

## The Split Majority Domination Number of a Graph

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**Abstract.** A majority dominating set  $D$  of a graph  $G = (V, E)$  is a split majority dominating set if the induced sub graph  $\langle V - D \rangle$  is disconnected. The split majority domination number  $\gamma_{SM}(G)$  of  $G$  is the minimum cardinality of a minimal split majority dominating set. In this paper, we study the split majority dominating set of a graph  $G$  and its number. Also some bounds of  $\gamma_{SM}(G)$  and the relationship of  $\gamma_{SM}(G)$  with other known parameters of  $G$  are obtained.

**Keywords:** Majority dominating set, split majority dominating set.

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### 1. Introduction

The graphs considered here are finite, undirected, without loops or multiple edges and have at least one component which is not complete or at least two components neither of which are isolated vertices. Unless otherwise stated, all graphs are assumed to have  $p$  vertices and  $q$  edges. For a vertex  $v \in V(G)$ , the open neighborhood of  $v$ ,  $N(v)$  is the set of vertices adjacent to  $v$  and the closed neighborhood  $N[v] = N(v) \cup \{v\}$ . Other graph theoretic terminology not defined here can be found in [2].

**Definition 1.** [3] A set  $D \subseteq V(G)$  of vertices in a graph  $G = (V, E)$  is a dominating set if every vertex  $v \in V$  is either an element of  $D$  or adjacent to an element of  $D$ . A dominating set  $D$  is called minimal dominating set if no proper subset of  $D$  is a dominating set. The minimum cardinality of minimal dominating set is called the domination number of a graph  $G$  and it is denoted by  $\gamma(G)$ . A set  $D \subseteq V(G)$  of vertices in a graph  $G$  is called an independent set if no two vertices in  $D$  are adjacent. An independent set is called a maximal independent set if any vertex set properly containing  $D$  is not independent. The minimum cardinality of a maximal independent set is called the lower independence number and also independent domination number and the maximum cardinality of a maximal independent set is called the independence number in a graph  $G$  and it is denoted by  $i(G)$  and  $\beta_o(G)$  respectively.

**Definition 2.** [7] A subset  $D \subseteq V(G)$  of vertices in a graph  $G = (V, E)$  is called a majority dominating set if at least half of the vertices of  $G$  are either in  $D$  or adjacent to

the vertices of  $D$ . i.e.  $|N[D]| \geq \left\lceil \frac{p}{2} \right\rceil$ . A majority dominating set  $D$  is minimal if no proper subset of  $D$  is a majority dominating set. The minimum cardinality of a minimal majority dominating set is called the majority domination number and it is denoted by  $\gamma_M(G)$ .

**Definition 3. [5]** A set  $D$  of vertices of a graph  $G$  is said to be a majority independent set if it induces a totally disconnected sub graph with  $|N[D]| \geq \left\lceil \frac{p}{2} \right\rceil$  and  $|pn[v, D]| \geq |N[D]| - \left\lceil \frac{p}{2} \right\rceil$ , for every  $v \in D$ . If any vertex set  $D'$  properly containing  $D$  is not majority independent then  $D$  is called maximal majority independent set. The maximum cardinality of a maximal majority independent set is called majority independence number and it is denoted by  $\beta_M(G)$ . The above two parameters have been studied by Swaminathan and JoselineManora.

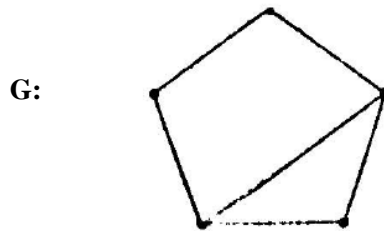
**Definition 4. [5]** A majority dominating set  $D$  of a graph  $G = (V, E)$  is called an independent majority dominating (IMD) set if the induced sub graph  $\langle D \rangle$  has no edges. The minimum cardinality of a maximal majority independent set is called lower majority independent set of  $G$  and it is also called the independent majority domination number of  $G$ , denoted by  $i_M(G)$ .

