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Eccentric Connectivity Index and Detour Eccentric Connectivity Index – A Comparison

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Abstract. In this paper we define detour eccentric connectivity index and detour connective eccentric index of graphs. We compare the properties of both these indices. The values of these indices for some common graphs are calculated. A relation connecting average eccentricity and these indices is also derived.

Keywords: detour distance in graphs, eccentric connectivity index, connective eccentric index, detour eccentric connectivity index, detour connective eccentric index

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1.Introduction

In theoretical chemistry molecular structure descriptor or topological indices, are used to compute properties of chemical compounds. In this paper, we discuss two topological indices namely, eccentric connectivity index and connective eccentric index [4,5,8]. We define these concepts analogously using the detour distance in graphs. We study the differences between both these indices using some examples. We also derive the relation between these indices and the average eccentricity. Throughout this paper, graph means simple connected graph. A graph is a pair G:(V,E) where V is a finite non-empty set and E isa symmetric binary relation on V . The following definitions are taken from the book by Harary [6]. In a graph, $\boldsymbol{G = (V, E)}$, V *(or* $V(G)$*)* and E*(or* $E(G)$*)* denote the vertex set and the edge set of G, respectively. A graph $\boldsymbol{G = (V, E)}$ is trivial, if it has only one vertex, i.e. $|V(G)| = 1$; otherwise G is nontrivial. The number of edges incident with a vertex u is called degree of a vertex and is denoted by $deg_G(u)$.The complete graph on $\boldsymbol{n}$ vertices is denoted by $\boldsymbol{K_n}$ and it is that graph in which there exist an edge between each pair of vertices u, v in G. For a set $S \subseteq V(G)$, $G[S]$ is the subgraph induced by S. A connected acyclic graph is called a tree. A graph G is said to be regular, if every vertex of G has the same degree. If this degree is equal to r, then G is r − regular or regular of degree r. If G is a graph and u, v are two vertices of G, length of shortest path between u and v in G is called the geodesic distance between u and v and is denoted by $d(u, v)$. The eccentricity of the vertex v is denoted by $e(v)$ and is defined as $e(v) = Max[d(u,v)/u\in$

G].If a graph G is disconnected, then $e(v) = \infty$ for all vertices $v \in G$. Radius of G is defined as r(G) = [$e_g(v)/v \in G$] Diameter of G is defined as $d(G) = Max[e(v)/v \in G]$ If $e(v) = r(G)$ then v is called a central vertex of G and the graph induced by all central vertices of *G* is called Centre of *G* and is denoted by *C(G).* A vertex *v* of *G* is called an eccentric vertex of *G,* if there exists a vertex u in G such that $d(u, v) = e(u)$. This means that if the vertex v is farthest from another vertex u then v is an eccentric vertex of u (as well as G) and is denoted by $u^* = v$. If $G = C(G)$ then it is called self centered graph.

2. Detour distance, eccentric connectivity index, connective eccentric index

The concept of Detour distance and related properties are discussed in [2]. If *u, v* are two vertices in the graph *G* the detour distance between these vertices denoted by *D(u, v)* is the length of a longest *u-v* path in *G* [2]. Note that there exist graphs in which $D(u, v) = d(u, v)\ \forall u, v \in V$. The detour eccentricity of the vertex v is denoted by $e_D(v)$ and is defined as $e_D(v) = Max\ [D(u,v)/\ u \in G]$. If a graph G is disconnected, then $e_D(v) = \infty$ for all vertices $v \in G$. Detour radius of G is defined as $r_D = Min\ [e_D(v)/\ v \in G$. Detour diameter of *G* is defined as$d_D(G) = Max\ [e_D(v)/\ v \in G$.If $e_D(v) = r_D(G)$ then *v* is called a Detour central vertex of *G* and the sub graph induced by all Detour central vertices of G is called Detour centre of G and is denoted by $C_D(G)$. A vertex v of *G* is called a Detour eccentric vertex of *u* (and that of G) if there exists a vertex *u* in *G* such that $D(u,v) = e_D(u)$ and is denoted by u= *v*. If $G = C_D(G)$ then it is called a detour self -centered graph. The eccentric connectivity index of the molecular graph G, $\xi^C(G)$, was proposed by Sharma, Goswami and Madan [8]. It is defined as $\xi^c(G) = \sum_{v \in V} \deg(v)\, e(v)$, where *deg(v)* denotes the degree of the vertex *v* in *G* and *e(v)* is the geodesic eccentricity of v. A broom graph $B_{n,d}$ is a specific kind of graph on n vertices, having a path P with d vertices and n−d pendant vertices, all of these being adjacent to either the origin u or the terminus v of the path Sriram and Ranganayakulu [1] derived the eccentric connectivity index and domination number of Broom graph $B_{n,d}$.

