

Type-2 Fuzzy Set Theory in Medical Diagnosis

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Abstract. The diagnostic decision in medicine is frequently encountered with uncertainties. Modeling of uncertainties in the process of diagnosis of disease under fuzzy environment is an important subject. Various efforts have been made to model the uncertainties in this area through fuzzy sets and its generalizations. The theory of type-2 fuzzy sets is an intuitive and computationally feasible in addressing uncertain and imprecise information in decision making. Present work proposes the application of type-2 fuzzy relations to such problems by extending the Sanchez's approach. A hypothetical example is discussed to illustrate the methodology.

Keywords: Type-1 fuzzy sets, type-2 fuzzy sets, medical diagnosis, type-2 fuzzy relations, secondary membership function

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1. Introduction

Membership grades of elements of type-1 fuzzy set are crisp numbers in $[0, 1]$. The utility of the concept of type-1 fuzzy set depends on the capability of the user to construct appropriate membership function. Further in a blurring situation, this estimation becomes poorer [1]. Interval valued fuzzy set is a generalization of type-1 fuzzy set that uses a closed interval contained in $[0, 1]$ for the membership grades of its elements. Although interval estimation of membership function in fuzzy sets covers the disadvantage of point estimation to some extent yet it gives same weight to all the possibilities of membership grade in the interval estimation.

Zadeh [2] initiated another important extension of the concept of type-1 fuzzy sets in the form of type-2 fuzzy sets. These sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets. Mendel and John [1] gave a simple representation for Type-2 fuzzy sets. Due to the dependence of the membership functions on available linguistic information and numerical data. Linguistic information (e.g., rules from experts), in general, does not give any information about the shapes of the membership functions when membership functions are determined or tuned based on numerical data, the uncertainty in the numerical data e.g., noise, translates into uncertainty in the membership functions. In all such cases, type-2 framework of fuzzy sets can be used to model information about the linguistic, numerical uncertainty very well [3].

To diagnose the patient for diseases carries various stages which are certainly filled with uncertainties upto some extent. Physicians generally collect information by

examining the patient physically and history of the patient. In physical examination, some symptoms may be overlooked and some important part of the history may not be revealed by the patient. Moreover physicians gather information from the laboratory tests which are often depend on the exact interpretation of the results which are rare. However techniques are available to measure some symptoms upto the extent of its occurrence yet a symptom may be the indicative of several diseases. More information available to the physician from different types of examinations and laboratory tests may increase the uncertainty (non-specificity).

One of the earliest works on medical diagnosis allowing fuzziness was proposed by Sanchez [4]. Thereafter some researchers have contributed in this area successfully. In this concern, Esogbue and Elder proposed fuzzy mathematical models [5,6,7]. An application of fuzzy set theory was given by Adlassing [8]. Chen [9] presented weighted fuzzy algorithms and Belacel proposed PROCFTN methodology to handle uncertainties [10]. In the same direction Yao and Yao used the concept of fuzzy number and compositional rule of inference to make decisions [11]. Roychowdhury et al. gave diagnostic decision model using a GA- fuzzy approach [12]. Roy and Biswas [13] defined compositions for interval valued fuzzy sets and used them for the same. Type-2 fuzzy set is a generalization of type-1 fuzzy set as well as of interval valued fuzzy sets. Quite recently, Own [14] proposed a switching function and type-2 fuzzy similarity and presented applications of these in medical diagnosis and pattern recognition. Pandey et al. [15] proposed diagnostic decision model using vague sets. Celik and Yamak [16] applied fuzzy soft set theory to medical diagnosis. They used the concept of fuzzy arithmetic operations through Sanchez's approach to make the decisions. Elizabeth and Sujatha [17] used interval valued fuzzy number matrices in medical diagnosis present paper extends the Sanchez's approach of medical diagnosis in the type-2 fuzzy atmosphere. This work uses the concept of type-2 fuzzy relations and is different from the work given by Own.

A brief sketch of the paper is as follows: Section 2 studies some relevant basics of type-2 fuzzy sets. Composition operation between type-2 fuzzy relations and a proposition have been discussed in section 3. Section 4 introduces the extended Sanchez's approach for type-2 fuzzy sets. To end with, an example and Discussion based conclusion of the work are presented in Sections 5 and 6 respectively.