**Definition 5. [9]** A dominating set  $D$  is said to be split dominating set if the induced sub graph  $\langle V \setminus D \rangle$  is disconnected. The split domination number  $\gamma_s(G)$  is the minimum cardinality of a split dominating set.

**2. Definition and example**

**Definition 2.1.** A majority dominating set  $D \subseteq V(G)$  is said to be split majority dominating set if the induced sub graph  $\langle V - D \rangle$  is disconnected. A split majority dominating set  $D$  is minimal if no proper subset of  $D$  is split majority dominating set. The split majority domination number  $\gamma_{SM}(G)$  of  $G$  is the minimum cardinality of a minimal split majority dominating set of a graph  $G$ .

**Example 2.2.** Consider the following graph  $G$ .



**Figure 2.1.** A graph with different  $\gamma_M(G) = 1$  and  $\gamma_{SM}(G) = 2$ .

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### 3. Split majority domination number for some standard graphs

1. Suppose  $G = P_p$ . Then  $\gamma_{SM}(G) = \left\lceil \frac{p}{6} \right\rceil$ .
2. Let  $G = W_p$ ,  $p \geq 3$ . Then  $\gamma_{SM}(G) = 3$ .
3. Let  $G = \overline{K_{m,n}} - \{e\}$ ,  $m \leq n$ . Then  $\gamma_{SM}(G) = 1$ .
4. Suppose  $G = F_p$ . Then  $\gamma_{SM}(G) = 2$ .
5. For  $G = K_{1,p-1}$ ,  $S(K_{1,p-1})$ ,  $D_{r,s}$ .  $\gamma_{SM}(G) = 1$ .
6. Let  $G$  be Hajos graph. Then  $\gamma_{SM}(G) = 2$ .
7. Suppose  $G$  is Petersen graph. Then  $\gamma_{SM}(G) = 3$ .
8. For any graph  $G = K_p - \{e\}$ ,  $\gamma_{SM}(G) = p - 2$ .
9. Let  $G = \overline{mK_2}$ . Then  $\gamma_{SM}(G) = p - 2$ .
10. Suppose  $G = K_{m,n}$ ,  $m \leq n$ . Then  $\gamma_{SM}(G) = m$ .

#### Observations 3.1.

1. For any graph  $G$ ,  $\gamma_{SM}(G) = \max \{ \gamma_M, |S| \}$ , where  $S$  is set of cut vertices.
2. If there exists a cut vertex  $v$  which is also a majority dominating vertex then  $\gamma_M(G) = \gamma_{SM}(G) = 1$ .
3. If there exists a cut vertex which is also a full degree vertex  $v$  in  $G$  then  $\gamma(G) = \gamma_S(G) = \gamma_M(G) = \gamma_{SM}(G) = 1$ .

**Theorem 3.2.** For any cycle  $C_p$ ,  $p \geq 4$ . Then,  $\gamma_{SM}(C_p) = \begin{cases} 2 & \text{if } 4 \leq p \leq 6. \\ \left\lceil \frac{p}{6} \right\rceil & \text{if } p > 6. \end{cases}$

#### Proof:

**Case 1:** Let  $G$  be  $C_p$ ,  $4 \leq p \leq 6$  and  $D$  be  $\gamma_{SM}$ -set of  $G$ . Assume  $p$  takes the at most

value 6. For any vertex  $v \in C_p$ ,  $|N[v]| = 3 = \left\lceil \frac{p}{2} \right\rceil$ . Since  $C_p$  is a closed walk,  $|D| \neq 1$ .

Therefore  $|D| \geq 2$ . Suppose  $|D| = 3$ . Then any set  $D - \{v\}$ , where  $v \in D$  would be split majority dominating set of  $G$  resulting  $D$  cannot be minimal. Therefore  $|D| \leq 2$  which implies  $|D| = 2$ . Therefore,  $\gamma_{SM}(G) \leq |D| = 2$  if  $4 \leq p \leq 6$ .