The connective eccentric index of a graph was defined by Gupta, Singh and Madan [4,5] as $C^\xi(G) = \sum_{v \in V} \frac{\deg(v)}{e(v)}$. The average eccentricity [3] of a graph is defined as $avec(G) = \frac{1}{n}[e(v_1) + e(v_2) + \cdots e(v_n)]$wherev$_1$, v$_2$,…. v$_n$areverticesof*G*. The thorny graphs of G are obtained by attaching a number of thorns i.e., degree one vertices to each vertex of G. De and Pal [7] have derived explicit expressions for the connective eccentricity index of some classes of thorny graphs.Similarly we have the average detour eccentricity of a graph as $Davec(G) = \frac{1}{n}[e_D(v_1) + e_D(v_2) + \cdots e_D(v_n)]$.We discuss some results concerned with the eccentric connectivity index, average eccentricity and connective eccentric index of graphs. In Section 3 we define the concepts of detour eccentric connectivity index and detour connective eccentric index of graphs.

Proposition 1. Let G be a connected graph on n vertices, n ≥ 2. Then $avec(G) < \xi^c(G)$.
Proof:

$$\text{avec(G)} = \frac{1}{n}\sum_{i=1}^{n} e(v_i) < \sum_{i=1}^{n} e(v_i) \le \sum_{i=1}^{n} \deg(v_i)\, e(v_i) = \xi^c(G)$$

Therefore $avec(G) < \xi^c(G)$.

Corollary 1. If G is self-centered then, $r(G) < \xi^c(G)$.

Proof: If G is self-centered, then $avec(G) = r(G)$. So by Proposition 1 we get $r(G) < \xi^c(G)$.

Proposition 2. Let G be a connected graph with $|V| = n$ and G is k- *regular,* then $\xi^c(G) = nk\; avec(G)$.

Proof:

$$\xi^c(G) = \sum_{i=1}^{n} \deg(v_i)\, e(v_i) = \sum_{i=1}^{n} k e(v_i)$$

$$= k \sum_{i=1}^{n} e(v_i) = \frac{nk}{n} \sum_{i=1}^{n} e(v_i) = nk\; avec(G).$$

Proposition 3. Let $G: (V,E)$ be a self- centered graph with $|V| = n$ with $e(v_i) = m\; \forall i$, then $\xi^c(G) = 2m|E|$.

Proof:

$$\xi^c(G) = \sum_{i=1}^{n} \deg(v_i)\, e(v_i) = \sum_{i=1}^{n} \mathrm{m.}\deg(v_i) = m \sum_{i=1}^{n} \deg(v_i) = m.2|E|$$

Therefore, $\xi^c(G) = 2m|E|$.

Proposition 4. Let $G: (V,E)$ be a self- centered graph with $|V| = n$ and $e(v_i) = m\; \forall i$, then $C^{\xi}(G) = \frac{2|E|}{m}$.

Proof:

$$\mathrm{C^{\xi}(G)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \frac{\deg(v_i)}{e(v_i)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \frac{\deg(v_i)}{m} = \frac{1}{m} \sum_{\mathrm{i=1}}^{\mathrm{n}} \deg(v_i) = \frac{2|E|}{m}.$$

Remark 1. From Proposition 3, if G is self – centered with $|V| = n$, we have $\xi^c(G) = 2m|E|$ where $m = e(v_i) \forall i$.

So $\xi^c(G) = 2m|E| = m.m.C^{\xi}(G) = m^2 C^{\xi}(G)$.

