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Reverse order Triangular, Trapezoidal and Pentagonal Fuzzy Numbers

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Abstract. In this paper we define reverse order triangular fuzzy number and reverse order pentagonal fuzzy number with the help of triangular fuzzy number. We also define reverse order trapezoidal fuzzy number with the help of trapezoidal fuzzy number. we include basic arithmetic operations like addition, subtraction for reverse order triangular fuzzy number and reverse order trapezoidal fuzzy number. We have also verified with examples for the above mentioned operations.

Keywords: Fuzzy numbers,triangular fuzzy number, reverse order triangular fuzzy number, reverse order trapezoidal fuzzy number, reverse order pentagonal fuzzy number, fuzzy operations

AMS Mathematics Subject Classification (2010): 94Dxx

1. Introduction

Zadeh introduced fuzzy set theory in 1965. Different types of fuzzy sets [3] are defined in order to clear the vagueness of the existing problems. A fuzzy number [9], is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. The concept of fuzzy numbers is the generalization of the concept of real numbers. D.Dubois and H.Prade has defined fuzzy number as a fuzzy subset of the real line [5,12]. So far fuzzy numbers like triangular fuzzy numbers [4], trapezoidal fuzzy numbers [2,10], Pentagonal fuzzy numbers [11], Hexagonal, Octagonal , pyramid fuzzy numbers and Diamond fuzzy number [14] have been introduced with its membership functions. These numbers have got many applications [7] like non-linear equations, risk analysis and reliability. Many operations [8,1,6] were done using fuzzy numbers.

In the case, of triangular fuzzy number, the membership function characterizing a fuzzy number has one maximum on the supporting interval but in the case of trapezoidal fuzzy number, the requirement of one maximum has been relaxed allowing a flat segment at level $\alpha = 1$.

In order to gain more diversity and to describe large number of linguistic variables [13], we allow minimum instead of maximum or a flat segment at level $\alpha = 0$. We give three functions, with respect to the axis α defined on bounded supporting interval . If a bounded interval is more suitable we can restrict x between its end points. A shift of the function to the left or right can be easily made. Also if the interest is in a large variable which is either positive or negative. Then correspondingly only values, $x \geq 0$ or $x \leq 0$ are to be considered.When the supporting interval considered is unbounded we get Bell shaped fuzzy numbers.

In this paper, we introduce reverse order triangular fuzzy number (rotfn)with its membership functions, reverse order trapezoidal fuzzy number and reverse order pentagonal fuzzy number with its membership functions. Section one is the introduction and section two presents the basic definitions of fuzzy numbers; section three presents reverse order triangular fuzzy number and its arithmetic operations; section four presents reverse order trapezoidal fuzzy number and its arithmetic operations, and finally section five presents reverse order pentagonal fuzzy number.

2. Preliminaries and notations

Definition 2.1.(Fuzzy set): A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set $\underset{\sim}{A}$ in a universe of discourse X is defined as the following set of pairs:

$$\underset{\sim}{A} = \{(x, \mu_{\underset{\sim}{A}}(x); x \in X\}$$

Here $\mu_{\underset{\sim}{A}} : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\underset{\sim}{A}$ and $\mu_{\underset{\sim}{A}}(x)$ is called the membership value of $x \in X$ in the fuzzy set $\underset{\sim}{A}$. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2. (Fuzzy Number): A Fuzzy number $\underset{\sim}{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_{\underset{\sim}{A}}(x_o)$ is piecewise continous

(ii) There exist atleast one $x_o \in R$ with $\mu_{\underset{\sim}{A}}(x_0) = 1$

(iii) $\underset{\sim}{A}$ must be normal and convex

Definition 2.3. (Triangular Fuzzy Number): Triangular Fuzzy Number is defined as $\underset{\sim}{A}$= {a,b,c}, where all a, b, c are real numbers and its membership function is given below.

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & for\, a \leq x \leq b \\ \dfrac{(c-x)}{(c-b)} & for\, b \leq x \leq c \\ 0 & otherwise \end{cases}$$

Definition 2.4. (Trapezoidal Fuzzy Number): A fuzzy set $\underset{\sim}{A}$ = (a, b, c, d) is said to trapezoidal fuzzy number if its membership function is given by where $a \leq b \leq c \leq d$

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & for\, x < a \\ \dfrac{(x-a)}{(b-a)} & for\, a \le x \le b \\ 1 & for\, b \le x \le c \\ \dfrac{(d-x)}{(d-c)} & for\, c \le x \le d \\ 0 & for\, x > d \end{cases}$$