2. Preliminaries

In the present section we discuss type-2 fuzzy set proposed in [1] and some relevant basic concepts related to type-2 fuzzy sets.

Definition 1. A type-2 fuzzy set \tilde{A} defined on a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}} : X \rightarrow F([0,1])$ and is expressed by the following set notation:

$$\tilde{A} = \{ \langle x, \tilde{\mu}_{\tilde{A}}(x) \rangle : x \in X \} \quad (1)$$

$F([0,1])$ denotes the set of all type-1 fuzzy sets that can be defined on the set $[0, 1]$. $\tilde{\mu}_{\tilde{A}}(x)$, itself is a type-1 fuzzy set for value of $x \in X$ and is characterized by a secondary membership function $f_x : J_x \rightarrow [0,1]$. Therefore, \tilde{A} can be represented as:

$$\tilde{A} = \{ \langle x, \{ \langle u, f_x(u) \rangle : u \in J_x \} \rangle : x \in X \} \quad (2)$$

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where $J_x \subseteq [0,1]$ is the set of all possible primary memberships corresponding to an element x . In discrete case, a type-2 fuzzy set \tilde{A} can also be expressed in the following ways:

$$\tilde{A} = \left\{ \left(x, \sum_{u \in J_x} f_x(u)/u \right) : x \in X, u \in J_x \subseteq [0,1] \right\} \quad (3)$$

$$\tilde{A} = \sum_{x \in X} \left[\sum_{u \in J_x} f_x(u)/u \right] / x, \text{ where } J_x \subseteq [0,1]. \quad (4)$$

The symbol $\sum \sum$ used in (4) indicates inclusion of all admissible values of x and u . Now, onwards, type-2 fuzzy set expression in (4) is used.

Example 1. A type-2 fuzzy set defined on a finite universal set and finite set of primary membership can be represented by a 3-dimensional picture given in Fig. 1.

Let $X = \{1, 2, 3, 4, 5\}$ be the universe of discourse and suppose

$$J_1 = \{0, 0.2, 0.4, 0.6, 0.8\} = J_2 = J_4 = J_5, \quad J_3 = \{0.6, 0.8\}$$

be the sets of primary membership for $x = 1, 2, 3, 4, 5$ respectively. The secondary membership function associated with $x = 4$ is represented by a fuzzy set

$$\mu_{\tilde{A}}(2) = 0.25/0 + 0.3/0.2 + 0.4/0.4 + 0.5/0.6 + 0.4/0.8.$$

This secondary membership function can also be viewed through the five vertical lines at points $(4,0), (4,0.2), (4,0.4), (4,0.6)$ and $(4,0.8)$ in the figure. Similarly we can define the secondary membership function for $x = 1, 3, 4, 5$. We have shown all secondary membership functions in the following figure. Shaded portion is called the footprint uncertainty.

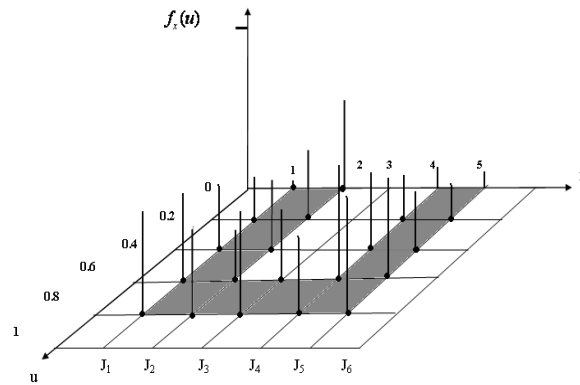


Figure 1:

Definition 2. Uncertainty in the primary memberships of a type-2 fuzzy set consists of a bounded region that we call the footprint uncertainty and is denoted by $\text{FOU}(\tilde{A})$. It is defined by

$$\text{FOU}(\tilde{A}) = \bigcup_{x \in X} J_x$$

The footprint uncertainty in the example 1 is

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$$\text{FOU}(\tilde{A}) = \{0, 0.2, 0.4, 0.6, 0.8\}$$

Definition 3. For every value $x = x'$, say, the 2-D plane whose axes are u and $f_{x'}(u)$ is called the vertical slice. A secondary membership function is thus a vertical slice. It can be represented by

$$\mu_{\tilde{A}}(x') = \sum_{u \in J_{x'}} f_{x'}(u)/u, J_{x'} \subseteq [0, 1], \text{ in which } 0 \leq f_{x'}(u) \leq 1, x' \in X.$$

The domain of a secondary membership function is called the primary membership of x and the amplitude of a secondary membership function is called a secondary grade. In equation (2), $f_x(u)$ is a secondary grade.

