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Another Decomposition of Fuzzy Continuity

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Abstract. There are various types of generalizations of fuzzy continuous functions in the development of fuzzy topology. Recently some decompositions of fuzzy continuity have been obtained by various authors with the help of generalized fuzzy continuous functions in fuzzy topological spaces. In this paper we obtain decomposition of fuzzy continuity by using a new generalized fuzzy continuity in fuzzy topological spaces.

Keywords: Fuzzy gs-closed set, fuzzy g'''-closed set, fuzzy χ^* -set, fuzzy g''' continuous function, fuzzy χ^*-continuous function

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1. Introduction

In the classical paper [19] of 1965, Zadeh generalized the usual notion of a set and introduced the important and useful notion of fuzzy sets. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors have introduced various types of fuzzy continuity.

Various types of generalizations of fuzzy continuous functions were introduced and studied by various authors in the recent development of fuzzy topology. The decomposition of fuzzy continuity is one of many problems in fuzzy topology. Tong [18] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy precontinuity. Rajamani [11] obtained a decomposition of fuzzy continuity. In this paper, we obtain decomposition of fuzzy continuity in fuzzy topological spaces by using fuzzy g'''-continuity.

2. Preliminaries

Definition 2.1. [16, 19] If X is a set, then any function A: X→[0, 1] (from X to the closed unit interval [0, 1]) is called a fuzzy set in X.

Definition 2.2. [11] If X is a set, then A, B : X →[0,1] are fuzzy sets in X.

(i) The complement of a fuzzy set A, denoted by A', is defined by A' (x) = 1 – A(x), $\forall x \in X$.

(ii) Union of two fuzzy sets A and B, denoted by A∨B, is defined by (A∨B) (x) = max{A(x), B(x)}, $\forall x \in X$.

(iii) Intersection of two fuzzy sets A and B, denoted by A∧B, is defined by (A∧B) (x) = min {A(x), B(x)}, $\forall x \in X$.

Definition 2.3. [16, 19] Let f: X→Y be a function from a set X into a set Y. Let A be a fuzzy subset in X and B be a fuzzy subset in Y. Then the Zadeh's functions f(A) and $f^{-1}(B)$ are defined by

(i) f(A) is a fuzzy subset of Y where

$$f(A)(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} A(z) & if \ \ f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

for each $y \in Y$.

(ii) $f^{-1}(B)$ is a fuzzy subset of X where $f^{-1}(B)(x) = B(f(x))$, for each $x \in X$.

Definition 2.4. [5, 16] Let X be a set and τ be a family of fuzzy sets in X. Then τ is called a fuzzy topology if τ satisfies the following conditions:

(i) $0, 1 \in \tau$.

(ii) If $A_i \in \tau$, $i \in I$ then $\cup_{i \in I} A_i \in \tau$ or $\vee_{i \in I} A_i \in \tau$.

(iii) If A, B $\in \tau$ then $A \cap B \in \tau$ or $A \wedge B \in \tau$.

The pair (X, τ) is called a fuzzy topological space (briefly fts.). The elements of τ are called fuzzy open sets. Complements of fuzzy open sets are called fuzzy closed sets.

Definition 2.5 .[16] Let A be a fuzzy set in a fts (X, τ). Then

(i) the closure of A, denoted by cl(A), is defined by

cl(A)= $\wedge$ { F : A $\leq$ F and F is a fuzzy closed };

(ii) the interior of A, denoted by int(A), is defined by

int(A)= $\vee${G : G $\leq$ A and G is a fuzzy open}.

Definition 2.6. A subset A of a fts (X, τ) is called:

(i) fuzzy semi-open set [1] if A ≤ cl(int(A));

(ii) fuzzy preopen set [4] if A ≤ int(cl(A));

(iii) fuzzy α -open set [4] if $A \le int(cl(int(A)))$.