Definition: 2.5. (Pentagonal Fuzzy Number): A Pentagonal Fuzzy Number (PFN) of a fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_P$ = {a, b, c, d, e}, and its membership function is given by,

$$\mu_{\underset{\sim}{A}_P}(x) = \begin{cases} 0 & for\, x < a, \\ \dfrac{(x-a)}{(b-a)} & for\, a \le x \le b \\ \dfrac{(x-b)}{(c-b)} & for\, b \le x \le c \\ 1 & x = c \\ \dfrac{(d-x)}{(d-c)} & for\, c \le x \le d \\ \dfrac{(e-x)}{(e-d)} & for\, d \le x \le e \\ 0 & for\, x > e \end{cases}$$

Definition: 2.6 (Diamond Fuzzy Number) A Diamond Fuzzy Number (DFN) of a fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_D$ = {a, b, c, (α_b β_b)}, and its membership function is given by,

$$\mu_{\underset{\sim}{A}_D}(x) = \begin{cases} 0 & for\, x < a, \\ \dfrac{(x-a)}{(b-a)} & for\, a \le x \le b \\ \dfrac{(c-x)}{(c-b)} & for\, b \le x \le c \\ & \\ \alpha_b \text{ - base} & \\ \dfrac{(a-x)}{(a-b)} & for\, a \le x \le b \\ \dfrac{(x-c)}{(b-c)} & for\, b \le x \le c \\ 1 & x = \beta_b \\ 0 & otherwise \end{cases}$$

where α_b is the base of the triangle a β_bc and also for the reverse order reflection of the above triangle, namely abc.

3. Reverse order Triangular fuzzy number

Definition 3.1. (Reverse order triangular fuzzy number). A fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_t$ = {-a,0,b} is said to be reverse order triangular fuzzy number(RoTFN) if its membership function is given by

$$\mu_{\underset{\sim}{A}_t}(x)=\begin{cases} 1 & for\ x \le -a \\ \frac{-1}{a}x & for -a \le x \le 0 \\ \frac{1}{b}x & for\ 0 \le x \le b \\ 1 & for\ x \ge b \end{cases}$$

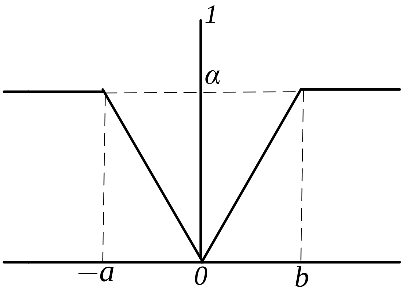

Figure 1: Graphical representation of reverse order triangular fuzzy number (RoTFN).

Example: An reverse order triangular fuzzy number, $\mu_{\underset{\sim}{A}_t}$ = (-3,0,2) and its membership is given as,

$$\mu_{\underset{\sim}{A}_t}(x)=\begin{cases} 1 & for\ x \le -3 \\ \frac{-1}{3}x & for -3 \le x \le 0 \\ \frac{1}{2}x & for\ 0 \le x \le 2 \\ 1 & for\ x \ge 2 \end{cases}$$

α- Cut of reverse order triangular fuzzy number

$$-\frac{1}{3}x=\alpha \;\Rightarrow\; -x=3\alpha$$

$$\frac{1}{2}x = \alpha \Rightarrow x = 2\alpha$$

$$\underset{\sim}{A}_{t_\alpha} = [-a^{\alpha}, b^{\alpha}] = [-3\alpha, 2\alpha]$$

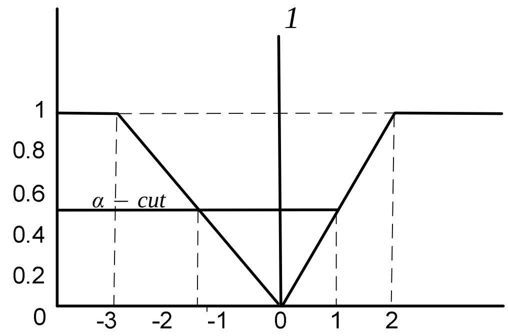

Figure 2: Graphical representation of α - Cut of reverse order triangular fuzzy number(RoTFN).

When (α=0.5), we get $A_{0.5}$ = [-1.5,1],

Also when (α=0), $\underset{\sim}{A}_{t_0} = [-a^0, b^0]$ = [-3,2].