Definition 4. A type-1 fuzzy set \tilde{A} can also be expressed as a type-2 fuzzy set. Its type-2 representation is $1/\mu_{\tilde{A}}(x)/x \quad \forall x \in X$. It means that the secondary membership function has only one value in its domain, namely the primary membership $\mu_{\tilde{A}}(x)$ at which the secondary grade equals to 1.

Example 2. Let $X = \{a, b, c, d\}$ if $J_a = \{0.4\}, J_b = \{0.6\}, J_c = \{1\}, J_d = \{0.5\}$ then

$$\tilde{A} = 1/0.4/a + 1/0.6/b + 1/1/c + 1/0.5/d$$

is a type-1 fuzzy set.

Definition 5. [3] Let \tilde{A} and \tilde{B} are two type-2 fuzzy sets in a discrete universe of discourse X . Let $\mu_{\tilde{A}}(x) = \sum_u f_x(u)/u$ and $\mu_{\tilde{B}}(x) = \sum_v f_x(v)/v$ are the membership

grades corresponding to every $x \in X$ for \tilde{A} and \tilde{B} respectively, where u, v are primary grades and $f_x(u), f_x(v)$ are secondary grades. Then

$$\tilde{A} \cup \tilde{B} = \sum_u \sum_v f_x(u) * g_x(v)/u \vee v, \quad (5)$$

$$\tilde{A} \cap \tilde{B} = \sum_u \sum_v f_x(u) * g_x(v)/u * v, \quad (6)$$

$$\tilde{\tilde{A}} = \sum_u f_x(u)/1 - u \quad (7)$$

where \vee represents the t-conorm and $*$ represents a t-norm. For computation max, min operations may be used for t-conorm and t-norm respectively. If more than one computation of u and v give the same point $u \vee v$ then in the union we keep the one with the largest membership grade. Similar logic will be adopted in case of intersection.

3. Type-2 fuzzy relations and their compositions

A n -ary type-2 fuzzy relation is a type-2 fuzzy set defined on the Cartesian product of the crisp sets X_1, X_2, \dots, X_n . Since the membership grade of association between elements of a type-1 fuzzy relation is a real number in $[0, 1]$ while in case of type-2 fuzzy relations, it is fuzzy set defined on $[0, 1]$. For simplicity, let us consider the case of binary

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relation i.e. $X \times Y$ where $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_n\}$. Then type-2 fuzzy relation \tilde{R} on $X \times Y$ is represented by type-2 fuzzy matrix in the following way

$$\tilde{R}(X, Y) = \begin{bmatrix} \tilde{\mu}_{\tilde{R}}(x_1, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_1, y_m) \\ \tilde{\mu}_{\tilde{R}}(x_2, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_2, y_m) \\ \dots & \dots & \dots & \dots \\ \tilde{\mu}_{\tilde{R}}(x_n, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_n, y_m) \end{bmatrix}$$

where each $\tilde{\mu}_{\tilde{R}}(x_i, y_j)$ is a type-1 fuzzy set, a membership grade.

Definition 6. [3] Let \tilde{R} and \tilde{S} are two type-2 fuzzy relations defined on $X \times Y$ and $Y \times Z$ respectively. Then compositions of relations is a type-2 fuzzy relation on $X \times Z$ and is given by

$$\tilde{\mu}_{\tilde{S} \circ \tilde{R}}(x, z) = \bigcup_{y \in Y} [\tilde{\mu}_{\tilde{R}}(x, y) \cap \tilde{\mu}_{\tilde{S}}(y, z)] \quad (8)$$

$(x, y) \in X \times Y, (y, z) \in Y \times Z, (x, z) \in X \times Z$ and $\forall y \in Y$,

\bigcup represents the union of two type-2 fuzzy sets and \bigcap means intersection of two type-2 fuzzy sets. The formulae for union and intersection are defined in (5) and (6). Now, we are able to prove the following proposition for type-2 fuzzy relations.