**Case 2:** Let  $G = C_p$ ,  $p > 6$ . Since  $C_p$  is closed walk,  $|D|$  must be at least 2 which is proved in case 1. Let  $D = \{v_1, v_2, \dots, v_t\}$  be a  $\gamma_{SM}$ -set of  $G$ , where  $|D| = t = \gamma_{SM}(G)$

. Then  $|N[D]| \geq \left\lceil \frac{p}{2} \right\rceil$ . Therefore  $|N[D]| \leq \sum_{i=1}^t (d(v_i) + \gamma_{SM}(G)) = \gamma_{SM}(G) (d(v) + 1)$ .

Then  $\left\lceil \frac{p}{2} \right\rceil \leq 3 \gamma_{SM}(G)$ . When  $p = 2r + 1$ ,  $3 \gamma_{SM}(G) \geq \frac{p}{2}$  and  $\gamma_{SM}(G) \geq \frac{p}{6}$  if  $p$  is

odd. When  $p = 2r$ ,  $3 \gamma_{SM}(G) \geq \frac{2p}{2}$  and  $\gamma_{SM}(G) \geq \frac{p}{6}$  if  $p$  is even. Thus in all cases

$\gamma_{SM}(G) \geq \frac{P}{6}$ . Conversely, Let  $S = \{ v_1, v_2, \dots, v_t \}$ , where  $t = \left\lceil \frac{P}{6} \right\rceil$  such that  $N[v_i] \cap N[v_j] = \emptyset$ ,  $i \neq j$  and  $|S| = \left\lceil \frac{P}{6} \right\rceil$ . Therefore,  $|N[S]| = 3 \left\lceil \frac{P}{6} \right\rceil$ . This implies that  $S$  is a split majority dominating set of  $G$ ,  $\gamma_{SM}(G) \leq |S| = \left\lceil \frac{P}{6} \right\rceil$ . Hence the result.

#### 4. Characterization of minimal split majority dominating sets

**Theorem 4.1.** A split majority dominating set  $D$  of  $G$  is minimal if and only if for each vertex  $v \in D$  one of the following holds:

- (i)  $|N[D]| > \left\lceil \frac{P}{2} \right\rceil$  and  $|pn[v, D]| > |N[D]| - \left\lceil \frac{P}{2} \right\rceil$ .
- (ii)  $|N[D]| = \left\lceil \frac{P}{2} \right\rceil$  and either  $v$  is an isolate of  $D$  or  $pn[v, D] \cap (V - D) \neq \emptyset$ .
- (iii)  $\langle (V - D) \cup \{v\} \rangle$  is connected.

**Proof:** Suppose  $D$  is minimal split majority dominating set. Then,  $|N[D]| \geq \left\lceil \frac{P}{2} \right\rceil$  and  $\langle V - D \rangle$  is disconnected. Suppose  $|N[D]| > \left\lceil \frac{P}{2} \right\rceil$ . Let  $v \in D$ . Therefore,  $D' = D - \{v\}$  is not a split majority dominating set of  $G$ . Then, either  $|N[D']| < \left\lceil \frac{P}{2} \right\rceil$  or  $\langle (V - D') \rangle$  is connected.

**Case 1:** When  $|N[D']| < \left\lceil \frac{P}{2} \right\rceil$ ,  $|N[D']| = |N[D]| - |pn[v, D]|$ . Therefore,  $|pn[v, D]| > |N[D]| - \left\lceil \frac{P}{2} \right\rceil$ . Condition (i) holds.

