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Certain Properties of Extended Wright Generalized Hypergeometric Function

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Abstract. In this paper, we obtain the extended Wright generalized Hypergeometric function using extended Beta function. We also obtain certain integral representations, Mellin transform and some derivative properties of extended Wright generalized Hypergeometric function. Further, we represent extended Wright generalized Hypergeometric function in the form of Laguerre polynomials and Whittaker function.

Keywords: Extended Wright generalized Hypergeometric function; Extended Beta function; Laguerre polynomials; Whittaker function; Mellin transform; Fractional derivative operator

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1. Introduction

The Wright function was introduced and investigated by British mathematician E. M. Wright in the form of power series [3-6]:

$$\phi(\alpha,\beta;z) = {}_0\psi_1\left[\left.\frac{}{(\alpha,\beta)}\right| z\right] = \sum_{k=0}^{\infty}\frac{1}{\Gamma(\alpha+k\beta)}\frac{z^k}{k!}\,;\ \beta>-1,\quad \alpha,z\in C \tag{1}$$

Fox and Wright investigated the more general function termed as Wright generalized Hypergeometric function given as [1-2]:

$${}_m\psi_n\left[\left.\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{matrix}\right| z\right] = \sum_{k=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{z^k}{k!};\ z,a_i,b_j\in C;\ \alpha_i,\beta_j\in R^+$$

$$(i=1,\dots,m;\ j=1,\dots,n) \tag{2}$$

where $\sum_{j=1}^{n}\beta_j - \sum_{i=1}^{m}\alpha_i > -1$, for large values of z.

In 1997, Chaudhary introduced extended Euler's Beta function [7-8], defined as

$$B_p(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}e^{-\frac{p}{t(1-t)}}dt;\ \mathrm{Re}(p),\mathrm{Re}(x),\mathrm{Re}(y)>0 \tag{3}$$

Here, we extend Wright generalized Hypergeometric function using extended Euler's Beta function and the fact $\frac{(\gamma)_k}{(c)_k}=\frac{B(\gamma+k,c-\gamma)}{B(\gamma,c-\gamma)}$ as follows:

$${}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]=\frac{1}{\Gamma(c-\gamma)}\sum_{k=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{B_p(\gamma+k,c-\gamma)z^k}{k!} \tag{4}$$

for $\mathrm{Re}(p)>0$; $\mathrm{Re}(c)>\mathrm{Re}(\gamma)>0$.

Extended Riemann-Liouville fractional derivative operator: Ozarslan and Ozergin defined the extended Riemann - Liouville fractional derivative operator as follows [10]:

$$D_z^{\mu,p}\{f(z)\}=\frac{1}{\Gamma(-\mu)}\int_0^z f(t)(z-t)^{-\mu-1}e^{\frac{-pz^2}{t(z-t)}}dt,\ \mathrm{Re}(\mu)<0,\ \mathrm{Re}(p)>0, \tag{5}$$

and for $m-1<\mathrm{Re}(\mu)<m(m=1,2,...)$.

$$D_z^{\mu,p}\{f(z)\}=\frac{d^m}{dz^m}D_z^{\mu-m}\{f(z)\}=\frac{d^m}{dz^m}\left\{\frac{1}{\Gamma(-\mu+m)}\int_0^z f(t)(z-t)^{-\mu+m-1}e^{\frac{-pz^2}{t(z-t)}}dt\right\}$$

For $p=0$, classical Riemann-Liouville fractional derivative operator can be obtained.

Lemma 1. If $\mathrm{Re}(\nu),\mathrm{Re}(p)>0$ then we have following integral representation [11]:

$$\int_0^1 t^{\mu-1}(1-t)^{\nu-1}e^{\frac{-p}{t}}dt=\Gamma(\nu)p^{\frac{\mu-1}{2}}e^{\frac{-p}{2}}W_{\frac{1-\mu-2\nu}{2},\frac{\mu}{2}}(p)$$

2. Main results

Theorem 1. Integral Representations of extended Wright generalized Hypergeometric function is given as:

$${}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]=\frac{1}{\Gamma(c-\gamma)}\int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}e^{-\frac{p}{t(1-t)}}\,{}_m\Psi_n\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{smallmatrix}\middle|tz\right]dt \tag{6}$$

where $\mathrm{Re}(p)>0$; $\mathrm{Re}(c)>\mathrm{Re}(\gamma)>0$; $z,a_i,b_j\in C$; $\alpha_i,\beta_j\in R^+$ $(i=1,...,m;\ j=1,...,n)$.