Proposition 1. If \tilde{R} and \tilde{S} are two type-2 fuzzy relations on $X \times Y$ and $Y \times Z$ respectively then

$$(i) ((\tilde{R})^{-1})^{-1} = \tilde{R} \quad (9)$$

$$(ii) (\tilde{S} \circ \tilde{R})^{-1} = \tilde{R}^{-1} \circ \tilde{S}^{-1} \quad (10)$$

Proof (i): The inverse type-2 fuzzy relation $\tilde{R}(X, Y)$, denoted by $\tilde{R}^{-1}(Y, X)$, is defined by $\tilde{\mu}_{\tilde{R}^{-1}}(y, x) = \tilde{\mu}_{\tilde{R}}(x, y) \quad \forall x \in X$ and $\forall y \in Y$. The inverse relation matrix is obtained by transposing the following relation matrix.

$$\tilde{R}(X, Y) = \begin{bmatrix} \tilde{\mu}_{\tilde{R}}(x_1, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_1, y_m) \\ \tilde{\mu}_{\tilde{R}}(x_2, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_2, y_m) \\ \dots & \dots & \dots & \dots \\ \tilde{\mu}_{\tilde{R}}(x_n, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_n, y_m) \end{bmatrix}$$

An inverse relation matrix is given as:

$$\tilde{R}^{(-1)}(Y, X) = \begin{bmatrix} \tilde{\mu}_{\tilde{R}}(x_1, y_1) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_n, y_1) \\ \tilde{\mu}_{\tilde{R}}(x_1, y_2) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_n, y_2) \\ \dots & \dots & \dots & \dots \\ \tilde{\mu}_{\tilde{R}}(x_1, y_m) & \dots & \dots & \tilde{\mu}_{\tilde{R}}(x_n, y_m) \end{bmatrix}$$

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It is obvious that

$$\tilde{R}^{(-1)}(X, Y) = \begin{bmatrix} \tilde{\mu}_{\tilde{R}}(x_1, y_1) & \cdot & \cdot & \cdot & \tilde{\mu}_{\tilde{R}}(x_1, y_m) \\ \tilde{\mu}_{\tilde{R}}(x_2, y_1) & \cdot & \cdot & \cdot & \tilde{\mu}_{\tilde{R}}(x_2, y_m) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \tilde{\mu}_{\tilde{R}}(x_n, y_1) & \cdot & \cdot & \cdot & \tilde{\mu}_{\tilde{R}}(x_n, y_m) \end{bmatrix}$$

Hence

$$((\tilde{R})^{-1})^{-1} = \tilde{R}.$$

Proof (ii):

Since, $\tilde{R}^{-1} : Y \rightarrow X$, $\tilde{S}^{-1} : Z \rightarrow Y$, $\tilde{S} \circ \tilde{R} : X \rightarrow Z$, $(\tilde{S} \circ \tilde{R})^{-1} : Z \rightarrow X$ and $\tilde{R}^{-1} \circ \tilde{S}^{-1} : Z \rightarrow X$.

Now we have, $(\tilde{S} \circ \tilde{R})^{-1}(z, x) = \tilde{S} \circ \tilde{R}(x, z) = \bigcup_{y \in Y} [\tilde{\mu}_{\tilde{R}}(x, y) \cap \tilde{\mu}_{\tilde{S}}(y, z)]$

Since, $\tilde{S}^{-1} : Z \rightarrow Y$ is defined by $\tilde{\mu}_{\tilde{S}^{-1}}(z, y) = \tilde{\mu}_{\tilde{S}}(y, z)$ and $\tilde{R}^{-1} : Y \rightarrow X$ is defined by $\tilde{\mu}_{\tilde{R}^{-1}}(y, x) = \tilde{\mu}_{\tilde{R}}(x, y)$, it is obvious that

$$\bigcup_{y \in Y} [\tilde{\mu}_{\tilde{R}}(x, y) \cap \tilde{\mu}_{\tilde{S}}(y, z)] = \bigcup_{y \in Y} [\tilde{\mu}_{\tilde{S}^{-1}}(z, y) \cap \tilde{\mu}_{\tilde{R}^{-1}}(y, x)] = \tilde{S}^{-1} \circ \tilde{R}^{-1}$$

Therefore, $(\tilde{S} \circ \tilde{R})^{-1} = \tilde{S}^{-1} \circ \tilde{R}^{-1}$.

