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Regular Weakly Generalized Homoeomorphism in Intuitionistic Fuzzy Topological Space

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Abstract. The purpose of this paper, is to introduce and study the concepts of intuitionistic fuzzy regular weakly generalized homeomorphism and intuitionistic fuzzy regular weakly generalized i* homeomorphism in intuitionistic fuzzy topological space. Some of their properties are explored.

***Keywords*:** Intuitionistic fuzzy topology, Intuitionistic fuzzy regular weakly generalized continuous mapping, intuitionistic fuzzy regular weakly generalized homeomorphism, regular weakly generalized i* homeomorphism.

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1. Introduction

Fuzzy set (FS) was proposed by Zadeh [17] in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which appeals more accurate to uncertainty quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy regular weakly generalized homeomorphism and intuitionistic fuzzy regular weakly generalized i* homeomorphism in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy regular weakly generalized homeomorphism and establish the relationships with other classes of early defined forms of intuitionistic fuzzy homeomorphisms.

2. Preliminaries

Definition 2.1. [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x)$: $X \to [0, 1]$ and $\nu_A(x)$: $X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2. [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$. Then
(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3. [3] An *intuitionistic fuzzy topology* (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:
(a) $0_\sim, 1_\sim \in \tau$
(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4. [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by
$\mathrm{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$
$\mathrm{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $\mathrm{cl}(A^c) = (\mathrm{int}(A))^c$ and $\mathrm{int}(A^c) = (\mathrm{cl}(A))^c$ [16].

Definition 2.5. An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ in an IFTS (X, τ) is said to be an
(a) [4] *intuitionistic fuzzy semi closed set* (IFSCS in short) if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$
(b) [4] *intuitionistic fuzzy α-closed set* (IFαCS in short) if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$
(c) [4] *intuitionistic fuzzy pre-closed set* (IFPCS in short) if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$
(d) [4] *intuitionistic fuzzy regular closed set* (IFRCS in short) if $\mathrm{cl}(\mathrm{int}(A)) = A$
(e) [15] *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
(f) [12] *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if $\mathrm{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS

(g) [10] *intuitionistic fuzzy α generalized closed set* (IFαGCS in short) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IαFGOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6. [7] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy regular weakly generalized closed set* (IFRWGCS in short) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

The family of all IFRWGCSs of an IFTS (X, τ) is denoted by IFRWGCS(X).

Definition 2.7. [7] An IFS A is said to be an *intuitionistic fuzzy regular weakly generalized open set* (IFRWGOS in short) in (X, τ) if the complement A^c is an IFRWGCS in X.

The family of all IFRWGOSs of an IFTS (X, τ) is denoted by IFRWGO(X).

Result 2.8. [7] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFRWGCS but the converses need not be true in general.

Definition 2.9. [8] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by

$rwgint(A) = \cup \{ G / G$ is an IFRWGOS in X and $G \subseteq A \}$

$rwgcl(A) = \cap \{ K / K$ is an IFRWGCS in X and $A \subseteq K \}$.

Definition 2.10. [3] Let f be a mapping from an IFS X to an IFS Y. If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the pre-image of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle / x \in X\}$.

If $A = \{\langle x, \lambda_A(x), \nu_A(x) \rangle / x \in X\}$ is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by $f(A) = \{\langle y, f(\lambda_A(y)), f\_(\nu_A(y)) \rangle / y \in Y\}$ where $f\_(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11. [9] A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy regular weakly generalized continuous* (IFRWG continuous in short) if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.12. [8] A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an *intuitionistic fuzzy regular weakly generalized irresolute* (IFRWG irresolute in short) if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFRWGCS B of (Y, σ).

Definition 2.13. A mapping f: (X, τ) → (Y, σ) from an IFTS (X, τ) into an IFTS (Y,σ) is said to be an
(a) [13] intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X.
(b) [4] intuitionistic fuzzy semi closed mapping (IFSCM for short) if f(A) is an IFSCS in Y for every IFCS A in X.
(c) [4] intuitionistic fuzzy pre-closed mapping (IFPCM for short) if f(A) is an IFPCS in Y for every IFCS A in X.
(d) [4] intuitionistic fuzzy α-closed mapping (IFαCM for short) if f(A) is an IFαCS in Y for every IFCS A in X.
(e) [11] intuitionistic fuzzy α-generalized closed mapping (IFαGCM for short) if f(A) is an IFαGCS in Y for every IFCS A in X.
(f) [16] intuitionistic fuzzy pre regular closed mapping (IFPRCM for short) if f(A) is an IFPCS in Y for every IFRCS A in X.