Proof. Using equation (3) in (4), we get:

$${}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]=\frac{1}{\Gamma(c-\gamma)}\sum_{k=0}^{\infty}\left\{\int_0^1 t^{\gamma+k-1}(1-t)^{c-\gamma-1}e^{-\frac{p}{t(1-t)}}dt\right\}\frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{z^k}{k!} \tag{7}$$

After interchanging the order of integration and summation and using equation (2), we obtain the desired result.

Corollary 2. If we take $t=\frac{u}{1+u}$ in Theorem 1, we obtain:

$$ {}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z;p)\right]=\frac{1}{\Gamma(c-\gamma)}\int_0^1\frac{u^{\gamma-1}}{(1+u)^c}e^{-\frac{p(1+u)^2}{u}}\,{}_m\Psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{matrix}\middle|\frac{uz}{1+u}\right]du \quad (8) $$

Corollary 3. If we take $t=\sin^2\theta$ in Theorem 1, we obtain:

$$ {}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z;p)\right] $$

$$ =\frac{2}{\Gamma(c-\gamma)}\int_0^1\sin^{2\gamma-1}\theta\cos^{2(c-\gamma)-1}\theta e^{-\frac{p}{\sin^2\theta\cos^2\theta}}\,{}_m\Psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{matrix}\middle|z\sin^2\theta\right]d\theta \quad (9) $$

For Wright generalized Hypergeometric function, we can easily obtain the following recurrence relations for $l=1,...,n$, as follows:

$$ {}_m\Psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{matrix}\middle|tz\right] $$

$$ =b_l\,{}_m\Psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n;\,j\neq l},(b_l+1,\beta_l)\end{matrix}\middle|tz\right]+\beta_l z\frac{d}{dz}\,{}_m\Psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n;\,j\neq l},(b_l+1,\beta_l)\end{matrix}\middle|tz\right] \quad (10) $$

using equation (10) in equation (6), we obtain recurrence relations for extended Wright generalized Hypergeometric function as follows:

Corollary 4. For extended Wright generalized Hypergeometric function, we obtain the following recurrence relations:

$$ {}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right]=b_l\,{}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n;\,j\neq l},(b_l+1,\beta_l),(c,1)\end{matrix}\middle|(z,p)\right] $$

$$ +\beta_l z\frac{d}{dz}\,{}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n;\,j\neq l},(b_l+1,\beta_l),(c,1)\end{matrix}\middle|(z,p)\right] \quad (11) $$

where $\mathrm{Re}(p)>0$; $\mathrm{Re}(c)>\mathrm{Re}(\gamma)>0$; $z,a_i,b_j\in C$; $\alpha_i,\beta_j\in R^+$ $(i=1,...,m;\, j,l=1,...,n)$.

Theorem 5. The Mellin Transform of extended Wright generalized Hypergeometric function is given as:

$$ M\left\{{}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right];s\right\} $$

$$ =\frac{\Gamma(s)\Gamma(c+s-\gamma)}{\Gamma(c-\gamma)}\,{}_{m+1}\Psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma+s,1)\\(b_j,\beta_j)_{1,n},(c+2s,1)\end{matrix}\middle|z\right] \quad (12) $$

where $\mathrm{Re}(s),\mathrm{Re}(p)>0$; $\mathrm{Re}(c)>\mathrm{Re}(\gamma)>0$; $z,a_i,b_j\in C$; $\alpha_i,\beta_j\in R^+$
$(i=1,...,m;\, j=1,...,n)$.

Proof: Taking the Mellin transform of extended Wright generalized Hypergeometric function and using equation (6), we obtain:

$$M\left\{{}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right];s\right\}$$

$$=\frac{1}{\Gamma(c-\gamma)}\int_0^\infty p^{s-1}\left[\int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}e^{-\frac{p}{t(1-t)}}\,{}_m\psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{matrix}\middle|tz\right]dt\right]dp \qquad (13)$$

Interchanging the order of integration and using the fact that $a^s\Gamma(s)=\int_0^\infty p^{s-1}e^{-ap}dp$, we obtain:

$$M\left\{{}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right];s\right\}$$

$$=\frac{\Gamma(s)}{\Gamma(c-\gamma)}\int_0^1 t^{\gamma+s-1}(1-t)^{c+s-\gamma-1}\,{}_m\psi_n\left[\begin{matrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{matrix}\middle|tz\right]dt \qquad (14)$$

using series representation of Wright generalized Hypergeometric function (2), after interchanging the order of integration and summation and using definition of Beta function, we obtain:

$$M\left\{{}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right];s\right\}$$

$$=\frac{\Gamma(s)\Gamma(c+s-\gamma)}{\Gamma(c-\gamma)}\sum_{k=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)\Gamma(\gamma+s+k)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)\Gamma(c+2s+k)}\frac{z^k}{k!} \qquad (15)$$

using definition of Wright Generalized Hypergeometric function, we obtain the required result.