3.2 Conditions on reverse order triangular fuzzy number.

An reverse order triangular Fuzzy Number $\underset{\sim}{A}_t$ should satisfy the following conditions;

(i) $\mu_{\underset{\sim}{A}_t}(x)$ is a continuous function in the interval [1,0]

(ii) $\mu_{\underset{\sim}{A}_t}(x)$ is strictly decreasing and continuous function on [-a, 0]

(iii) $\mu_{\underset{\sim}{A}_t}(x)$ is strictly increasing and continuous function on [0,b]

3.3. Arithmetic operations on reverse order triangular fuzzy number (ROTFN)

3.3.1. Addition of two reverse order triangular fuzzy numbers

If $\underset{\sim}{A}_t, = (-a_1, 0, b_1)$ and $\underset{\sim}{B}_t = (-a_2, 0, b_2)$; are two reverse order fuzzy numbers then,

$\underset{\sim}{A}_t + \underset{\sim}{B}_t = (-a_1 - a_2, b_1 + b_2)$.

For example;

$If, \underset{\sim t}{A} = (-2,0,1); \underset{\sim t}{B} = (-3,0,2);$. $Then, \underset{\sim}{A}_t + \underset{\sim}{B}_t = (-5,0,3)$

3.3.2. Subraction of two reverse order triangular fuzzy numbers

If $\underset{\sim}{A}_t, = (a_1, 0, b_1)$ *and* $\underset{\sim}{B}_t = (a_2, 0, b_2)$; *are two reverse order fuzzy numbers then,*

$\underset{\sim}{A}_t - \underset{\sim}{B}_t = (a_1 - b_2, b_1 - a_2)$

For example;

$$If,\ \underset{\sim}{A}_t = (-4,0,2);\ \underset{\sim}{B}_t = (-3,0,1)\ then,\ \underset{\sim}{A}_t - \underset{\sim}{B}_t = (-1,0,1)$$

3.3.3 Perfect reverse order triangular fuzzy number

A perfect reverse order triangular fuzzy number(PROTFN) is definsd as $\underset{\sim}{A}_{P_t}$ = $(-a,o,a)$, Where the numerical value of -a and a both are equal. In other words Perfect Reverse order triangular fuzzy number(PROTFN) preserves symmetry.
For example; $\underset{\sim}{A}_{P_t} = (-2,0,2)$

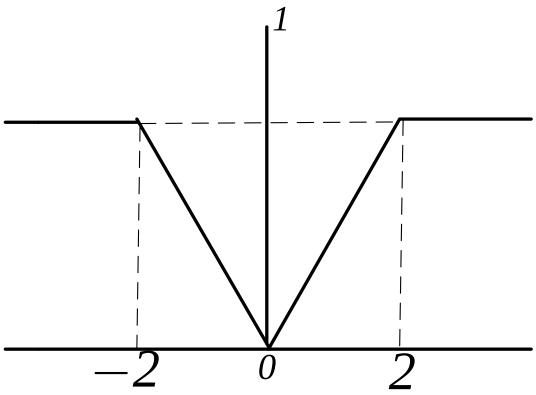

Figure 3: Graphical representation of perfect reverse order triangular fuzzy number (PRoTFN).

4. Reverse order Trapezoidal fuzzy number.

Definition: 4.1. (Reverse order trapezoidal fuzzy number). A fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_{tr}$ = {-a,-b,c,d}is said to be reverse order trapezoidal fuzzy number(ROTRFN) if its membership function is given by

$$\mu_{\underset{\sim}{A}_{tr}}(x) = \begin{cases} 1 & for\ x \le -a \\ \dfrac{-1(x+b)}{a-b} & for -a \le x \le -b \\ 0 & for -b \le x \le c \\ \dfrac{1(x-c)}{d-c} & for\ c \le x \le d \\ 1 & for\ x \ge d \end{cases}$$

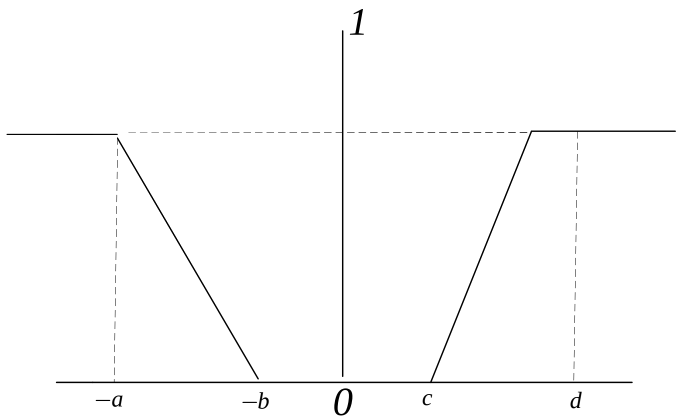

Figure 4: Graphical representation of reverse order trapezoidal fuzzy number (RoTRFN).