Definition 2.14. [5] A subset A of a space (X,τ) is called a r* g*-closed set if rcl(A) ⊆ U whenever A⊆ U and U is g-open.

Definition 2.15. [6] A subset A of a space (X,τ) is called a weakly *g-closed set (briefly w*g closed) if cl(int(A)) ⊆ U whenever A ⊆ U and U is $\hat{g}$ open.

Definition 2.16. [7] An IFTS (X, τ) is said to be an intuitionistic fuzzy $IF_{rw}T_{1/2}$ space if every IFRWGCS in X is an IFCS in X.

Definition 2.17. [7] An IFTS (X, τ) is said to be an intuitionistic fuzzy $IF_{rwg}T_{1/2}$ space if every IFRWGCS in X is an IFPCS in X.

3. Intuitionistic fuzzy regular weakly generalized homeomorphism

Definition 3.1. A bijective mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy regular weakly generalized homeomorphism (IFRWG homeomorphism in short) if f and f^{-1} are IFRWG continuous mappings.

Example 3.2. Let X = {a, b, c}, Y = {u, v, w} and G_1 = <x, (0.2/a, 0.3/b, 0.3/c), (0.5/u, 0.6/v, 0.7/w)>, G_2 = <y, (0.9/a, 0.8/b, 0.9/c) , (0.1/u, 0.1/v, 0.1/w)>. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Consider a bijective mapping f: (X, τ) → (Y, σ) defined as f(a)= u f(b) = v and f(c) = w. Then f and f^{-1} are IFRWG continuous mappings. Hence f is an IFRWG homeomorphism.

Theorem 3.3. Every IF homeomorphism is an IFRWG homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF homeomorphism. Then f and f^{-1} are IF continuous mappings. This implies f and f^{-1} are IFRWG continuous mappings. Hence f is an IFRWG homeomorphism.

Example 3.4. Let X = {a, b, c}, Y = {u, v, w} and G_1 = <x, (0.2/a, 0.3/b, 0.4/c), (0.7/u, 0.7/v, 0.6/w)>, G_2 = <y, (0.9/a, 0.6/b, 0.7/c), (0.1/u, 0.3/v, 0.3/w)>. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Consider a bijective mapping f: (X, τ) → (Y, σ) defined as f(a)= u f(b) = v and f(c) = w. Then f is an IFRWG homeomorphism but not an IFhomeomorphism, since f and f^{-1} are not IF continuous mappings.

Theorem 3.5. Let f: (X, τ) → (Y, σ) be an IFRWG homeomorphism from an IFTS (X, τ) into an IFTS (Y, σ). Then f is an IFhomeomorphism if (X, τ) and (Y, σ) are $IF_{rw}T_{1/2}$ spaces.

Proof: Let B be an IFCS in Y. By hypothesis, $f^{-1}(B)$ is an IFRWGCS in X. Since (X, τ) is an $IF_{rw}T_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping. Also by hypothesis, f^{-1}: (Y, σ) → (X, τ) is an IFRWG continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A)$ = f (A) is an IFRWGCS in Y, by hypothesis. Since (Y, σ) is an $IF_{rw}T_{1/2}$ space, f (A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Thus f is an IF homeomorphism.

Theorem 3.6. Every IFα homeomorphism is an IFRWG homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFα homeomorphism. Then f and f^{-1} are IFα continuous mappings. This implies f and f^{-1} are IFRWG continuous mappings. Hence f is an IFRWG homeomorphism.

Example 3.7. X = {a, b, c}, Y = {u, v, w} and G_1 = <x, (0.4/a, 0.4/b, 0.5/c), (0.6/u, 0.6/v, 0.5/w)>, G_2= <y, (0.7/a, 0.8/b, 0.6/c), (0.3/u, 0.2/v, 0.2/w)>. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Consider a bijective mapping f : (X, τ) → (Y, σ) defined as f(a)= u f(b) = v and f(c) = w. Then f is an IFRWG homeomorphism but not an IFα homeomorphism, since f and f^{-1} are not IFα continuous mappings.