**Case 2:** When  $\langle (V - D) \rangle$  is connected. Then,  $\langle (V - (D - \{v\})) \rangle$  is connected. This implies that  $\langle (V - D) \cup \{v\} \rangle$  is connected. Condition (iii) holds. Let  $|N[D]| = \left\lceil \frac{P}{2} \right\rceil$ . Suppose that  $v$  is a neither an isolate of  $D$  nor has a private neighbor in  $\langle V - D \rangle$ . That is,  $pn[v, D] = \emptyset$ . Then,  $|N[D - v]| = |N[D]| - |pn[v, D]| = \left\lceil \frac{P}{2} \right\rceil$  which implies that  $(D - \{v\})$  is a split majority dominating set of  $G$  which is a contradiction to the assumption that  $D$  is minimal split majority dominating set. Hence condition (ii) holds. Conversely, suppose any one of the above conditions holds. Let  $D$  be a split majority dominating set. Suppose  $D$  is not minimal. Then,  $D' = D - \{v\}$  is a split majority dominating set of  $G$ , for some  $v$

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$\in D$ . This implies that  $|N[D']| \geq \left\lceil \frac{p}{2} \right\rceil$  or  $\langle V - D' \rangle$  is disconnected -(\*) Suppose (i) holds for  $v$ . Then,  $|pn[v,D]| > |N[D]| - \left\lceil \frac{p}{2} \right\rceil$  and  $|N[D]| - |N[D - \{v\}]| = |pn[v,D]|$ . This implies that,  $|N[D]| < \left\lceil \frac{p}{2} \right\rceil$  which is a contradiction. Therefore,  $|N[D]| \geq \left\lceil \frac{p}{2} \right\rceil$ . Suppose (ii) holds for  $v$ . Then,  $|N[D]| = \left\lceil \frac{p}{2} \right\rceil$  and either  $v$  is an isolate of  $D$  or  $|pn[v,D]| \cap (V - D) \neq \emptyset$ . By (\*),  $|N[D']| \geq \left\lceil \frac{p}{2} \right\rceil$  for some  $v \in D$ . This implies that,  $|N[D]| - |pn[v,D]| = |N[D']| \geq \left\lceil \frac{p}{2} \right\rceil$ . That is,  $|N[D]| \geq |pn[v,D]| + \left\lceil \frac{p}{2} \right\rceil$ . If  $v$  is an isolate of  $D$ , then  $v \in pn[v,D]$  and  $|pn[v,D]| \geq 1$ . If  $pn[v,D] \cap (V - D) \neq \emptyset$  then  $|pn[v,D]| \geq 1$ . Hence,  $|N[D]| \geq \left\lceil \frac{p}{2} \right\rceil + 1$  which is a contradiction to  $|N[D]| = \left\lceil \frac{p}{2} \right\rceil$ . Suppose (iii) holds for  $v \in D$ . Then  $\langle (V - D) \cup \{v\} \rangle$  is connected and hence  $\langle V - D' \rangle$  is connected which is contradiction to the assumption  $\langle V - D' \rangle$  is disconnected. Hence,  $D$  is minimal split majority dominating set of  $D$ .

**Proposition 4.2.** Let  $G$  be a graph. Then,  $\gamma_{SM}(G) = 1$  if and only if there exists a cut vertex  $v$  in  $G$  with  $d(v) \geq \left\lceil \frac{p}{2} \right\rceil - 1$ .

**Proof:** Let  $\gamma_{SM}(G) = 1$  and  $D = \{v\}$  is a  $\gamma_{SM}$ -set of  $G$ . It is clear that  $\langle V - \{v\} \rangle$  is disconnected and  $v$  dominates at least half of the vertices of  $G$ . This implies that  $v$  is a cut vertex with  $d(v) \geq \left\lceil \frac{p}{2} \right\rceil - 1$ . The converse part is obvious.

**Theorem 4.3.** If the graph  $G$  has no cut vertex, then  $\gamma_{SM}(G) \geq 2$ .

**Proof:** Let  $G$  be a graph without cut vertex. Let  $D$  be  $\gamma_{SM}$ -set of  $G$ . Suppose  $\gamma_{SM}(G) = 1$ . This implies that  $D = \{v\}$ . It is possible only when  $v$  is a cut vertex such that  $V - \{v\}$  is disconnected. This is contradiction to the fact that  $G$  is graph without cut vertex. Therefore  $\gamma_{SM}(G) \geq 2$ .