Example: An reverse order trapezoidal fuzzy number, $\mu_{\underset{\sim}{A}_{tr}}$ = (-3,-2 ,1,3,) and its membership is given as,

$$\mu_{\underset{\sim}{A}_{tr}}(x) = \begin{cases} 1 & for\ x \leq -3 \\ \dfrac{-1(x+2)}{3-2} & for -3 \leq x \leq -2 \\ 0 & for -2 \leq x \leq 1 \\ \dfrac{1(x-1)}{3-1} & for\, 1 \leq x \leq 3 \\ 1 & for\ x \geq 3 \end{cases}$$

α- Cut of reverse order trapezoidal fuzzy number

$$\frac{-1(x+2)}{3-2} = \alpha \Rightarrow x = -\alpha - 2\ ,$$

$$\frac{1(x-1)}{3-1} = \alpha \Rightarrow x = 2\alpha + 1$$

$\underset{\sim}{A}_{tr_\alpha}$ =[$-\alpha-2, 2\alpha+1$]

When (α=0.5), we get $A_{0.5}$ = [-2.5,2],

Also when (α=0), $\underset{\sim}{A}_{t_0}$ = [-3,-2,1,3].

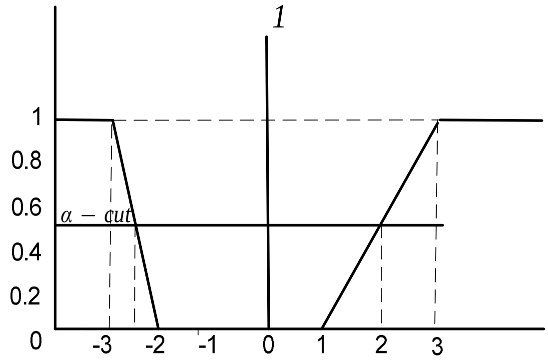

Figure 5: Graphical representation of α - Cut of reverse order trapezoidal fuzzy number (RoTRFN).

4.2. Conditions on reverse order trapezoidal fuzzy number.

An reverse order trapezoidal Fuzzy Number $\underset{\sim}{A}_{tr}$ should satisfy the following conditions;

(i) $\mu_{\underset{\sim}{A}_{tr}}(x)$ is a continuous function in the interval [1,0]

(ii) $\mu_{\underset{\sim}{A}_{tr}}(x)$ is strictly decreasing and continuous function on [-a,-b]

(iii) $\mu_{\underset{\sim}{A}_{tr}}(x)$ is strictly increasing and continuous function on [c,d]

4.3. Arithmetic operations on reverse order trapezoidal fuzzy number (ROTRFN)
4.3.1. Addition of two reverse order trapezoidal fuzzy numbers

If $\underset{\sim}{A}_{tr}, = (-a_1, -b_1, c_1, d_1)$ and $\underset{\sim}{B}_{tr} = (-a_2, -b_2, c_2, d_2)$; are two reverse order trapezoidal fuzzy numbers then, $\underset{\sim}{A}_{tr} + \underset{\sim}{B}_{tr} = (-a_1 - a_2, -b_1 - b_2, c_1 + c_2, d_1 + d_2)$.

For example;

$$If, \underset{\sim}{A}_{tr} = (-4,-2,1,2);\ \underset{\sim}{B}_{tr} = (-3,-1,2,3);.$$

$$Then, \underset{\sim}{A}_{tr} + \underset{\sim}{B}_{tr} = (-7,-3,3,5).$$

4.3.2. Subraction of two reverse order trapezoidal fuzzy numbers

If $\underset{\sim}{A}_{tr}, = (-a_1, -b_1, c_1, d_1)$ and $\underset{\sim}{B}_{tr} = (-a_2, -b_2, c_2, d_2)$; are two reverse order trapezoidal fuzzy numbers then,

$$\underset{\sim}{A}_{tr} - \underset{\sim}{B}_{tr} = (-a_1 - d_2, -b_1 - c_2, c_1 + b_2, d_1 + a_2).$$

For example;

$$If, \underset{\sim}{A}_{tr} = (-4,-1,3,4);\ \underset{\sim}{B}_{tr} = (-3,-2,2,3)\ then, \underset{\sim}{A}_{tr} - \underset{\sim}{B}_{tr} = (-7,-3,5,7)$$

4.3.3. Perfect reverse order trapezoidal fuzzy number

A perfect reverse order trapezoidal fuzzy number(PRTRFN) is definsd as $\underset{\sim}{A}_{P_{tr}}$ = $(-a,-b,b,a)$, Where the numerical value of -a and a both are equal. In other words Perfect Reverse order trapezoidal fuzzy number(PRoTRFN) preserves symmetry.
For example;