5. Proposed approach for medical diagnosis based on type-2 fuzzy relations

This section extends Sanchez's approach for medical diagnosis using the concept of type-2 fuzzy sets.

Let $S = \{s_1, s_2, \dots, s_r\}$, $D = \{d_1, d_2, \dots, d_m\}$ and $P = \{p_1, p_2, \dots, p_n\}$ denote the sets of symptoms, diseases and patients respectively. We define that the physician medical knowledge be represented as type-2 fuzzy relation showing the association between symptoms and diseases. Type-2 fuzzy relation representing the association between patients and symptoms is another knowledge that represents the severity of different symptoms to patients. These associations are represented by type-2 fuzzy matrices whose entries are type-1 fuzzy sets. Therefore, medical diagnosis on the basis of type-2 fuzzy sets involves development of type-2 fuzzy relations. To get the diagnostic decision appropriate composition rules can be used.

Let \tilde{A} be the type-2 fuzzy relation showing the relationship between patients and symptoms which is obtained using the type-1 fuzzy sets as entries for the linguistic terms. It can be defined as the following type-2 fuzzy matrix:

$$\tilde{A} = \begin{bmatrix} \tilde{\mu}_{\tilde{A}}(p_1, s_1) & \cdot & \cdot & \cdot & \tilde{\mu}_{\tilde{A}}(p_1, s_r) \\ \tilde{\mu}_{\tilde{A}}(p_2, s_1) & \cdot & \cdot & \cdot & \tilde{\mu}_{\tilde{A}}(p_2, s_r) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \tilde{\mu}_{\tilde{A}}(p_n, s_1) & \cdot & \cdot & \cdot & \tilde{\mu}_{\tilde{A}}(p_n, s_r) \end{bmatrix}$$

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and \tilde{R} be type-2 fuzzy relation showing the relationship between symptoms and diseases, is obtained by the previous physician medical knowledge. It is expressed as:

$$\tilde{R} = \begin{bmatrix} \tilde{\mu}_{\tilde{R}}(s_1, d_1) & \dots & \tilde{\mu}_{\tilde{R}}(s_1, d_m) \\ \tilde{\mu}_{\tilde{R}}(s_2, d_1) & \dots & \tilde{\mu}_{\tilde{R}}(s_2, d_m) \\ \vdots & \ddots & \vdots \\ \tilde{\mu}_{\tilde{R}}(s_r, d_1) & \dots & \tilde{\mu}_{\tilde{R}}(s_r, d_m) \end{bmatrix}$$

Use the composition rule of inference defined in (8) we have

$$\tilde{T} = \tilde{R} \circ \tilde{A} = \begin{bmatrix} \tilde{\mu}_{\tilde{T}}(p_1, d_1) & \dots & \tilde{\mu}_{\tilde{T}}(p_1, d_m) \\ \tilde{\mu}_{\tilde{T}}(p_2, d_1) & \dots & \tilde{\mu}_{\tilde{T}}(p_2, d_m) \\ \vdots & \ddots & \vdots \\ \tilde{\mu}_{\tilde{T}}(p_n, d_1) & \dots & \tilde{\mu}_{\tilde{T}}(p_n, d_m) \end{bmatrix}$$

membership grades of \tilde{T} are given by

$$\tilde{\mu}_{\tilde{T}}(p_i, d_j) = \bigcup_r \left[\tilde{\mu}_{\tilde{A}}(p_i, s_r) \bigcap \tilde{\mu}_{\tilde{R}}(s_r, d_j) \right] \quad \forall p_i \in P \text{ and } d_j \in D. \quad (11)$$

Thus \tilde{T} is a type-2 fuzzy matrix, showing the relationship between patients and diseases.