Proposition 5.

$C^{\xi}(K_n) = n(n-1)$, $C^{\xi}(C_{2n}) = 4, C^{\xi}(C_{2n-1}) = 2\left(2 + \frac{1}{n}\right), C^{\xi}(K_{m,n}) = mn \; and\; C^{\xi}(S_n) = \frac{3n-1}{2}$.

Proof:

$$C^{\xi}(K_n) = \sum_{i=1}^{n} \frac{\deg(v_i)}{e(v_i)} = \sum_{i=1}^{n} \frac{\mathrm{n}-1}{1} = n(n-1)$$

$$C^{\xi}(C_{2n}) = \sum_{i=1}^{2n} \frac{\deg(v_i)}{e(v_i)} = \sum_{i=1}^{2n} \frac{2}{n} = \frac{2.2n}{n} = 4$$

$$C^{\xi}(C_{2n+1}) = \sum_{i=1}^{2n+1} \frac{\deg(v_i)}{e(v_i)} = \sum_{i=1}^{2n+1} \frac{2}{n} = \frac{2.(2n+1)}{n} = 2\left(2 + \frac{1}{n}\right)$$

Now $K_{m,n}$ is self- centered and $e(v_i) = 2$ ∀i. Using proposition 4 we get,
$C^{\xi}(K_{m,n}) = \frac{2|E|}{e(v_i)} = \frac{2mn}{2} = mn.$

The central vertex of S_n has eccentricity 1 and all the remaining vertices are having eccentricity 2. Thus

$$C^{\xi}(S_n) = \sum_{i=1}^{n} \frac{\deg(v_i)}{e(v_i)} = \frac{n}{1} + \sum_{i=1}^{n-1} \frac{1}{2} = n + \frac{n-1}{2} = \frac{3n-1}{2}$$

3. Detour eccentric connectivity index and detour connective eccentric index

The concept of detour distance and related properties are discussed in [8]. If u,v are two vertices in the graph G, the detour distance between these vertices denoted by D(u,v) is the length of a longest u-v path in G [8]. We have defined the average detour eccentricity of a graph as $Davec(G) = \frac{1}{n}[e_D(v_1) + e_D(v_2) + \cdots e_D(v_n)]$. We now extend the definition of eccentric connectivity index and connective eccentric index using detour distance. The relations between average detour eccentricity and these indices are also studied.

Definition 1. The detour eccentric connectivity index of a graph G, $D\xi^c(G)$ is defined as $D\xi^c(G) = \sum_{v\in V} \deg(v)\, e_D(v)$.

Definition 2. The detour connective eccentric index of a graph is defined as $DC^{\xi}(G) = \sum_{v\in V} \frac{\deg(v)}{e_D(v)}$.

In the below results we discuss some properties of these indices.

Proposition 6. If G is a connected graph on n vertices then $Davec(G) < D\xi^c(G)$. Also $\xi^c(G) \le D\xi^c(G)$, $C^{\xi}(G) \ge DC^{\xi}(G)$.

Proof:

For a connected graph G, $e(v) \le e_D(v) \forall v \in V$. Hence

$$\text{Davec(G)} = \frac{1}{n}\sum_{i=1}^{n} e_D(v_i) < \sum_{i=1}^{n} e_D(v_i) \le \sum_{i=1}^{n} \deg(v_i)\, e_D(v_i) = D\xi^c(G)$$

Therefore $Davec(G) < D\xi^c(G)$

$$\xi^c(G) = \sum_{i=1}^{n} \deg(v_i)\, e(v_i) \le \sum_{i=1}^{n} \deg(v_i)\, e_D(v_i)$$

Therefore, $\xi^c(G) \le D\xi^c(G)$.

Now $C^{\xi}(G) = \sum_{i=1}^{n} \frac{\deg(v_i)}{e(v_i)} \geq \sum_{i=1}^{n} \frac{\deg(v_i)}{e_D(v_i)}$

Therefore, $C^{\xi}(G) \geq DC^{\xi}(G)$.