$\underset{\sim}{A}_{P_{tr}} = (-2,-1,1,2)$

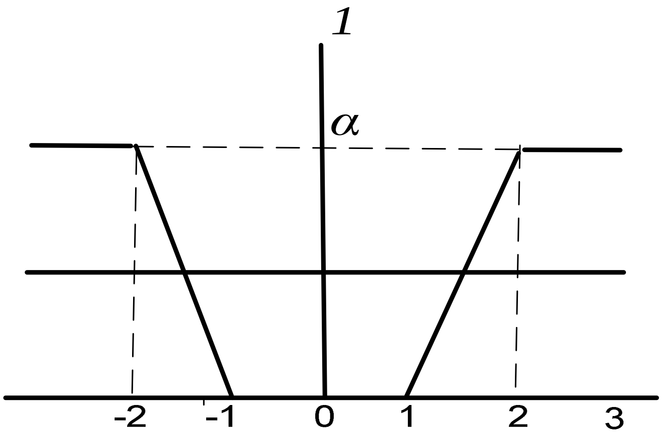

Figure 6: Graphical representation of perfect reverse order trapezoidal fuzzy number (PRoTRFN).

5. Reverse order Pentagonal fuzzy number.

Definition 5.1. (Reverse order Pentagonal fuzzy number). A fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_P$ = {-a,-b,0,c,d}is said to be reverse order Pentagonal fuzzy number(ROPFN) if its membership function is given by

$$\mu_{\underset{\sim}{A}_P}(x)=\begin{cases} 1 & for\ x \le -a \\ \frac{-1}{a}x & for -a \le x \le -b \\ \frac{-1}{b}x & for -b \le x \le 0 \\ \frac{1}{c}x & for\, 0 \le x \le c \\ \frac{1}{d}x & for\, c \le x \le d \\ 1 & for\ x \ge d \end{cases}$$

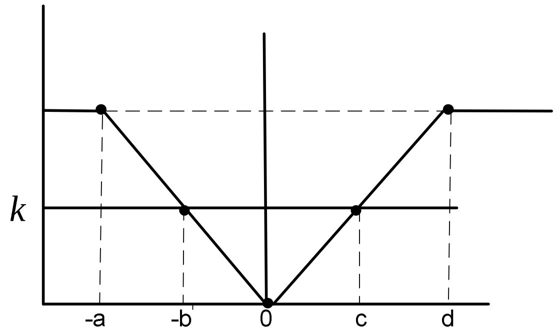

Figure 7: Graphical representation of reverse order pentagonal fuzzy number (RoPFN).

Example: A perfect reverse order pentagonal fuzzy number, $\mu_{\underset{\sim}{A}_P}$ = (-5,-2 ,0,2,5,) and its membership is given as,

$$\mu_{\underset{\sim}{A}_P}(x) = \begin{cases} 1 & \text{for } x \le -5 \\ \frac{-1}{5}x & \text{for } -5 \le x \le -2 \\ \frac{-1}{2}x & \text{for } -2 \le x \le 0 \\ \frac{1}{2}x & \text{for } 0 \le x \le 2 \\ \frac{1}{5}x & \text{for } 2 \le x \le 5 \\ 1 & \text{for } x \ge 5 \end{cases}$$

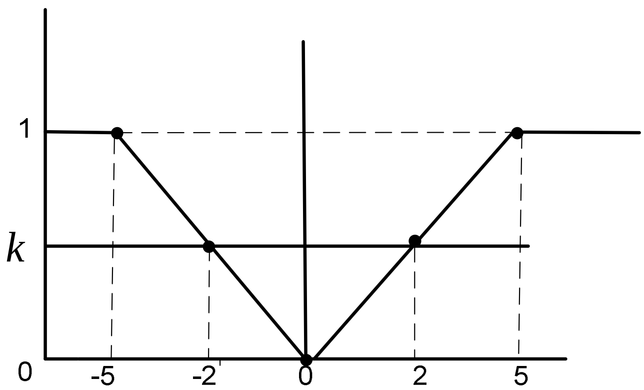

Figure 8: Graphical representation of perfect reverse order pentagonal fuzzy number (RoPFN).

6. Conclusion

The concept of fuzzy numbers and their utility aspects have been studied by researchers in recent times. Categorization of fuzzy numbers and their related properties have initiated new notions and approaches. In this paper, the Reverse Order Triangular Fuzzy Number and Reverse Order Trapezoidal Fuzzy Number has been introduced with arithmetic operations. Using a few examples,we have explained the relevant arithmetic operations. Also we defined Reverse Order Pentagonal Fuzzy Number. We observe that fuzzy number concepts could be applied to many real life problem.

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