The type-2 fuzzy relation, \tilde{R} used in the composition defined by (11) is the solution of the following type-2 fuzzy equation

$$\tilde{D} = \tilde{R} \circ \tilde{B} \quad (12)$$

\tilde{B} , a type-2 fuzzy relation that represents the severity of the symptoms and \tilde{D} , the type-2 fuzzy relation showing the diagnoses of the known patients. By solving the type-2 fuzzy relation equation in (12) for \tilde{R} , an accumulated medical knowledge can be obtained to associate the symptoms and diseases which is to be used in the composition defined in (11). Yan et al. [18] proposed semi tensor product of matrices to solve the type-2 fuzzy relation equations.

Property 1. Select an appropriate defuzzification method, Defuzzify the fuzzy sets (entries of the type-2 fuzzy matrix \tilde{T}) to obtain crisp entries, that are representative of corresponding entries of the matrix. Defuzzified crisp matrix is written as

$$T_D = \begin{pmatrix} t_{11} & \dots & t_{1m} \\ \vdots & \ddots & \vdots \\ t_{n1} & \dots & t_{nm} \end{pmatrix}$$

If $\max_l t_{il} = t_{ij}$, where $1 \leq l \leq m$ then patient p_i 's diagnosis as d_j . If $\max_l t_{il}$ occurs or closely occurs for more than one value. Symptoms can be reassessed and weights can be given to the symptoms.

Algorithm:

Step 1: Introduce type-2 fuzzy relation (type-2 fuzzy matrix), \tilde{A} to model the uncertainties of the state of the patient

Step 2: Using equation (12), obtain the physician medical knowledge in the form of type-2 fuzzy relation \tilde{R} which shows the association between symptoms and diseases

Step 3: Use an appropriate compositional rule to infer the association between patients and diseases. Here composition rule defined in equation (8) has been used to perform the analysis.

Step 4: Property 1 is used to make diagnostic decision.

6. An example

Here we present an example to illustrate the methodology defined in the section 5 numerically. This example has been already discussed in [11, 15].

$$\text{Let } S = \{S_1(\text{Headace}), S_2(\text{Fever}), S_3(\text{Phlegm})\}$$

$$D = \{d_1(\text{Cold}), d_2(\text{Pulmonary tuberculosis}), d_3(\text{Pertusis}), d_4(\text{Pneumonia})\}$$

$$P = \{P_1, P_2\}$$

The state of the patients is transformed to the following type-2 fuzzy relation whose entries are linguistic terms:

$$\tilde{A} = \begin{bmatrix} \text{High} & \text{High} & \text{High} \\ \text{Medium} & \text{Low} & \text{Very low} \end{bmatrix}.$$

These linguistic terms can be written in the form of type-2 fuzzy sets as follows:

$$\tilde{A} = \begin{bmatrix} \frac{0.7}{0.8} + \frac{1}{0.9} + \frac{0.95}{1} & \frac{0.7}{0.8} + \frac{1}{0.9} + \frac{0.9}{1} & \frac{0.7}{0.8} + \frac{1}{0.9} + \frac{0.95}{1} \\ \frac{0.6}{0.4} + \frac{1}{0.5} + \frac{0.9}{0.6} & \frac{0.7}{0.1} + \frac{1}{0.2} + \frac{0.8}{0.3} & \frac{1}{0} + \frac{0.9}{0.1} \end{bmatrix}$$

The type-2 fuzzy relation \tilde{R} , the association between symptoms and diseases is obtained on the basis of previous medical knowledge. This relation can also be constructed more logically by the approach given in [17].

$$\tilde{R} = \begin{bmatrix} \frac{0.6}{0.3} + \frac{1}{0.4} + \frac{0.8}{0.5} & \frac{0.2}{0.1} + \frac{1}{0.2} + \frac{0.9}{0.3} & \frac{0.3}{0.0} + \frac{1}{0.1} + \frac{0.9}{0.2} & \frac{0.6}{0.2} + \frac{1}{0.3} + \frac{0.9}{0.4} \\ \frac{0.7}{0.2} + \frac{1}{0.3} + \frac{0.9}{0.4} & \frac{0.6}{0.3} + \frac{1}{0.4} + \frac{0.9}{0.5} & \frac{0.6}{0.1} + \frac{1}{0.2} + \frac{0.9}{0.3} & \frac{0.3}{0.0} + \frac{1}{0.1} + \frac{0.8}{0.2} \\ \frac{0.3}{0.0} + \frac{1}{0.1} + \frac{0.8}{0.2} & \frac{0.7}{0.5} + \frac{1}{0.6} + \frac{0.9}{0.7} & \frac{0.3}{0.0} + \frac{1}{0.1} + \frac{0.9}{0.2} & \frac{0.6}{0.1} + \frac{1}{0.2} + \frac{0.95}{0.3} \end{bmatrix}$$