Theorem 3.8. Every IFG homeomorphism is an IFRWG homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFG homeomorphism. Then f and f^{-1} are IFG continuous mappings. This implies f and f^{-1} are IFRWG continuous mappings. Hence f is an IFRWG homeomorphism.

Example 3.9. X = {a, b, c}, Y = {u, v, w} and G_1 = <x, (0.2/a, 0.2/b, 0.1/c), (0.8/u, 0.7/v, 0.8/w)>, G_2= <y, (0.9/a, 0.8/b, 0.9/c), (0.1/u, 0.1/v, 0.1/w)>. Then τ = {0~, G_1,

$1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping f: $(X, \tau) \to (Y, \sigma)$ defined as f(a)= u f(b) = v and f(c) = w. Then f is an IFRWG homeomorphism but not an IFG homeomorphism, since f and f^{-1} are not IFG continuous mappings.

Theorem 3.10. Every IFαG homeomorphism is an IFRWG homeomorphism but not conversely.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFαG homeomorphism. Then f and f^{-1} are IFαG continuous mappings. This implies f and f^{-1} are IFRWG continuous mappings.

Example 3.11. X = {a, b, c}, Y = {u, v, w} and G_1 = <x, (0.5/a, 0.6/b, 0.7/c), (0.5/u, 0.4/v, 0.3/w)>, G_2 = <y, (0.6/a, 0.5/b, 0.5/c), (0.3/u, 0.5/v, 0.4/w)>. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a bijective mapping f:$(X, \tau) \to (Y, \sigma)$ defined as f(a)= u f(b) = v and f(c) = w. Then f is an IFRWG homeomorphism but not an IFαG homeomorphism, since f and f^{-1} are not IFαG continuous mappings.

Theorem 3.12. Let f: $(X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.

(a) f is an IFRWGOM,
(b) f is an IFRWGCM,
(c) f^{-1}: $(Y, \sigma) \to (X, \tau)$ is an IFRWG continuous mapping.

Proof:

(a) ⇒ (b): Let A be an IFCS in X, then A^c is an IFOS in X. By hypothesis, $f(A^c) = (f(A))^c$ is an IFRWGOS in Y. Therefore f(A) is an IFRWGCS in Y. Hence f is an IFRWGCM.

(b) ⇒ (c): Let B be an IFCS in X. Since f is an IFWGCM, $f(A) = (f^{-1})^{-1}(A)$ is an IFRWGCS in Y. Hence f^{-1} is an IFRWG continuous mapping.

(c) ⇒ (a): Let A be an IFOS in X. By hypothesis, $(f^{-1})^{-1}(A) = f(A)$ is an IFRWGOS in Y. Hence f is an IFRWGOM.

Corollary 3.13. Let f: $(X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ). If f is an IFRWG continuous mapping, then the following statements are equivalent.

(a). f is an IFRWGCM,
(b). f is an IFRWGOM,
(c). f is an IFRWG homeomorphism.

Proof: Obvious.

Theorem 3.14. The composition of two IFRWG homeomorphism need not be an IFRWG homeomorphism in general.

Proof: Let $X = \{a, b\}$, $Y = \{c, d\}$ and $Z = \{u, v\}$ and $G_1 = \langle x, (0.8/a, 0.6/b), (0.2/c, 0.4/d)\rangle$, $G_2 = \langle y, (0.5/c, 0.6/d), (0.5/u, 0.4/v)\rangle$, $G_3 = \langle z, (0.6/a, 0.5/b), (0.3/u, 0.5/v)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$, $\sigma = \{0\sim, G_2, 1\sim\}$ and $\delta = \{0\sim, G_3, 1\sim\}$ are IFTs on X, Y and Z respectively. Consider a bijective mapping $f: (X, \tau) \to (Y, \sigma)$ defined as $f(a) = c$, $f(b) = d$ and $g: (Y, \sigma) \to (Z, \delta)$ by $g(c) = u$, $g(d) = v$. Then f and f^{-1} are IFRWG continuous mappings. Also g and g^{-1} are IFRWG continuous mappings. Hence f and g are IFRWG homeomorphism. But the composition gof: $X \to Z$ is not an IFRWG homeomorphism, since gof is not an IFRWG continuous mapping.

