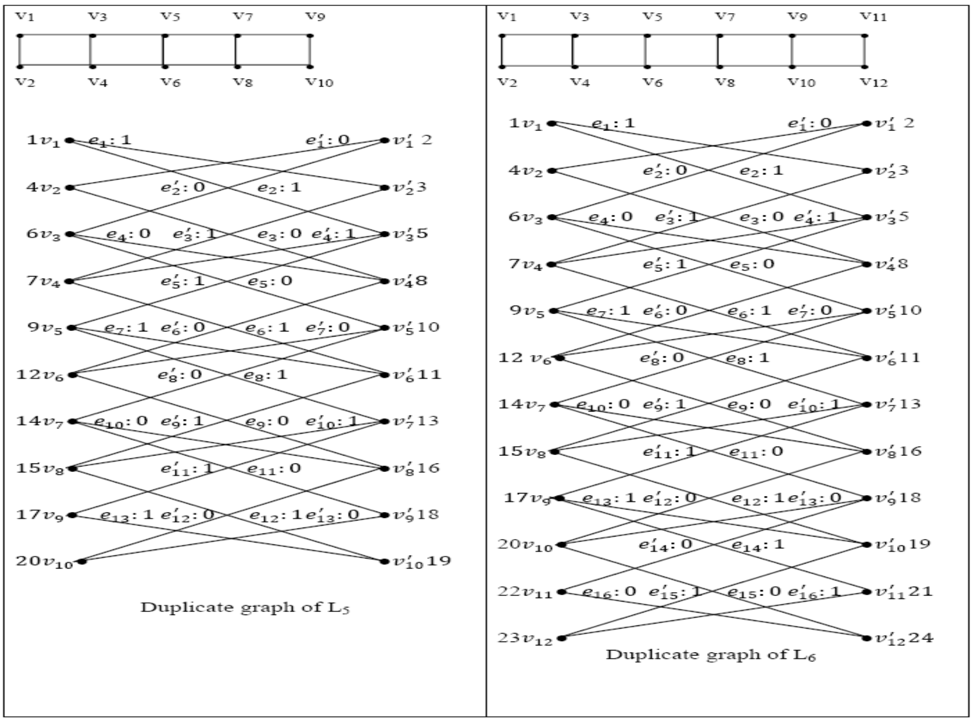

Figure 4:

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