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

For a subset A of a fuzzy topological space X, the fuzzy α-closure (resp. fuzzy semi-closure, fuzzy pre-closure) of A, denoted by αcl(A) (resp. scl(A), pcl(A)), is the intersection of all fuzzy α-closed (resp. fuzzy semi-closed, fuzzy preclosed) subsets of X containing A. Dually, the fuzzy α-interior (resp. fuzzy semi-interior, fuzzy pre-interior) of A, denoted by αint(A) (resp. sint(A),
pint(A)), is the union of all fuzzy α-open (resp. fuzzy semi-open, fuzzy preopen) subsets of X contained in A.

Definition 2.7. [5, 11] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy continuous if $f^{-1}(\lambda)$ is fuzzy open in X for each fuzzy open set λ in Y.

Definition 2.8. Let (X, τ) be a fuzzy topological space. A fuzzy set λ in X is called:

(i) a fuzzy generalized-semi closed (briefly fuzzy gs-closed) set [3] if $scl(\lambda) \le \mu$ whenever $\lambda \le \mu$ and μ is fuzzy open in (X, τ). The complement of fuzzy gs-closed set is called fuzzy gs-open set;

(ii) a fuzzy g'''-closed set [9] if $cl(\lambda) \le \mu$ whenever $\lambda \le \mu$ and μ is a fuzzy gs-open in (X, τ). The complement of fuzzy g''' -closed set is called fuzzy g''' -open.

Definition 2.9. [10] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy g''' -continuous if for each fuzzy closed set λ of Y, $f^{-1}(\lambda)$ is fuzzy g'''-closed in X.

Proposition 2.10. [9] Every fuzzy closed set is fuzzy g''' -closed but not conversely.

Example 2.11. [9] Let X= {a, b} with $\tau = \{0_X, A, 1_X\}$ where A is a fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=1 is a fuzzy g''' -closed set but not fuzzy closed in (X, τ).

Proposition 2.12. [10] Every fuzzy continuous function is fuzzy g''' -continuous but not conversely.

Example 2.13. [10] Let X = Y = {a, b} with $\tau = \{0_X, A, 1_X\}$ where A is a fuzzy set in X defined by A(a)=1, A(b)=0 and $\sigma = \{0_Y, B, 1_Y\}$ where B is fuzzy set in Y defined by B(a)=0.5, B(b)=0. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let $f : (X, \tau) \to (Y, \sigma)$ be the fuzzy identity function. Clearly f is fuzzy g''' -continuous but not fuzzy continuous.

3. Decomposition of fuzzy continuity

We introduce the following definition.

Definition 3.1. A subset λ in a fuzzy topological space (X, τ) is called fuzzy χ^*-set if $\lambda = \alpha \wedge \beta$ where α is fuzzy gs-open and β is fuzzy closed in X.

Example 3.2. Let X= {a, b} with τ ={0_X, β_1, 1_X} and β_1 and β_2 are fuzzy sets in X defined by β_1(a)=0.6, β_1(b)=0.5; β_2(a)=0.4, β_2(b)=0.4. Then (X, τ) is a fuzzy topological space. Clearly β_2 is fuzzy χ^*-set.

Remark 3.3. Every fuzzy closed set is fuzzy χ^*-set but not conversely.

Example 3.4. Let X= {a, b} with τ={0_X, β_1, 1_X} and β_1 and β_2 are fuzzy sets in X defined by β_1(a)=0.6, β_1(b)=0.5; β_2(a)=0.4, β_2(b)=0.4. Then (X, τ) is a fuzzy topological space. Clearly β_2 is fuzzy χ^*-set but not fuzzy closed in (X, τ).

Remark 3.5. Fuzzy g'''-closed sets and fuzzy χ^*-sets are independent of each other.

Example 3.6. Let X= {a, b} with τ ={0_X, A, 1_X} where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=1 is a fuzzy g'''-closed set but not fuzzy χ^*-set in (X, τ).

Example 3.7. Let X={a, b} with τ ={0_X, β_1, 1_X} and β_1 and β_2 are fuzzy sets in X defined by β_1(a)= 0.6, β_1(b)= 0.5; β_2(a)= 0.4, β_2(b)= 0.4. Then (X, τ) is a fuzzy topological space. Clearly β_2 is fuzzy χ^*-set but not fuzzy g'''-closed set in (X, τ).