Theorem 3.15. Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \delta)$ be two IFRWG homeomorphisms and (Y, σ) an $IF_{rw}T_{1/2}$ space. Then $g \circ f$ is an IFRWG homeomorphism.

Proof: Let A be an IFCS in Z. Since $g: (Y, \sigma) \to (Z, \delta)$ is an IFRWG continuous mapping, $g^{-1}(A)$ is an IFRWGCS in Y. Then $g^{-1}(A)$ is an IFCS in Y as (Y, σ) is an $IF_{rw}T_{1/2}$ space. Also since $f: (X, \tau) \to (Y, \sigma)$ is an IFRWG continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFRWGCS in X. Hence gof is an IFRWG continuous mapping.

Let A be an IFCS in X. Since $f^{-1}: (Y, \sigma) \to (X, \tau)$ is an IFRWG continuous mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFRWGCS in Y. Then f(A) is an IFCS in Y as (Y, σ) is an $IF_{rw}T_{1/2}$ space. Also since $g^{-1}: (Z, \delta) \to (Y, \sigma)$ is an IFRWG continuous mapping, $(g^{-1})^{-1}(f(A)) = g(f(A)) = (gof)(A)$ is an IFRWGCS in Z. Therefore $((gof)^{-1})^{-1}(A) = (gof)(A)$ is an IFRWGCS in Z. Hence $(gof)^{-1}$ is an IFRWG continuous mapping. Thus gof is an IFRWG homeomorphism.

In Figure 1 by "A $\to$ B" we mean A implies B but not conversely

4. Intuitionistic fuzzy regular weakly generalized i* homeomorphism

Definition 4.1. A bijective mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy regular weakly generalized i* homeomorphism (IFWRGi* homeomorphism in short) if f and f^{-1} are IFRWG irresolute mappings.

Theorem 4.2. Every IFRWGi* homeomorphism is an IFRWG homeomorphism but not conversely.

Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be an IFRWGi* homeomorphism. Let B be an IFCS in Y. This implies B is an IFRWGCS in Y. By hypothesis, $f^{-1}(B)$ is an IFRWGCS in

X. Hence f is an IFRWG continuous mapping. Similarly we can prove f^{-1} is an IFRWG continuous mapping. Hence f and f^{-1} are IFRWG continuous mapping. Thus f is an IFRWG homeomorphism.