Corollary 6. If we take $s=1$ in equation (12), we obtain:

$$\int_0^\infty {}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right]dp=(c-\gamma)\,{}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma+1,1)\\(b_j,\beta_j)_{1,n},(c+2,1)\end{matrix}\middle|z\right] \qquad (16)$$

Corollary 7. If we take Inverse Mellin transform on both sides of equation (12), we obtain:

$${}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{matrix}\middle|(z,p)\right]$$

$$=\frac{1}{\Gamma(c-\lambda)}\int_{\nu-i\infty}^{\nu+i\infty}\Gamma(s)\Gamma(c+s-\gamma)\,{}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,m},(\gamma+s,1)\\(b_j,\beta_j)_{1,n},(c+2s,1)\end{matrix}\middle|z\right]p^{-s}ds \qquad (17)$$

where $\nu>0$.

Theorem 8. For $\mathrm{Re}(p)>0$; $\mathrm{Re}(c)>\mathrm{Re}(\gamma)>0;\ z,a_i,b_j\in C;\ \alpha_i,\beta_j\in R^+$ $(i=1,...,m;\ j=1,...,n)$, we have:

$$D_z^{\lambda-\mu,p}\left\{z^{\lambda-1}{}_m\Psi_n\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{smallmatrix}\middle|z\right]\right\}=z^{\mu-1}{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right].$$

Proof: From the definition of extended Riemann-Liouville fractional derivative (5), we obtain:

$$D_z^{\lambda-\mu,p}\left\{z^{\lambda-1}{}_m\Psi_n\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{smallmatrix}\middle|z\right]\right\}=\frac{1}{\Gamma(\mu-\lambda)}\int_0^z t^{\lambda-1}{}_m\Psi_n\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m}\\(b_j,\beta_j)_{1,n}\end{smallmatrix}\middle|t\right](z-t)^{-\lambda+\mu-1}e^{\frac{-pz^2}{t(z-t)}}dt \tag{18}$$

using $t=uz$ in (18) and equation (6), we obtain the required result:

Theorem 9. For extended Wright generalized Hypergeometric function, we obtain the following derivative formula:

$$\frac{d^r}{dz^r}\left\{{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]\right\}={}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i+r\alpha_i,\alpha_i)_{1,m},(\gamma+r,1)\\(b_j+r\beta_j,\beta_j)_{1,n},(c+r,1)\end{smallmatrix}\middle|(z;p)\right],\quad r\in N \tag{19}$$

Proof. Using (2) in equation (6) and differentiating it with respect to *z*, we obtain

$$\frac{d}{dz}\left\{{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]\right\}$$

$$=\frac{1}{\Gamma(c-\gamma)}\int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}e^{-\frac{p}{t(1-t)}}\sum_{k=1}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{t^k z^{k-1}}{(k-1)!}dt$$

By taking $k-1=l$ and using (6), we obtain:

$$\frac{d}{dz}\left\{{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]\right\}={}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i+\alpha_i,\alpha_i)_{1,m},(\gamma+1,1)\\(b_j+\beta_j,\beta_j)_{1,n},(c+1,1)\end{smallmatrix}\middle|(z;p)\right]$$

Continuing this process r times, we obtain the required result.

Theorem 10. For extended Wright generalized Hypergeometric function, we obtain the following derivative formula:

$$\frac{d^r}{dp^r}\left\{{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]\right\}$$

$$=\frac{(-1)^r}{(c-\gamma-1)(c-\gamma-2)...(c-\gamma-r)}\,{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma-r,1)\\(b_j,\beta_j)_{1,n},(c-2r,1)\end{smallmatrix}\middle|(z;p)\right] \tag{20}$$

Proof: Differentiating equation (6) with respect to *p* and using (2), we obtain:

$$\frac{d}{dp}\left\{{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]\right\}=\frac{(-1)}{(c-\gamma-1)}\,{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma-1,1)\\(b_j,\beta_j)_{1,n},(c-2,1)\end{smallmatrix}\middle|(z;p)\right]$$

Continuing this process r times, we obtain the required result.

Theorem 11. For extended Wright generalized Hypergeometric function, we have:

$$e^{2p}{}_{m+1}\Psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]$$

$$=\frac{1}{\Gamma(c-\gamma)}\sum_{r,s,k=0}^{\infty}\frac{L_r(p)L_s(p)\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{z^k}{k!}B(r+k+\gamma+1,s+c-\gamma+1) \tag{21}$$

where $\mathrm{Re}(p)>0$; $\mathrm{Re}(c)>\mathrm{Re}(\gamma)>0$; $z,a_i,b_j\in C$; $\alpha_i,\beta_j\in R^+$ $(i=1,...,m;\, j=1,...,n)$.