Using composition rule of type-2 fuzzy relations \tilde{A} and \tilde{R} defined in (8), we get a type-2 fuzzy relation \tilde{T} showing the relationship between patients and diseases.

$$\tilde{T} = \begin{bmatrix} \frac{0.6}{0.3} + \frac{1}{0.4} + \frac{0.8}{0.5} & \frac{0.7}{0.5} + \frac{1}{0.6} + \frac{0.9}{0.7} & \frac{0.6}{0.1} + \frac{1}{0.2} + \frac{0.9}{0.3} & \frac{0.6}{0.2} + \frac{1}{0.3} + \frac{0.9}{0.4} \\ \frac{0.6}{0.3} + \frac{1}{0.4} + \frac{0.9}{0.5} & \frac{0.2}{0.1} + \frac{1}{0.2} + \frac{0.9}{0.3} & \frac{0.7}{0.1} + \frac{1}{0.2} + \frac{0.8}{0.3} & \frac{0.6}{0.2} + \frac{0.8}{0.3} + \frac{0.8}{0.4} \end{bmatrix}.$$

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To make diagnostic decision, the notion of defuzzification for each entry of the type-2 fuzzy matrix has been taken which gives a defuzzified matrix and using the probability distribution rule, expected percentages of chances to suffer with different diseases for the patients are obtained. For speciality, centre of area method has been adopted for defuzzification. This results matrix, \tilde{T}_D

$$\tilde{T}_D = \begin{bmatrix} 0.4083 & 0.6077 & 0.2120 & 0.3120 \\ 0.4120 & 0.2333 & 0.2040 & 0.2720 \end{bmatrix}.$$

Normalizing the above matrix, we get

$$\tilde{T}_N = \begin{bmatrix} 0.2651 & 0.3946 & 0.1377 & 0.2026 \\ 0.3674 & 0.2081 & 0.1819 & 0.2426 \end{bmatrix}.$$

Now, using probability distribution rule, we get the expected percentage of diagnosis as follows:

For patient P_1 : D_1 is 26.51%, D_2 is **39.46%**, D_3 is 13.77%, D_4 is 20.26%,
For patient P_2 : D_1 is **36.74%**, D_2 is 20.81%, D_3 is 18.19%, D_4 is 24.26%

4.7. Discussion and conclusion

It is clear from the result of the example discussed in the previous section using proposed methodology that patient P_1 is diagnosed to have disease D_2 and patient P_2 is diagnosed to have disease D_1 . This example has already been discussed in [11, 15] via fuzzy relations and vague sets respectively. In the present methodology, it has been discussed in type-2 fuzzy environment by allowing more fuzziness in the state of the patients and the relation between symptoms and diseases. Diagnoses of [11, 15] and present paper for both the patients are same but expected percentage for possible diseases certainly deviated. Thus, the role of type-2 fuzzy environment introduced in diagnostic decisions may affect the result with more authenticity.

In the process of medical diagnosis, state of patient are given by the patient through linguistic terminology like as high headache, much more vomiting, medium pain in backbone etc., consideration of type-1 fuzzy sets as grades for association instead of membership grades in [0,1] is more advantageous to model the state of the patient. Similarly type-2 fuzzy relation has been introduced representing the association between symptoms and diseases. Sanchez's approach has been extended for medical diagnosis in this reference. The approach used to form type-2 fuzzy matrix showing the association of symptoms and diseases is based on the work mentioned in [18]. This is computationally costly. For future work, comparison may be made of present approach with the approach given by Own [14]. It would also be interesting to extend the existing work in the framework of interval type-2 fuzzy sets.

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