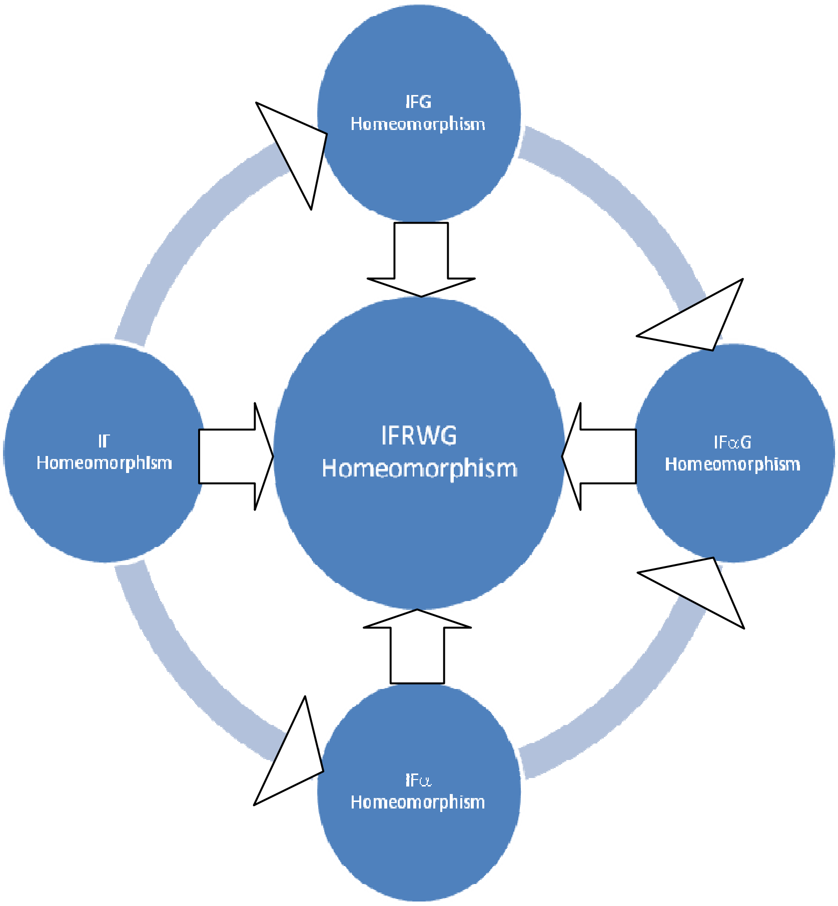

Figure 1: Relations between intuitionistic fuzzy regular weakly generalized homeomorphism and other existing intuitionistic fuzzy homeomorphisms.

Example 4.3. X = {a, b, c}, Y = {u, v, w} and G_1 = ‹x, (0.3/a, 0.5/b, 0.5/c), (0.7/u, 0.5/v, 0.5/w)>, G_2 = ‹y, (0.8/a, 0.7/b, 0.7/c), (0.2/u, 0.2/v, 0.2/w)>. Then τ = {0~, G_1, 1~} and σ = {0~, G_2 , 1~} are IFTs on X and Y respectively. Consider a bijective mapping f: (X, τ) → (Y, σ) defined as f(a)= u f(b) = v and f(c) = w. Then f is an IFWG homeomorphism. Let A = ‹y, (0.3/a, 0.5/b, 0.5/c), (0.7/u, 0.5/v, 0.5/w)> be an IFS in Y. Clearly A is an IFRWGCS in Y. But f^{-1}(A) is not an IFRWGCS in X. This implies f is not an IFWG irresolute mapping. Hence f is not an IFRWGi* homeomorphism.

Theorem 4.4. Let f: (X, τ) → (Y, σ) be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.
(a). f is an IFRWGi* homeomorphism,
(b). f is an IFRWG irresolute and IFRWGi*OM,
(c). f is an IFRWG irresolute and IFRWGi*CM.
Proof:
(a) => (b): Let f be an IFRWGi* homeomorphism. Then f and f^{-1} are IFRWG irresolute mappings. To prove that f is an IFRWGi*OM, let A be an IFRWGOS in X. Since f^{-1}: (Y, σ) → (X, τ) is an IFRWG irresolute mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFRWGOS in Y. Hence f is an IFRWGi*OM.

(b) => (a): Let f be an IFRWG irresolute and IFRWGi*OM. To prove that f^{-1}: (Y, σ) → (X, τ) is an IFRFWG irresolute mapping, let A be an IFRWGOS in X. Since f is an IFRWGi*OM, f(A) is an IFRWGOS in Y. Now $(f^{-1})^{-1}(A) = f(A)$ is an IFRWGOS in Y. Therefore f^{-1}: (Y, σ) → (X, τ) is an IFRWG irresolute mapping. Hence f is an IFRWGi* homeomorphism.

(b) => (c): Let f be an IFRWG irresolute and IFRWGi*OM. To prove that f is an IFRWG*C, let B be an IFRWGCS in X. Then B^c is an IFRWGOS in X. Since f is an IFRWG*OM, $f(B^c) = (f(B))^c$ is an IFRWGOS in Y. Therefore f(B) is an IFRWGCS in Y. Hence f is an IFRWGi*CM.

(c) => (b): Let f be an IFRWG irresolute and IFRWGi*CM. To prove that f is an IFRWGi*OM, let A be an IFRWGOS in X. Then A^c is an IFRWGCS in X. Since f is an IFRWGi*CM, $f(A^c) = (f(A))^c$ is an IFRWGCS in Y. Therefore f(A) is an IFRWGOS in Y. Hence f is an IFRWGi*OM.