**Theorem 4.4.** A majority dominating set  $D$  of  $G$  is a split majority dominating set if and only if there exists two vertices  $w_1, w_2$  from two components of  $V - D$  such that every  $w_1 - w_2$  path contains a vertex of  $D$ .

**Proof:** Suppose  $D$  is split majority dominating set of  $G$ . Then  $\langle V - D \rangle$  is disconnected and it must contain at least two components  $G_1$  and  $G_2$ . Let  $w_1 \in G_1, w_2 \in G_2$ . Now,  $w_1 - w_2$  would be a path through a vertex  $v \in D$ . This path contains a vertex  $u$  of  $D$ . Conversely, Let  $D$  be a majority dominating set such that  $V - D$  is disconnected. This implies that  $D$  is split majority dominating set of  $G$ .

**Theorem 4.5.** If  $G$  has one cut vertex  $v$  and at least two blocks  $H_1$  and  $H_2$  with  $v$  adjacent to all vertices of  $H_1$  and  $H_2$ , then  $v$  is in every  $\gamma_{SM}$  set of  $G$ .

**Proof:** Let  $D$  be a  $\gamma_{SM}$  set of  $G$ . Suppose  $v \in V - D$ . Then, each of  $H_1$  and  $H_2$  contributes at least one vertex to  $D$  say  $u$  and  $w$  respectively. This implies that  $D - \{u, w\}$  is a split majority dominating set of  $G$ , a contradiction as  $v$  is adjacent to all vertices of  $H_1$  and  $H_2$ . Hence,  $v$  is in every  $\gamma_{SM}$  set of  $G$ .

**Theorem 4.6.** A tree  $T$  has a majority dominating vertex adjacent to more than one pendant vertex or  $T$  has a non support vertex if and only if every  $\gamma_M$  set of  $T$  is also a  $\gamma_{SM}$  set of  $T$ .

**Proof:** Let  $S$  be a  $\gamma_M$  set of a tree  $T$ .

**Case 1:** Suppose  $T$  has a majority dominating vertex  $v$  adjacent to more than one pendant vertex. Then  $v$  must be in  $S$  and so  $S$  is a  $\gamma_{SM}$  set of  $T$

**Case 2:** Suppose  $T$  has a non support  $v$ . Then  $S$  contains either  $v$  or at least one support adjacent to  $v$  or a non support adjacent to  $v$ . In this case,  $\langle V - S \rangle$  is disconnected and so  $S$  is a  $\gamma_{SM}$ -set of  $T$ . Conversely, Suppose every  $\gamma_M$ -set  $S$  of  $T$  is also a  $\gamma_{SM}$ -set of  $T$ . Then, every  $\langle V - S \rangle$  is disconnected.

**Case 1:** Suppose  $\gamma_M = \gamma_{SM} = 1$ . Then,  $T$  has a majority dominating vertex  $v$  and  $S = \{v\}$ . Since  $\langle V - S \rangle$  is disconnected,  $v$  is adjacent to more than one pendant vertex.

**Case 2:** Suppose  $\gamma_M = \gamma_{SM} \geq 2$ . Then  $T$  has no majority dominating vertex  $v$ . So, the following cases arise.

- (i)  $S$  contains only supports.
- (ii)  $S$  contains only non – support vertices and
- (iii)  $S$  contains non support and support vertices.

Thus the proof.

**Theorem 4.7.** If a majority dominating set  $S$  of  $G$  is also a split majority dominating set then there exists two vertices  $v_1, v_2$  in different components of  $(V-S)$  such that  $d(v_1, v_2) \geq 2$ .

**Proof:** If not, assume that for any two vertices  $v_1, v_2$  in different components of  $V - S$ ,  $d(v_1, v_2) = 1$ . Then  $\langle V - S \rangle$  is connected which is a contradiction to  $S$  is a split majority dominating set of  $G$ .