Next we investigate the values of detour eccentric connectivity index and detour connective eccentric index of some common graphs. The following propositions give the values of these indices for some common graphs.

Proposition 7.

1. $D\xi^c(K_n) = n(n-1)^2$ and $DC^{\xi}(K_n) = n$
2. $D\xi^c(C_n) = 2n(n-1)$ and $DC^{\xi}(C_n) = \frac{2n}{n-1}$
3. $D\xi^c(S_n) = \xi^c(S_n) = 3(n-1)$ and $DC^{\xi}(S_n) = C^{\xi}(S_n) = \frac{3n-1}{2}$

Proof:

1. $D\xi^c(K_n) = \sum_{i=1}^{n} \deg(v_i)\, e_D(v_i) = \sum_{i=1}^{n} (n-1)(n-1) = n(n-1)^2$
$$DC^{\xi}(K_n) = \sum_{i=1}^{n} \frac{\deg(v_i)}{e_D(v_i)} = \sum_{i=1}^{n} \frac{n-1}{n-1} = n$$
2. $D\xi^c(C_n) = \sum_{i=1}^{n} \deg(v_i)\, e_D(v_i) = \sum_{i=1}^{n} 2(n-1) = 2n(n-1)$
$$DC^{\xi}(C_n) = \sum_{i=1}^{n} \frac{\deg(v_i)}{e_D(v_i)} = \frac{2n}{n-1}$$
3. The star graph is tree. Both geodesic and detour distance are same in trees. Hence from proposition 5 we get $D\xi^c(S_n) = \xi^c(S_n) = 3(n-1)$ and $DC^{\xi}(S_n) = C^{\xi}(S_n) = \frac{3n-1}{2}$.

Remark 2. Proposition 3 and Proposition 4 also hold good with respect to detour distance. Hence these results will be true for detour connectivity index and detour connective eccentric index. Hence if G is self centered then

$D\xi^c(G) = 2m|E|$ and $DC^{\xi}(G) = \frac{2|E|}{m}$ where $e(v_i) = m \;\forall\; v_i \in G$.

Proposition 8. The detour eccentric connectivity index and detour connective eccentric index of a wheel graph on n vertices W_n are $2(n-1)(2n-1)$ and $\frac{2(2n-1)}{n-1}$ respectively.

Proof: The wheel graph W_n is detour self centered and has detour radius n-1. The number of edges in W_n is 2n-1. Using remark 2 we get

$D\xi^c(W_n) = 2m|E| = (n-1).2.(2n-1) = 2(n-1)(2n-1)$
$$DC^{\xi}(W_n) = \frac{2|E|}{m} = \frac{2(2n-1)}{n-1}$$

Proposition 9. The detour eccentric connectivity index of a complete bi partite graph $K_{m,n}$ is $D\xi^c(K_{m,n}) = 2n^2(2n-1), if\ m = n\ = mn[2(m+n)-3], if\ m \neq n.$

Proof: Consider the complete bi partite graph $K_{m,n}$. Let the bipartition be $S = \{u_1, u_2, \ldots, u_m\}\ and\ T = \{v_1, v_2, \ldots, v_n\}$.

Case* 1: *m* = *n

$$D\xi^{c}(K_{m,n}) = \sum_{i=1}^{n} \deg(u_i)\, e_D(u_i) + \sum_{j=1}^{n} \deg(v_j)\, e_D(v_j)$$

$$= \sum_{i=1}^{n} n(n+n-1) + \sum_{j=1}^{n} n(n+n-1) = \sum_{i=1}^{n} n(2n-1) + \sum_{j=1}^{n} n(2n-1)$$

$$= 2n^2(2n-1)$$

***Case* 2: *m* ≠ *n* (*say m* < *n*)**

As m < n, $e_D(u_i) = m+n-2$ $and e_D(v_i) = m+n-1$

$$D\xi^{c}(K_{m,n}) = \sum_{i=1}^{m} \deg(u_i)\, e_D(u_i) + \sum_{j=1}^{n} \deg(v_j)\, e_D(v_j)$$