Proof: We will use the known identity [10]

$$e^{\frac{-p}{t(1-t)}}=e^{-2p}\sum_{r,s=0}^{\infty}L_r(p)L_s(p)t^{r+1}(1-t)^{s+1}, \quad 0<t<1,$$

using above result and series representation of Wright generalized Hypergeometric function (2) in (6) and after interchanging the order of integration and summation, we obtain:

$${}_{m+1}\psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]$$

$$=\frac{e^{-2p}}{\Gamma(c-\gamma)}\sum_{r,s,k=0}^{\infty}L_r(p)L_s(p)\frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{z^k}{k!}\int_0^1 t^{r+k+\gamma+1-1}(1-t)^{c+s-\gamma+1-1}dt$$

Multiplying both sides of above equation by e^{2p} and using definition of Euler Beta function, we obtain required result.

Theorem 12. For extended Wright generalized Hypergeometric function, we have:

$$e^{\frac{3p}{2}}{}_{m+1}\psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]$$

$$=\frac{1}{(c-\gamma-1)}\sum_{r,k=0}^{\infty}\frac{L_r(p)\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{z^k}{k!}p^{\frac{r+k+\gamma-1}{2}}W_{\frac{\gamma-2c-r-k-1}{2},\frac{r+k+\gamma}{2}}(p) \tag{22}$$

Proof: Using $e^{\frac{-p}{t(1-t)}}=e^{\frac{-p}{1-t}}e^{\frac{-p}{t}}$ and the generating function of Laguerre polynomials, we can easily obtain:

$$e^{\frac{-p}{t(1-t)}}=e^{-p}e^{\frac{-p}{t}}(1-t)\sum_{r=0}^{\infty}L_r(p)t^r$$

using above result and series representation of Wright generalized Hypergeometric function (2) in (6) and after interchanging the order of integration and summation, we obtain:

$${}_{m+1}\psi_{n+1}\left[\begin{smallmatrix}(a_i,\alpha_i)_{1,m},(\gamma,1)\\(b_j,\beta_j)_{1,n},(c,1)\end{smallmatrix}\middle|(z;p)\right]=\frac{e^{-p}}{\Gamma(c-\gamma)}\sum_{r,k=0}^{\infty}\frac{L_r(p)\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{j=1}^{n}\Gamma(b_j+k\beta_j)}\frac{z^k}{k!}\int_0^1 t^{\gamma+r+k-1}(1-t)^{c-\gamma}e^{\frac{-p}{t}}dt \tag{23}$$

Finally, using Lemma 1, we obtain the required result.

3. Special case

If we take $m = n = 1, a_1 \to c, \alpha_1 \to 1, b_1 \to \beta$ and $\beta_1 \to \alpha$ in above theorems and corollaries, we will obtain the results obtained by Ozarslan and Yilmaz [9].

REFERENCES

1. A.A.Kilbas, M.Saigo and J.J.Trujillo, On the generalized Wright function, *Fract. Calc. Appl. Anal.*, 5 (2004) 437-460.
2. A.Erdelyi, W.Magnus, F.Oberhettinger and F.G.Tricomi, *Higher Transcendental Functions*, McGraw-Hill, New York, 1954.
3. E.M.Wright, On the coefficients of power series having exponential singularities, *J. London Math. Soc.*, 8 (1933) 71-79.
4. E.M.Wright, The asymptotic expansion of the generalized Hypergeometric function, *Proc. London Math. Soc.*, 10 (1935) 286–293.
5. E.M.Wright, The asymptotic expansion of the generalized hypergeometric function, *Proc. London Math. Soc.*, 2(46) (1940) 389-408.
6. E.M.Wright, The generalized Bessel function of order greater than one, *Quart. J. Math. Oxford series*, 2 (1940) 36-48.
7. M.A.Chaudhry and S.M.Zubair, *On a Class of Incomplete Gamma functions with Applications*, CRC Press, Boca Raton, 2002.
8. M.A.Chaudhry, A.Qadir, H.M.Srivastava and R.B.Paris, Extended Hypergeometric and Confluent Hypergeometric functions, *Appl. Math. Comput.*, 159 (2004) 589-602.
9. M.A.Ozarslan and B.Yilmaz, The extended Mittag – Leffler function and its properties, *Journal of Inequalities and Applications*, 85 (2014) 1-10.
10. M.A.Ozarslan and E.Ozergin, Some generating relations for extended Hypergeometric functions via generalized fractional derivative operator, *Math. Comput. Model*, 52 (2010) 1825-1833.
11. M.A.Ozarslan, Some remarks on extended Hypergeometric, extended Confluent Hypergeometric and extended Appell's functions, *J. Comput. Anal. Appl.*, 14(6) (2012) 1148-1153.