**Theorem 4.8.** Let  $G_1$  and  $G_2$  be two connected graphs and  $(G_1 \circ G_2)$  be the corona of  $G_1$  and  $G_2$ . If  $\gamma_M$  set contains at least one vertex of  $G_1$  then  $\gamma_M(G_1 \circ G_2) = \gamma_{SM}(G_1 \circ G_2)$ .

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**Proof:** Let  $D$  be the  $\gamma_M$  set of  $(G_1 \circ G_2)$  containing at least one vertex  $v$  of  $G_1$ . Since the removal of  $v$  from  $G_1$  makes the graph  $(G_1 \circ G_2)$  disconnected,  $\langle V - D \rangle$  is disconnected. Then  $D$  is the  $\gamma_{SM}$  set of  $G$ . Therefore,  $\gamma_M(G_1 \circ G_2) = \gamma_{SM}(G_1 \circ G_2)$ .

#### 5. Bounds on $\gamma_{SM}$ - set

**Proposition 5.1.** For any graph  $G$ ,  $1 \leq \gamma_{SM}(G) \leq p - 2$ .

**Proof:** Let  $G$  be any graph with a cut vertex  $v$  such that  $d(v) \geq \left\lceil \frac{p}{2} \right\rceil - 1$ . Then,

$\gamma_{SM}(G) = 1$ . Suppose  $G$  is a graph with vertices  $v_i$  whose  $d(v_i) \geq p - 2$ . Let  $D = \{v_1, v_2, \dots, v_{p-2}\}$  be majority dominating set of  $G$ . Then,  $\langle V - D \rangle$  is disconnected. This implies that  $D$  is split majority dominating set of  $G$  implying  $\gamma_{SM}(G) \leq |D| = p - 2$ . These bounds are sharp for  $K_{1, n-1}$  and  $\overline{mK_2}$ .

**Theorem 5.2.** For any graph  $G$ ,  $\gamma_{SM}(G) \leq \alpha_M(G)$ .

**Proof:** Let  $D$  be a majority vertex covering set of  $G$  such that  $d(v_i, v_j) \geq 2$ ,  $i \neq j$  for every  $v_i, v_j \in D$ . Since  $D$  covers at most  $\frac{p(p-1)}{4}$  edges,  $|N[D]| \geq \left\lceil \frac{p}{2} \right\rceil$ ,  $D$  would be a majority dominating set of  $G$ . Since  $d(v_i, v_j) \geq 2$ ,  $\langle V - D \rangle$  is disconnected and  $D$  would be a split majority dominating set of  $G$ . Therefore,  $\gamma_{SM}(G) \leq |D| = \alpha_M(G)$ . Thus the result.

**Theorem 5.2.** For any graph  $G$ ,  $\gamma_M(G) + \gamma_{SM}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1$ .

**Proof:** Since  $\gamma_M(G) \leq \beta_M(G)$  and  $\gamma_{SM}(G) \leq \alpha_M(G)$ ,  $\gamma_M(G) + \gamma_{SM}(G) \leq \beta_M(G) + \alpha_M(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1$ .

**Theorem 5.3. [4]** For any graph  $G$ ,

- (i)  $\left\lceil \frac{p}{2(\Delta(G)+1)} \right\rceil \leq \gamma_M(G)$ .
- (ii)  $\gamma_M(G) \leq \left\lceil \frac{p}{2} \right\rceil - \Delta(G)$  if  $\Delta(G) < \left\lceil \frac{p}{2} \right\rceil - 1$ .
- (iii)  $\gamma_M(G) = 1 \leq \left\lceil \frac{p - \Delta(G)}{2} \right\rceil$  if  $\Delta(G) \geq \left\lceil \frac{p}{2} \right\rceil - 1$ .