$$= \sum_{i=1}^{m} n(m+n-2) + \sum_{j=1}^{n} m(n+n-1)$$

$$= mn(m+n-2) + mn(m+n-1) = \mathrm{mn}[2(\mathrm{m}+\mathrm{n})-3]$$

Proposition 10.
The detour connective eccentric index of a complete bi partite graph $K_{m,n}$ is

$$DC^{\xi}(K_{m,n}) = \frac{2n^2}{2n-1}\ , if\ \ m = n$$

$$= mn\, \frac{2(m+n)-3}{(m+n-2)(m+n-1)}\ , if\ m \neq n$$

Proof:
Consider the complete bi partite graph $K_{m,n}$. Let the bipartition be $S = \{u_1, u_2, \dots, u_m\}$ and $T = \{v_1, v_2, \dots, v_n\}$.

Case* 1: *m* = *n

$$\mathrm{D}C^{\xi}(K_{m,n}) = \sum_{i=1}^{n} \frac{\deg(u_i)}{e_D(u_i)} + \sum_{j=1}^{n} \frac{\deg(v_i)}{e_D(v_i)} = \sum_{i=1}^{n} \frac{\mathrm{n}}{n+n-1} + \sum_{j=1}^{n} \frac{\mathrm{n}}{n+n-1}$$

$$= \sum_{i=1}^{n} \frac{\mathrm{n}}{2n-1} + \sum_{j=1}^{n} \frac{\mathrm{n}}{2n-1} = \frac{2n^2}{2n-1}$$

***Case* 2: *m* ≠ *n* (*say m* < *n*)**

As m < n, $e_D(u_i) = m+n-2$ and $e_D(v_i) = m+n-1$

$$\mathrm{D}\xi^{\mathrm{c}}(\mathrm{K}_{\mathrm{m,n}}) = \sum_{\mathrm{i}=1}^{\mathrm{m}} \frac{\deg(\mathrm{u_i})}{\mathrm{e_D}(\mathrm{u_i})} + \sum_{\mathrm{j}=1}^{\mathrm{n}} \frac{\deg(\mathrm{v_i})}{\mathrm{e_D}(\mathrm{v_i})}$$

$$= \sum_{i=1}^{m} \frac{n}{(m+n-2)} + \sum_{j=1}^{n} \frac{m}{(m+n-1)}$$
$$= \frac{mn}{m+n-2} + \frac{mn}{m+n-1}$$
$$= \mathrm{m}n \frac{2(m+n)-3}{(m+n-2)(m+n-1)}$$

4. Conclusion

This chapter has introduced new concepts like detour eccentric connectivity index and detour connective eccentric index of a graph. The values of these indices are obtained for some class of graphs. We hope that the new concepts introduced in this chapter will be useful in further classification of chemical compounds according to their molecular structure.

REFERENCES

1. A.Sriram, D.Ranganayakulu,I.Venkat andK.G.Subramanian,On eccentric graphs of broom graphs, *Annals of Pure and Applied Mathematics,* 5(2) (2014) 146–152.
2. Chartrand and P.Zhang, Distance in graphs - taking the long view, *AKCE Journal of Graphs and Combinatorics,* 1 (2004) 1–13.
3. P.Dankelmann, W.Goddard and C.S.Swart, The average eccentricity of a graph and its subgraphs, *UtilitasMathematica,* 65 (2004) 41–51.
4. M.Ghorbani,Connective eccentric index of fullerenes, *Journal of MathematicalNanoscience,*1 (2011) 43–50.
5. S.Gupta, M.Singh and A.K.Madan, Connective eccentricity index: A novel topological descriptor for predicting biological activity, *Journal of Molecular Graphics and Modelling,* 18 (2000) 18–25.
6. F.Harary, *Graph Theory,* Addison Wesley, 1969.
7. N.De, S.M.A.Nayeem and A.Pal, Connective eccentric index of some Thorny graphs, *Annals of Pure and Applied Mathematics,*7(1) (2014) 59–64.
8. V.Sharma,R.Goswami and A.K.Madan, Eccentric connectivity index: A novel highly discriminating topological descriptor for structure property and structure activity studies, *Journal of Chemical Informatics and Computer Science,* 37 (1997) 273–282.