Proposition 3.8. Let (X, τ) be a fuzzy topological space. Then a fuzzy subset λ of (X, τ) is fuzzy closed set if and only if it is both fuzzy g'''-closed set and fuzzy χ^*-set.

Proof: Necessity is trivial. To prove the sufficiency, assume that λ is both fuzzy g'''-closed and fuzzy χ^*-set. Then $\lambda = \alpha \wedge \beta$, where α is fuzzy gs-open and β is fuzzy closed in (X, τ). Therefore, $\lambda \leq \alpha$ and $\lambda \leq \beta$ and so by hypothesis, cl(λ) $\leq \alpha$ and cl(λ) $\leq \beta$. Thus cl(λ) $\leq \alpha \wedge \beta = \lambda$ and hence cl(λ)= λ i.e., λ is fuzzy closed in (X, τ).

Definition 3.9. A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be fuzzy χ^*-continuous if for each fuzzy closed set μ of (Y, σ), $f^{-1}(\mu)$ is a fuzzy χ^*-set in (X, τ).

Example 3.10. Let X=Y={a, b} with τ ={0_X, α_1, 1_X} where α_1 is a fuzzy set in X defined by α_1 (a)=0.4, α_1 (b)=0.5 and σ={0_Y, α_2, 1_Y} where α_2 is a fuzzy set in Y defined by α_2 (a)=0.6, α_2 (b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)$\rightarrow$ (Y, σ) be the fuzzy identity function. Clearly f is fuzzy χ^*continuous.

Proposition 3.11. Every fuzzy continuous function is fuzzy χ^*-continuous but not conversely.

Example 3.12. Let X=Y={a, b} with τ ={0_X, α_1, 1_X} where α_1 is a fuzzy set in X defined α_1 (a)=0.4, α_1 (b)=0.5 and σ ={0_Y, α_2, 1_Y} where α_2 is a fuzzy set in Y defined by α_2 (a)=0.6, α_2 (b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→ (Y, σ) be the fuzzy identity function. Clearly f is fuzzy χ^*-continuous but not fuzzy continuous.

Remark 3.13. Fuzzy g'''-continuity and fuzzy χ^* - continuity are independent of each other.

Example 3.14. Let X=Y={a, b} with τ ={0_X, A, 1_X} where A is a fuzzy set in X defined by A(a)=1, A(b)=0 and σ={0_Y, B, 1_Y} where B is a fuzzy set in Y defined by B(a)=0.5, B(b)=0. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→ (Y, σ) be the fuzzy identity function. Clearly f is fuzzy g'''-continuous but not fuzzy χ^* - continuous.

Example 3.15. Let X=Y={a, b} with τ ={0_X, α_1, 1_X} where α_1 is a fuzzy set in X defined by α_1 (a)=0.4, α_1 (b)=0.5 and σ={0_Y, α_2, 1_Y} where α_2 is a fuzzy set in Y defined by α_2 (a)=0.6, α_2 (b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ)→ (Y, σ) be the fuzzy identity function. Clearly f is fuzzy χ^*-continuous but not fuzzy g''' - continuous.

Theorem 3.16. A function f: (X, τ) → (Y, σ) is fuzzy continuous if and only if it is both fuzzy g''' - continuous and fuzzy χ^* -continuous.

Proof: Assume that f is fuzzy continuous. Then by Proposition 2.12 and Proposition 3.11, f is both fuzzy g'''-continuous and fuzzy χ^* -continuous.

Conversely, assume that f is both fuzzy g'''-continuous and fuzzy χ^* -continuous. Let V be a fuzzy closed subset of (Y, σ). Then $f^{-1}(V)$ is both fuzzy g'''-closed set and fuzzy χ^*-set. By Proposition 3.8, $f^{-1}(V)$ is a fuzzy closed set in (X, τ) and so f is fuzzy continuous.

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