**Theorem 5.4.** For any graph  $G$ ,  $\gamma_{SM}(G) \geq \left\lceil \frac{p}{2(\Delta(G)+1)} \right\rceil$ .

**Proof:** Since  $\gamma_{SM}(G) \geq \gamma_M(G)$  and  $\left\lceil \frac{p}{2(\Delta(G)+1)} \right\rceil \leq \gamma_M(G)$ ,

$$\gamma_{SM}(G) \geq \left\lceil \frac{p}{2(\Delta(G)+1)} \right\rceil.$$

**Theorem 5.5.** For any graph G, (i)  $\gamma_{SM}(G) \leq \Delta(G) + 1$  if  $\Delta(G) < \left\lceil \frac{p}{2} \right\rceil - 1$ .

$$(ii) \gamma_{SM}(G) \leq \left\lceil \frac{p}{2} \right\rceil \text{ if } \Delta(G) \geq \left\lceil \frac{p}{2} \right\rceil - 1.$$

**Proof:** (i) When  $\Delta(G) < \left\lceil \frac{p}{2} \right\rceil - 1$ ,  $\gamma_M(G) + \gamma_{SM}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1$  and  $\gamma_M(G) \leq \left\lceil \frac{p}{2} \right\rceil - \Delta(G)$ , we have  $\gamma_{SM}(G) \leq \Delta(G) + 1$ . The bound is sharp for G being  $P_{13}$ .

(ii) When  $\Delta(G) \geq \left\lceil \frac{p}{2} \right\rceil - 1$ ,  $\gamma_M(G) + \gamma_{SM}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1$  and  $\gamma_M(G) = 1$ , We have  $\gamma_{SM}(G) \leq \Delta(G)$ . This bound is sharp for G being  $K_5 - \{e\}$ .

**Theorem 5.6.** For any graph G,  $\gamma_{SM}(G) \leq \begin{cases} \frac{p(2\Delta+1)}{2(\Delta+1)} \text{ if } p \text{ is even.} \\ \frac{p(2\Delta+1)}{2(\Delta+1)} + 1 \text{ if } p \text{ is odd.} \end{cases}$

where  $\Delta(G)$  is the maximum degree of G.

**Proof:** Let D be a  $\gamma_{SM}$  set in G. Since D is minimal,  $\gamma_M(D) \leq |V - D|$  and so

$$\gamma_M(D) \leq p - \gamma_{SM}(G). \text{ Since } \gamma_M(G) \geq \left\lceil \frac{p}{2(\Delta+1)} \right\rceil, \text{ Two cases arise.}$$

**Case 1: When p is even.**

$$\frac{p}{2(\Delta+1)} \leq \gamma_M(G) \leq p - \gamma_{SM}(G) \text{ and } \frac{p}{2(\Delta+1)} \leq p - \gamma_{SM}(G).$$

This implies that  $\gamma_{SM}(G) \leq p - \frac{p}{2(\Delta+1)}$  which yields  $\gamma_{SM}(G) \leq \frac{p(2\Delta+1)}{2(\Delta+1)}$ .

**Case 2: When p is odd.**  $\frac{p}{2(\Delta+1)} + 1 \leq p - \gamma_{SM}(G)$  and  $\gamma_{SM}(G) \leq p - \frac{p+2(\Delta+1)}{2(\Delta+1)}$ .

On simplification, we get  $\gamma_{SM}(G) \leq \frac{p(2\Delta+1)}{2(\Delta+1)} + 1$ .

## 6. Relationship of $\gamma_{SM}(G)$ with other domination parameters of G

**Theorem 6.1.** For any tree T,  $\gamma_{SM}(T) = \gamma_M(T)$ .



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**Proof:** Suppose  $D_M$  and  $D_{SM}$  are the majority dominating set and split majority dominating set of  $T$ . Since  $\gamma_M(G) \leq \gamma_{SM}(G)$ , we have  $D_M \subseteq D_{SM}$ . Since  $T$  is minimally connected,  $|D_{SM}|$  cannot be greater than  $|D_M|$ . Therefore,  $D_{SM} \subseteq D_M$ . Hence,  $\gamma_{SM}(T) = \gamma_M(T)$ .