Theorem 4.6. The composition of two IFRWGi* homeomorphism is an IFRWGi*homeomorphism in general.
Proof: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be any two IFRWGi* homeomorphisms. Let A be an IFRWGCS in Z. Since g: (Y, σ) → (Z, δ) is an IFRWG irresolute mapping, $g^{-1}(A)$ is an IFWGCS in Y. Also since f: (X, τ) → (Y, σ) is an IFRWG irresolute mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an IFRWGCS in X. Hence gof is an IFRWG irresolute mapping. Again, let A be an IFWGCS in X. Since f^{-1}: (Y, σ) → (X, τ) is an IFWG irresolute mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFRWGCS in Y. Also since g^{-1}: (Z, δ) → (Y, σ) is an IFRWG irresolute mapping, $(g^{-1})^{-1}(f(A)) = g(f(A)) = (gof)(A)$ is an IFRWGCS in Z. Therefore $((gof)^{-1})^{-1}(A) = (gof)(A)$ is an IFRWGCS in Z.
Hence $(gof)^{-1}$ is an IFRWG irresolute mapping. Thus gof is an IFRWGi* homeomorphism.

Remark 4.7. The family of all IFRWGi* homeomorphism from (X, τ) onto itself is denoted by IFRWGi*(X, τ).

Theorem 4.8: The set IFRWGi*(X, τ) forms a group under composition of mappings.
Proof:
(i). Operation is closed.
(ii). The composition of two IFRWGi* homeomorphism is an IFRWGi* homeomorphism in general. Hence associative axiom is satisfied.
(iii). Since the identity is IFRWGi* homeomorphism, it is an identity element of IFRWGi*(X, τ).
(iv). As the element of IFRWGi*(X, τ) are bijection f^{-1} exist in IFRWGi*(X, τ).
Hence IFRWG*(X, τ) forms a group under composition of mappings.

Theorem 4.9. If f: X → Y is IFRWGi* then it is induces an isomorphism f* from the group IFRWGi*(X, τ) onto IFRWGi*(Y, σ) given by $f^*(h)=f.h.f^{-1}$ for every h є IFRWGi*(X, τ).
Proof: By usual arguments the prrof follows.

Theorem 4.10. Let X and Y be topological spaces and let f be a bijective mapping from X onto Y. Then f is rwg open and rwg continuous if and only if f is rwg homeomorphism.
Proof: Let f be rwg open and rwg continuous. Let A be an open set in X. Then f(A) is rwg open in Y. i.e $(f^{-1})^{-1}(A) = f(A)$ is rwg open in Y. Hence f^{-1} is rwg continuous.

Conversely, assume that f be a rwg homeomorphisms and $f^{-1} = f$. Since f is bijective, g is also bijective. If A is an open set $g^{-1}(A)$ is a rwg open set for g is rwg continuous. That is f(A) is rwg open. Hence f is rwg open.

Theorem 4.11. Let X and Y be topological spaces and let f be a bijective mapping from X onto Y. Then f is rwg homeomorphism if and only if f is rwg closed and rwg continuous.
Proof: Assume that f is rwg homeomorphisms, let A be a closed set in X. then X-A is open and since $f = g^{-1}$ is rwg continuous, $g^{-1}(X\text{-}A)$ is rwg pen. That is $g^{-1}(X\text{-}A) = Y\text{-}g^{-1}(F)$ is rwg open. Thus $g^{-1}(F)$ is rwg closed, that is f(F) is rwg closed. Hence f is rwg closed map.

Conversely assume that f is rwg closed and rwg continuous. Let B be an open set. Then X-B is closed. Since f is closed f(X-B) is rwg closed. That is $g^{-1}(X\text{-}B) = Y - g^{-1}$ is rwg closed. That is $g^{-1}(X\text{-}B) = Y - g^{-1}$ is rwg closed, implies $g^{-1}(G)$ is rwg open. Thus inverse image under g of every open set is rwg open. That is $g = f^{-1}$ is rwg continuous. Thus f is rwg homeomorphisms.

5. Conclusion

In this paper, we introduce the notion of intuitionistic fuzzy regular weakly generalized homeomorphism and intuitionistic fuzzy regular weakly generalized *

homeomorphism in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy regular weakly generalized homeomorphism and establish the relationships with other classes of early defined forms of intuitionistic fuzzy homeomorphisms.

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