**Theorem 6.2.** For any graph  $G$ ,

- (i)  $\kappa(G) \leq \gamma_{SM}(G) \leq \gamma_S(G)$ , where  $\kappa$  is vertex connectivity.
- (ii)  $\gamma_M(G) \leq \gamma_{SM}(G)$ .
- (iii)  $\gamma_M(G) \leq \gamma_{SM}(G) \leq \gamma(G) \leq \gamma_S(G)$ .

**Proof:** (i) Let  $D$  be a  $\gamma_S$ -set of a graph  $G$ . Then,  $D$  is also a split majority dominating set of  $G$ . Therefore,  $\gamma_{SM}(G) \leq |D| = \gamma_S(G)$ . Let  $S$  be a  $\gamma_{SM}$ -set of a graph  $G$ . Then,  $\langle V - S \rangle$  is disconnected. Therefore, the minimum number of vertices  $S$  would disconnect  $G$  and hence  $\kappa(G) \leq |S| = \gamma_{SM}(G)$ .

(ii) Since every split majority dominating set  $S$  of  $G$  is a majority dominating set of  $G$ ,  $\gamma_M(G) \leq |S| = \gamma_{SM}(G)$ .

(iii) Since  $\gamma_{SM}(G) \leq \gamma(G)$ ,  $\gamma_M(G) \leq \gamma_{SM}(G)$  and  $\gamma(G) \leq \gamma_S(G)$ , we have  $\gamma_M(G) \leq \gamma_{SM}(G) \leq \gamma(G) \leq \gamma_S(G)$ .

**Theorem 6.3.** For any tree  $T$  with a vertex of degree  $k$ ,  $\gamma_{SM}(G) \leq p - k$ . This bound is sharp if  $G = K_{1,p-1}$ .

**Proof:** Let  $v$  be a vertex in tree  $T$  with degree  $k$ . Then  $\langle N(v) \rangle$  is disconnected and  $V - N(v)$  is split majority dominating set. Therefore,  $|V - N(v)| \geq \gamma_{SM}(G)$  and hence  $\gamma_{SM}(G) \leq p - k$ .

**Theorem 6.4.** If  $\text{diam}(G) = 2$ , then  $\gamma_{SM}(G) \leq \delta(G)$  where  $\delta(G)$  is the minimum degree of  $G$ .

**Proof:** Since,  $\gamma_{SM}(G) \leq \gamma_S(G)$  and  $\gamma_S(G) \leq \delta(G)$ , we have  $\gamma_{SM}(G) \leq \delta(G)$ .

**Theorem 6.5.** For any graph  $G$ ,

- (i)  $\alpha_M(G) + \beta_M(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1 [5]$ ,
- (ii)  $\gamma_M(G) + \gamma_{SM}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1$ .
- (iii)  $i_M(G) + \gamma_{SM}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1$ .

**Proof:** Since  $\gamma_M(G) \leq \beta_M(G)$  and  $\gamma_{SM}(G) \leq \alpha_M(G)$ ,

$$\gamma_M(G) + \gamma_{SM}(G) \leq \alpha_M(G) + \beta_M(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1.$$

(iii) Since  $i_M(G) \leq \beta_M(G)$  and  $\gamma_{SM}(G) \leq \alpha_M(G)$ ,

$$i_M(G) + \gamma_{SM}(G) \leq \alpha_M(G) + \beta_M(G) \leq \left\lceil \frac{p}{2} \right\rceil + 1.$$

### 7. Conclusion

In this article, we have extended the notion of majority dominating set  $D$  of a graph  $G$  to its complement  $V-D$  such that  $\langle V-D \rangle$  is disconnected. We have characterized some theorems for a majority dominating set to be a split majority dominating set and established  $\gamma_M(T) = \gamma_{SM}(T)$  for any tree  $T$ . It would be interesting to prove that for which graphs  $\gamma_M(G) = \gamma_{SM}(G)$ .

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